3rd URSI AT-AP-RASC, Gran Canaria, 29 May – 3 June 2022

Phase-Accurate Analytical Transmission Line Model and for a 1–50 GHz Millimeter-Wave Textile-Based Wearable Goubau Single Wire Transmission Line (SWTL)

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Abstract

Goubau-Sommerfeld Single Wire Transmission Lines (SWTLs) are a feasible approach for low-loss high-speed transmission. However, the long length of an SWTL makes full-wave electromagnetic simulations too computationally intensive. In this paper, a transmission line model is proposed for a state-of-the-art textile-based wearable broadband millimeter-wave SWTL, which represents the first broadband closed-form analytical model for a wearable surface wave transmission line. The inductance and capacitance are calculated per unit length using closed-form formulas and used to construct a simple SWTL equivalent circuit based on the lossy transmission line model. The analytically calculated phase exhibits under 7% error compared to vector network analyzer (VNA) measurements from 2 to 50 GHz. The VNA measurements are then used to fit the loss terms in the model leading to a good agreement between calculations, full-wave simulations, and experiments. It is anticipated that the developed model and low-cost mmWave SWTL will empower future integration of low-loss SWTLs in different applications without the need for complex numerical simulations.

1 Introduction

Low-loss Radio Frequency (RF) transmission through to the millimeter-wave (mmWave) bands represents a significant challenge in body area networks (BANs) and wearable applications [1]. In many healthcare, fitness tracking, and telemetry applications, low-loss high-data rate links are needed between wearables situated on the same user [2, 3], where RF power could be harvested from an off-body source using textile rectennas [4], and used to power many co-located wearables [2]. To this end, several "on-body" antennas have been realized with extensive characterization of the on-body wireless channel. However, both spherical spreading and human body absorption significantly hinder a high channel gain betwen on-body antennas [5].

To address the challenge of low-loss on-body communication, there has been a surge in the recent research in mechanisms other than conventional on-body antennas for wireless or quasi-wireless transmission over the human body with low insertion losses. 2.4 GHz Spoof Surface Plasmons (SSPs) [2], 13.56 MHz magneto-inductive waveguides using near-field communication (NFC) [3], microstrip patch-fed stripline [6], and textile integrated waveguides [7], are among the various implementations which aim to reduce the on-body loss for high-speed RF links. Nevertheless, a truly broadband solution was yet to be demonstrated for wearable on-body links. Moreover, none of the aforementioned quasi-wireless solutions, not based on TEM or Quasi-TEM transmission lines, were demonstrated covering the 5G mmWave bands as well as approaching the 60 GHz Wireless BAN band.

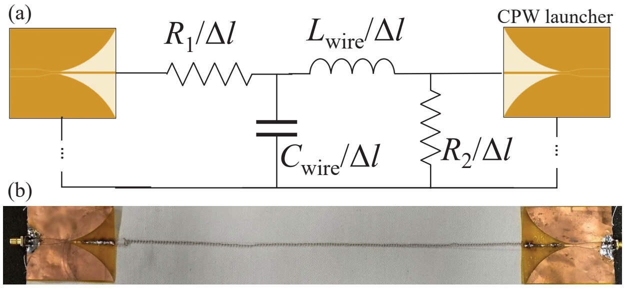

Figure 1. The proposed shielded on-body SWTL: (a) equivalent circuit model per unit length, showing the CPW launchers; (b) photograph of the line prototype.

This paper follows on the experimental characterization of a broadband Coplanar Waveguide (CPW)-fed Single Wire Transmission Line (SWTL) implemented using e-textile materials and demonstrated on-body [5]. Herein, a simple closed-form transmission line model is presented for the first time for a shielded Goubau line. The SWTL represents the lowest loss reported mechanism for on-body signalling. The closed-form model proposed in this work closely matches the measured phase-response and group delay of the SWTL, as well as empirically models the measured insertion losses, agreeing with both full-wave simulations and practical vector network analysis.

2 Transmission Line Model

Considering the low complexity of the proposed SWTL over similar SSPs which include a number of capacitive and inductive elements [8], the simple transmission line model shown in Fig. 1(a) can be used to model the transmission line. Given the 50 GHz maximum frequency of operation of the SWTL, as in [5], and to maintain a relatively low model complexity, the SWTL is modelled per mm ($\Delta l = 1$ mm); a finer model could be developed using the same technique for SWTLs operating at higher frequencies.

In practice, the SWTL is fed using a 50 Ω coaxial connector with a CPW to surface wave launcher implemented on a flexible substrate, as shown in Fig. 1(b). The line is implemented using a conductive thread formed of a Litz wire with approximately 20 µm radius r. To shield the line from the human body losses, a conductive textile metal plane backs the SWTL, separated by the approximately 0.5 mm-thick polyester cotton fabric substrate with ε_r=1.7; the dimensions of the launcher are detailed in [5].

First, the inductance of the SWTL is calculated using the simple wire inductance formula, where L_{wire} is given by

$$L_{\text{wire}} = 2l\left\{\ln\left[\left(\frac{2\Delta l}{r}\right)\left(1+\sqrt{1+\left(\frac{r}{2\Delta l}\right)^2}\right)\right]- \sqrt{1+\left(\frac{r}{2\Delta l}\right)^2}+\frac{\mu}{4}+\left(\frac{r}{2\Delta l}\right)\right\} \quad (1)$$

where μ is the permeability of the propagation medium and is approximately equal to that of free space; Δl=1 mm [9]. While an unshielded SWTL exists far away its launcher's ground, and does not include a current return path [10], its capacitance can still be treated as a single metal plate for the purpose of extracting an analytical circuit model [8]. Previously, an analytical model was proposed for a planar SSP slow-wave transmission line, with no shielding, where it was assumed that the capacitance could be calculated based on the electrostatic capacitance of a single-plate which reaches infinity [8], which is given by

$$C = 8\varepsilon r. \quad (2)$$

On the other hand, the proposed SWTL includes a shielding plane. Shielded surface wave transmission lines, using a conductive "perfect electrical conductor (PEC)" backing, have previously been investigated using SSPs at 2.4 GHz, demonstrating that the surface wave mode could still be maintained along the line [2]. In the proposed model, the unconnected shielding PEC plane is assumed to act as a ground plane, where the capacitance between the SWTL and the shielding plane can be calculated using the wire over a PEC plane formula given by

$$C_{\text{wire}} = \frac{2\pi\Delta l\varepsilon_r\varepsilon_0}{\cosh^{-1}(h/r)}, \quad (3)$$

where h is the 0.5 mm substrate height [11].

Using (1) and (3), the inductance and capacitance per unit length were calculated as L_{wire}=0.775 nH/mm and C_{wire}=17.5 fF. Given that for a loss-less line

$$Z_0 = \sqrt{\frac{L}{C}}, \quad (4)$$

the proposed SWTL maintains Z_0=210.4 Ω, in line with the typical $200 < Z_0 < 350$ Ω range discussed by Goubau *et al.* for an SWTL [10]. Therefore, the high Z_0 of the SWTL contributes to reducing the insertion losses over a standard Z_0=50 Ω transmission line [10], where the attenuation per unit length is normally given by

$$\alpha[\text{dB}/\Delta l] = 8.686 \times \frac{R/\Delta l}{2Z_0}. \quad (5)$$

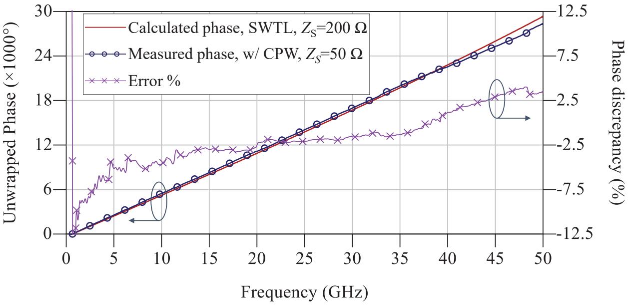

Figure 2. The analytical and experimental unwrapped phase of the SWTL, and calculated discrepancy between them.

In the next section, the calculated loss-less model is compared to VNA measurements, and a lossy model is fitted based on the s-parameters of the prototype in Fig. 1(b).

3 Results and Discussion

3.1 Phase Delay and the Loss-Les Model

Using the developed loss-less LC model of the SWTL, the characteristic impedance, phase delay, and group delay can be calculated. Based on the calculated Z_0 of 210.4 Ω, a source impedance of Z_S=200 Ω is assumed in the analytical circuit model simulations. As a result, the proposed circuit model only deals with the SWTL as opposed to the mode converters, which could be realized using coplanar [5], microstrip [8], or horn-like [10] launchers to cover different frequency bands and satisfy the different size requirements. The response of the analytical circuit model was calculated in Keysight ADS up to 50 GHz using the Z_S=200 Ω port.

Fig. 2 shows the calculated unwrapped phase delay along a 420 elements (i.e. 42 cm)-long SWTL model. The phase response of an SWTL of a similar length (excluding the 50 Ω coax to CPW feeds) was measured experimentally using a SOLT-calibrated Agilent E8361A PNA up to 50 GHz. The measured unwrapped phase response is shown alongside the closed-form model, in Fig. 2.

The discrepancy percentage between the analytical and experimental phase delay is shown on the secondary axis of Fig. 2. Below 2 GHz, a high discrepancy is observed, which could be attributed to the imperfect surface wave excitation influencing the measured phase at lower frequencies. However, across the entire bandwidth of the SWTL and up to

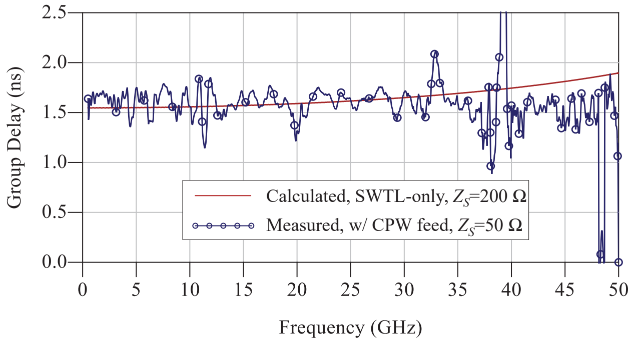

Figure 3. The analytical and experimental group delay of the shielded SWTL.

50 GHz, the average error is under 3%. Therefore, the proposed model which uses the capacitance between the wire and its shielding plane is validated for phase-accurate modelling of the shielded SWTL.

A key advantage of the SWTL over wireless on-body transmission is the stable phase response resulting in a consistent group delay across its entire bandwidth. Such stable group delay is key for high datarate communication over a broad bandwidth. Fig. 3 shows the calculated group delay of the circuit model alongside the VNA-measured group delay of the SWTL, exhibiting a very good agreement. Therefore, the proposed circuit model could be used to model the SWTL alongside transceivers and other components to reliably estimate the propagation delay of the signal.

3.2 The Lossy SWTL Model

The next step in modelling the SWTL is a broadband lossy model. Given the high surface roughness of an e-textile conductor [12], calculating the frequency-dependent series resistance of the non-uniform wire using a closed-form formula represents a significant challenge beyond the scope of this work. R_1 and R_2 in Fig. 1(a) are represented as frequency-dependent resistors; the measured S_{21} in Fig. 4 to fit the frequency-dependent loss terms given by

$$R_1/\Delta l = a\sqrt{f} \tag{6}$$

$$R_2/\Delta l = b/f \tag{7}$$

where f is the frequency, and a and b are the curve fitting parameters.

Fig. 4 shows the calculated S_{21} for a the SWTL with a==3.16×10^{-6} and b=435×10^{12}. The SWTL was also simulated using full-wave electromagnetic simulation (finite difference time domain) in CST Microwave Studio for further validation of the fitted model. The CST model uses the conductivity of copper (σ=5.8×10^{7} S/m), as well as includes the dielectric loss due to the tanδ=0.017 of the fabric substrate. The full-wave model also includes the 50 Ω CPW to surface wave launcher and is excited using a waveport. The simulated response is shown alongside the analytical and experimental results in Fig. 4, where it can be seen that the calculated response of the equivalent circuit model closely matches that from full-wave simulations.

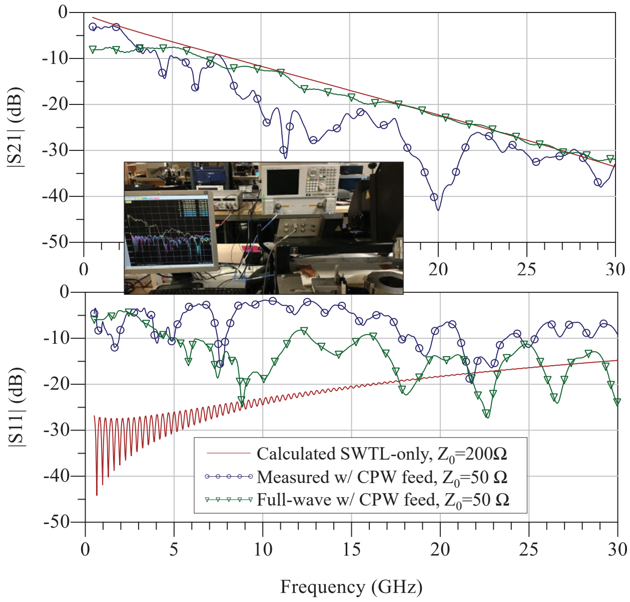

Figure 4. The analytically calculated, full-wave, simulated, and experimentally measured s-parameters of the shielded SWTL; the inset shows a photograph of the measurement setup.

On the other hand, the S_{11} of the lossy SWTL in Fig. 4, excluding the launchers and excited using a source of Z_S=200 Ω, is different from the simulated and measured response. This is due to the non-ideal excitation by the CPW launcher in [5], which can be further improved by redesigning the launcher as previously demonstrated in the THz spectrum [13]. While the simulated response exhibits a reflection coefficient under -10 dB from 5 to 50 GHz, the SWTL can be well matched with over 20 dB return loss to the source, should a mode converter be realized with an improved impedance match to the source.

3.3 Attenuation and Microstrip Comparison

The measured attenuation in dB/mm is shown alongside the calculated attenuation of a single unit cell of the proposed closed-form lossy model in Fig. 5. For benchmarking, the attenuation of a printed microstrip line implemented on the same substrate is included in Fig. 5 [12]. 3% smoothing has been applied to the measured response of the SWTL to improve the clarity of the graph, as the observed ripple is mostly due to the impedance mismatch at the input of the CPW launcher [13].

Comparing the microstrip and SWTL responses in Fig. 5, it can be seen that the attenuation of the microstrip line (in dB/mm) at 50 GHz is around 4.5× higher than that of the SWTL, with the ratio of microstrip to SWTL attenuation ranging between 2–4:1 across the full bandwidth of both lines. Recalling the analytically calculated Z_0=210.4 of the

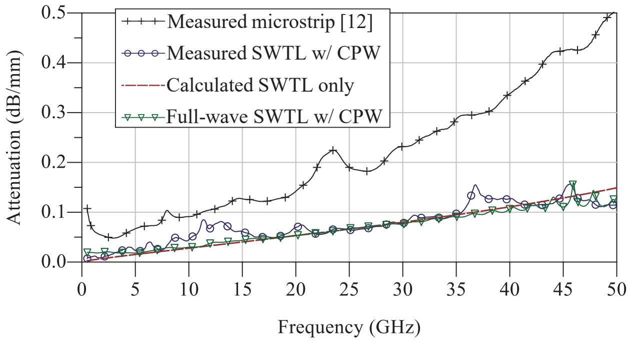

Figure 5. The analytical, measured, and simulated attenuation per mm of the SWTL, shown alongside a reference microstrip line on the same substrate [12].

SWTL vs. a microstrip $Z_0 \approx 50\ \Omega$, it can be seen that the experimentally measured attenuation follows the trend in (5), demonstrating that the lower attenuation could be attributed in part to the high Z_0 of the SWTL, reducing the conductive losses in the SWTL. The SWTL also utilizes a higher conductivity Litz wire than the printed silver traces of the microstrip in [12], further contributing to the lower attenuation.

4 Conclusion

In this paper, a simple analytical model is proposed for a shielded Goubau-Sommerfeld SWTL implemented using low-cost e-textile materials for wearable applications, which represents the first broadband equivalent circuit model of a wearable surface wave transmission line. The proposed model is phase-accurate and uses closed-form inductance and capacitance terms for a wire over a ground plane, exhibiting under 10% phase error compared to VNA measurements up to 50 GHz. A lossy model is then extracted based on the measured s-parameters, showing good agreement with full-wave simulations of the SWTL.

It is concluded that the simple transmission line model is suitable for rapid analysis of SWTLs based on closed-form equations. The simple and all-analytical phase-accurate loss-less model is expected to enable SWTLs to be realized for a range of broadband long-range and low-loss RF communication applications. In future work, a full analytical model could be developed to incorporate the coax to SWTL launcher for improved design and characterization of future SWTLs.

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