A Quantum Graph Approach to Meta-Material Design via Scattering Matrix Construction and Band Engineering

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Over the past 20 years meta-materials have gained a large amount of attention due to their potential for possessing highly nontrivial and exotic properties, such as cloaking or perfect lensing [1]. There has been a great push to create simple models that accurately describe the required material composition that would yield such properties. For example the use of a transmission line model to give the property of an effective negative index of refraction [2]. Following suit we introduce a novel quantum graph approach to meta-material design.

Since their introduction in 1997 [2], quantum graphs have become a most valuable model for studying quantum and wave effects. Their versatility and ease of constructions as well as the finite dimensionality and the exactness of 'semiclassical' expressions, make them ideal toy models to test ideas and research hypothesis', see [3] for an overview. The theory has been applied to many physical domains beyond quantum chaos, namely modelling vibrations of coupled plates [5] as well as being used to model power transport though through networks of cables [6].

As of the time of presenting this work, quantum graph theory has never been used as a design tool for meta-material construction. To address this, a toy model for periodic meta-materials constructed from resonant elements is introduced in the language of quantum graph theory. The meta-materials are theoretically designed and explored through scattering matrix construction and investigated through the engineering of resulting band diagrams. To showcase the engineered material properties, multiple periodic meta-materials are married together, and the required boundary conditions are solved across them. To exemplify the versatility of the model the properties of wave collimation, positive and negative refraction and total internal reflection are presented. The proposed quantum graph technique is very flexible and can be easily and quickly adjusted, making it an ideal design tool for creating meta-materials with exotic band diagram properties or testing promising multi-layer set ups.

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