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Phase Noise Measurements May Not Be What They Seem

— Invited —

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Abstract

Measuring an oscillator with different instruments may give the false impression of high accuracy because some regions of the spectrum (typically white and flicker PM, above 140 dBc/Hz in the case of RF oscillators) are highly reproductible, with differences of a small fraction of a dB across instruments. Serious problems arise in the low-noise regions of the spectrum, chiefly white PM noise below –150 dBc/Hz in RF oscillators, where most instruments may fail. Some artifacts are the tip of the tip of the iceberg, which may hide gross errors and blatant nonsenses like *negative* phase noise. To our understanding, most of the problem arise from the unclear definition of the measurand, and from the lack of analysis of the B-Type (systematic) uncertainty.

Introduction

Virtually all phase noise analyzers exploit the correlation scheme, where two equal channels measure simultaneously the phase noise $S_\varphi(f)$ of the oscillator under test in order to average out the background noise present in the cross spectrum $S_{yx}(f)$. The two channels may have separate references for improved rejection of the background.

For historical reason, the result is displayed as

$$\mathcal{L}(f) = (1/2)\, S_\varphi(f) \quad \text{dBc/Hz} \qquad (1)$$

The traditional analyzers use a saturated double-balanced mixer as the phase detector, while newer analyzers detect the phase after direct digitization of the input RF signal. Aliasing may be used to extend the frequency range beyond the first Nyquist zone of the ADCs, and down-conversion may extend the range to microwaves.

Inside the Instruments

Let us denote with a, b and c the random phase of the two references and the DUT, respectively, and with d any unwanted or unpredicted random phase hitting on the two channels with sign $\varsigma = \varsigma_x \varsigma_y = \pm 1$. As the problem is stated, a, b, c and d are statistically independent. All signals are truncated over a time T, the upper case denotes the Fourier transform of the lowercase counterpart, and time and frequency are implied

$$x = c - a + \varsigma_x d \;\leftrightarrow\; X = C - A + \varsigma_x D \qquad (2)$$

$$y = c - b + \varsigma_y d \;\leftrightarrow\; Y = C - B + \varsigma_y D \qquad (3)$$

The single-sided cross spectrum is

$$S_{yx} = \frac{2}{T}\,\mathbb{E}\{YX^*\} \qquad f > 0\,, \qquad (4)$$

where $\mathbb{E}\{\,\}$ is the mathematical expectation and $*$ stands for complex conjugate. While with $d = 0$ we get $S_{yx} = S_\varphi$, in the real world (4) gives $S_{yx} = S_\varphi + \varsigma S_d$.

The theory suggests the use of the estimator

$$\widehat{S_\varphi} = \langle \Re\{S_{yx}(f)\} \rangle_m\,, \qquad (5)$$

where $\langle \ldots \rangle_m$ is the average on m realizations, because (i) it discards the unnecessary background noise in $\Im\{S_{yx}(s)\}$ and (ii) is unbiased. Sadly, most commercial instruments use the estimator

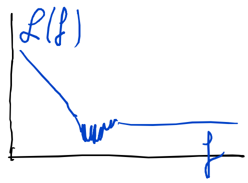
$$\widehat{S_\varphi} = \left| \langle S_{yx}(f) \rangle_m \right|\,. \qquad (6)$$

The $|\ldots|$ operator takes in the unnecessary noise from $\Im\{S_{yx}(s)\}$, which results in 4-fold increased measurement time for the same rejection of the background, and makes errors and artifacts more difficult to identify.

Where Things Can Go Wrong

A common belief is that the instrument can only *add* noise. A related belief is that the *lowest* noise observed in a series of measures is the trusted one. Both beliefs are wrong because the cross spectrum $S_{yx}(f)$ is a form of covariance, thus the contribution of background and artifacts ςDD^* may be negative (under estimation of noise). It is obvious from (4) that even $S_{yx}(f)$ may be negative, which is a blatant nonsense.

We have evidence that V-shaped artifacts like the figure aside are the signature that $\langle \Re\{S_{yx}(f)\} \rangle_m < 0$ in a large interval of f. Other instruments use $\langle |\langle S_{yx}(f) \rangle_m| \rangle_n$, where the outer average is used to smooth the spectrum. The interplay of the two averages is even more



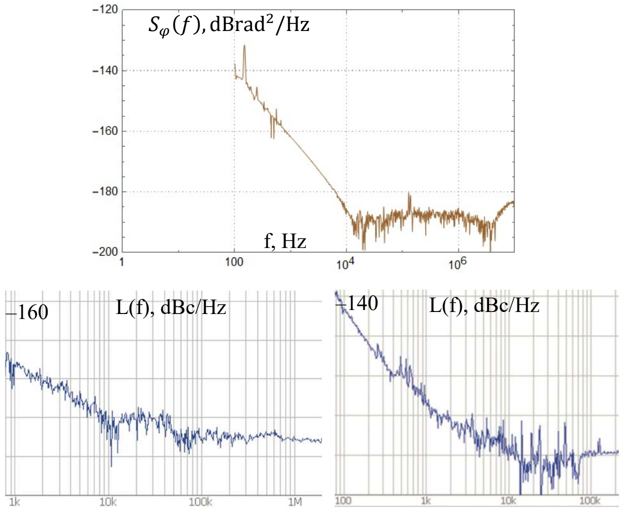


Figure 1. Examples of artifacts found in the spectrum of low-noise oscillators. The pictures are intentionally anonymized to prevent the reader from ascribing them to any brand or author.

difficult to understand because the inner average may still not be converging. More example are shown in Figure 1.

Another weird and grossly wrong outcome, yet well explained analytically, is a lower phase noise, observed systematically on certain oscillators when an attenuator is inserted between the oscillator and the noise analyzer.

A Lesson Learned

At conceptual level, errors and misunderstandings arise from the following reasons

- ❖ The measurand is not defined clearly. For example,
  - do we measure each bin of $S_\varphi(f)$ or the coefficients of the polynomial approximation?
  - Are the spurs detected and removed?
  - Is impedance matching specified?
- ❖ The uncertainty is not described appropriately in terms of *Type A* and *Type B* evaluation, and the *Zero uncertainty*, as in the VIM (International Vocabulary of Metrology), available from the BIPM.

At more technical level, we have identified the following problems.

1. Deterministic spurs (the one and only minor problem, easy identified by experienced experimentalists)
2. Signal processing inside the instrument is not sufficiently documented.
3. The estimator $\left|\langle S_{yx}(f)\rangle_m\right|$ or $\langle\left|\langle S_{yx}(f)\rangle_m\right|\rangle_n$ is used, instead of $\langle\Re\{S_{yx}(f)\}\rangle_m$
4. AM noise may result in measurement bias with $+/-$ sign. This seems to have a serious impact only on the instruments based on the saturated mixer.
5. Thermal noise in the dark port of the input power splitter. This results always in a negative bias, decently easy to correct, but not accounted for in any commercial instrument we know.
6. Impedance matching. When the oscillator under test has a narrow filter at the output, impedance matching is quite different at the carrier frequency and in the noise sidebands,
7. Crosstalk is a plague affecting virtually all RF and microwave circuits. In the case of phase noise analyzers, crosstalk between the two channels may result in measurement bias of unpredictable sign and amount.
8. Having more than one oscillator at the same frequency on the desk, oscillator under test and references, may result in loose lock or false lock. In turn, we may experience artificially-reduced phase noise or erratic behavior.

A further, serious problem is that the phase noise analyzers are often used *beyond the specifications*, under the wrong belief that the background noise of the instrument can only be a positive bias. The *typical background noise* often published by the manufacturers fuels the wrong belief mentioned, and encourages the users to trust the phase noise spectrum even when it is below the specs. The correct interpretation is that the spectra, or the portions of, which fall below the minimum detectable noise must be discarded.

Most people know about the minimum detectable quantity from general experience in everyday life. Below a critical speed of the order of 1 m/s (3.6 km/h, or 2.3 mph), the speedometer of an elderly car cannot say if the car is moving or is at the rest. And a cooking scale cannot say whether a post stamp has a mass or not. Oddly, such awareness is lost when it comes to phase noise.

Phase noise analyzers may be very accurate in the regions where the noise is rather high. In different occasions we notice that, measuring an HF/VHF OCXO with different phase noise analyzers, the spectra overlap perfectly in the low-frequency region, where $S_\varphi \gtrsim -140\ \mathrm{dBrad^2/Hz}$, with discrepancy within a fraction of the dB even if the calibration is void. With this experience, it is easy to over-trust the instrument.

Digital instruments have higher background than the saturated-mixer counterpart. This goes with all the problem related to higher rejection of the background, first of which the longer averaging time. However, digital instruments offer new options with the oscillator under test and the references all at different frequencies, provided the input channels have separate NCOs.

We study these new options to identify some of the problems mentioned, and possibly to correct the result. As a fringe outcome, we have observed that the frequency stability of some high-end OCXOs improves when the frequency is intentionally set off the nominal value.

Where Knowledge Comes From

The experience we report comes from our obsession about phase noise, frequency stability and precision oscillators for more than 30 years. We had stimulating discussions with numerous colleagues we don't have room to mention, and three workshops specific on the cross spectrum [1-3].

The following references are more about a suggestion about where to learn than about giving credit in the usual academic style. The reader is encouraged to follow the references cited in these works.

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Acknowledgments

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Appendix: A Free Bonus

The ***Enrico's Chart for Phase Noise and Two-Sample Variances*** is a reference card collecting the most useful concepts, formulas and plots in a single A4/A-size sheet, intended to be a staple on the desk of whoever works with these topics. It is available from Zenodo under Creative Commons 4.0 license CC-BY-NC-ND, downloadable at the URL https://doi.org/10.5281/zenodo.4399218, or flashing the QR code aside. The reader may also ne interested in the forthcoming article *The Companion of the Enrico's Chart for Phase Noise and Two-Sample Variances*, soon available on https://arXiv.org.