Two Co-Planar Resistive Half-Planes: a Uniform Asymptotic Solution for the Plane Wave Diffraction by the Discontinuity

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Abstract

The Uniform Asymptotic Physical Optics approach is here applied to study the plane wave diffraction by the discontinuity between two co-planar resistive half-sheets. These last possess different surface resistivity, thus causing discontinuities of reflected and transmitted fields. The related coefficients for parallel and perpendicular polarizations are determined by means of the resistive boundary conditions in correspondence of the screens and then applied to the approach. The resulting diffracted field can be used under the rules of the Uniform Geometrical Theory of Diffraction by taking properly into account that the Physical Optics approximation is implemented for the equivalent sources.

1 Introduction

The core of this work refers to the plane wave diffraction by the discontinuity between two co-planar half-sheets under the validity of the resistive boundary conditions. The Uniform Asymptotic Physical Optics (UAPO) approach is here adopted to solve the above diffraction problem in the case of an arbitrarily polarized electric field at skew incidence with respect to the rectilinear discontinuity. Such an approach is suitable when considering junctions, since it takes advantage from the linearity of the scattering integral at the beginning of the analytical method. Reference systems in Fig. 1 are used.

The plane wave diffraction by a junction of equal resistive sheets having a non-planar configuration has been studied by means of the UAPO approach in [1], where a two-dimensional propagation model has been considered in the case of incidence normal to the junction and incident electric field parallel to the discontinuity. The use of the Boundary Element Method (BEM) has assessed the effectiveness of the proposed approach.

The UAPO approach has been recently applied in [2] to isolated resistive half-planes when studying arbitrarily polarized electric fields at skew incidence with respect to the edge. The resistive boundary conditions [3] have been accounted for evaluating the reflection and transmission coefficients of the electric field components that are parallel and perpendicular to the ordinary plane of incidence. As well known, such conditions characterize the behavior of electric and magnetic fields in correspondence of a thin lossy dielectric sheet. The related surface resistivity R_e contains information about its geometric and electric parameters (e.g., the sheet is perfectly conducting if $R_e = 0$). Obviously, further methods have been offered in literature to solve diffraction problems involving resistive half-planes (see [4]-[8] for a non-exhaustive list of references).

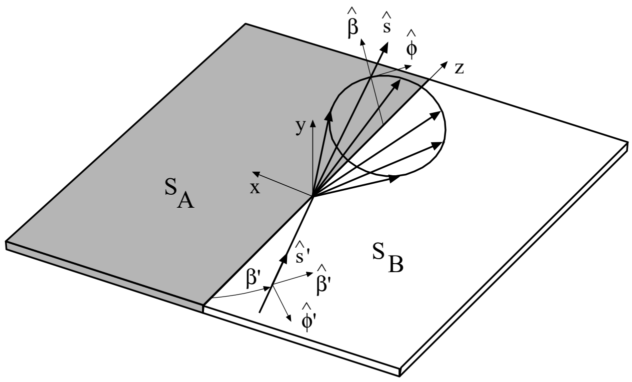

Figure 1. Geometry relevant to the plane wave diffraction by the junction between co-planar resistive half-planes.

2 PO Approximation for Radiating Sources

An incident electric field

$$\underline{E}^i = \left[E_\perp^i \, \hat{u}_\perp + E_\parallel^i \left(\hat{u}_\perp \times \hat{s}' \right) \right] \exp\left(-jk_0 \hat{s}' \cdot \underline{r} \right) \qquad (1)$$

propagates in the free space and interacts with a resistive sheet S. Parallel ($\parallel$) and perpendicular ($\perp$) field components are used in (1). Moreover, k_0 is the free-space propagation constant, $\underline{r}$ represents the position vector of the observation point P, the unit vector $\hat{s}'$ fixes the incidence direction and $\hat{u}_\perp = (\hat{s}' \times \hat{n}) / |\hat{s}' \times \hat{n}|$ if $\hat{n}$ is the unit vector normal to S.

The resistive boundary conditions imply that electric and magnetic fields satisfy the following relations on S [3]:

$$\hat{n}\times\left[\underline{E}^{+}-\underline{E}^{-}\right]\Big|_{S}=0 \tag{2}$$

$$\hat{n}\times\left[\hat{n}\times\underline{E}\right]\Big|_{S}=-R_e\,\hat{n}\times\left[\underline{H}^{+}-\underline{H}^{-}\right]\Big|_{S} \tag{3}$$

The superscripts + and – identify the fields on the upper and lower parts of S and the surface resistance is:

$$R_e=-\frac{j\zeta_0}{k_0 d(\varepsilon_r-1)} \tag{4}$$

where ζ_0 is the free-space impedance, d and ε_r are the thickness and the relative permittivity of the sheet, respectively.

Accounting for (1)-(3), the electric ($\underline{J}_s$) and magnetic ($\underline{M}_s$) PO surface currents to be used in the radiation integral of the UAPO approach can be so expressed:

$$\begin{aligned}\underline{J}_s&=\hat{n}\times\left[\underline{H}^{+}-\underline{H}^{-}\right]\Big|_{S}=\\&\frac{1}{\zeta_0}\Big[\left(1-R_{\perp}-T_{\perp}\right)E_{\perp}^{i}\cos\theta^{i}\,\hat{u}_{\perp}+\\&\left(1+R_{\|}-T_{\|}\right)E_{\|}^{i}\left(\hat{n}\times\hat{u}_{\perp}\right)\Big]\exp\left(-jk_0\hat{s}'\cdot\underline{r}'\right)\end{aligned} \tag{5}$$

$$\underline{M}_s=\left[\underline{E}^{+}-\underline{E}^{-}\right]\Big|_{S}\times\hat{n}=0 \tag{6}$$

where $\underline{r}'$ is the position vector on S and θ^{i} is the standard angle of incidence. The reflection (R) and transmission (T) coefficients are determined by means of the boundary conditions (2) and (3):

$$R_{\|}=\frac{\cos\theta^{i}}{\gamma+\cos\theta^{i}} \tag{7}$$

$$T_{\|}=\frac{\gamma}{\gamma+\cos\theta^{i}} \tag{8}$$

$$R_{\perp}=-\frac{1}{1+\gamma\cos\theta^{i}} \tag{9}$$

$$T_{\perp}=\frac{\gamma\cos\theta^{i}}{1+\gamma\cos\theta^{i}} \tag{10}$$

with $\gamma=2R_e/\zeta_0$.

3 UAPO Diffraction Matrix

Figure 1 shows the adopted reference systems ($\hat{s}'=-\sin\beta'\cos\phi'\,\hat{x}-\sin\beta'\sin\phi'\,\hat{y}+\cos\beta'\,\hat{z}$ from this point on). Let us denote by the uppercase letter A (B) the resistive half-plane for $x>0$ ($x<0$). The values of R_e related to the joining half-planes cause discontinuities of reflected and transmitted fields.

When considering a junction, the integral formulation permits to separate the contributions to the scattered electric field $\underline{E}^{S}$, i.e.,

$$\begin{aligned}\underline{E}^{S}=&-jk_0\iint_{S_A}\left[\zeta_0\left(\underline{\underline{I}}-\hat{u}_R\hat{u}_R\right)\underline{J}_{s_A}\right]g\left(\underline{r},\underline{r}'\right)\mathrm{d}S+\\&-jk_0\iint_{S_B}\left[\zeta_0\left(\underline{\underline{I}}-\hat{u}_R\hat{u}_R\right)\underline{J}_{s_B}\right]g\left(\underline{r},\underline{r}'\right)\mathrm{d}S\end{aligned} \tag{11}$$

with $g\left(\underline{r},\underline{r}'\right)=\exp\left(-jk_0\left|\underline{r}-\underline{r}'\right|\right)/4\pi\left|\underline{r}-\underline{r}'\right|$. The symbol $\underline{\underline{I}}$ identifies the 3-D identity matrix and $\hat{u}_R$ is the unit vector from the source point P' to P. According to (11) and accounting for the UAPO approach as applied in [2], the field diffracted by the junction writes as:

$$\begin{aligned}\begin{pmatrix}E_{\beta}^{d}\\E_{\phi}^{d}\end{pmatrix}&=\left[\underline{\underline{A}}\,I_A^{d}+\underline{\underline{B}}\,I_B^{d}\right]\begin{pmatrix}E_{\beta'}^{i}\\E_{\phi'}^{i}\end{pmatrix}\\&=\underline{\underline{D}}\begin{pmatrix}E_{\beta'}^{i}\\E_{\phi'}^{i}\end{pmatrix}\frac{\exp(-jk_0 s)}{\sqrt{s}}\end{aligned} \tag{12}$$

where s is the distance from the diffraction point to P. The functions depend on the incidence and diffraction directions, and contain the transition function $f_t(\cdot)$ of the Uniform Geometrical Theory of Diffraction (UTD) [9], i.e.,

$$I_{A,B}^{d}=\frac{\exp(-j\pi/4)}{2\sqrt{2\pi k_0}}\,\frac{f_t\left(2k_0 s\sin^2\beta'\cos^2\left(\dfrac{\phi_{A,B}\pm\phi'_{A,B}}{2}\right)\right)}{\sin^2\beta'\left[\cos\phi_{A,B}+\cos\phi'_{A,B}\right]}\cdot\frac{\exp(-jk_0 s)}{\sqrt{s}} \tag{13}$$

where $\phi'_A=\phi'$, $\phi'_B=\pi-\phi'$, $\phi_A=\phi$ and $\phi_B=\pi-\phi$. The sign + (–) is used if $0<\phi_{A,B}<\pi$ ($\pi<\phi_{A,B}<2\pi$).

Matrices $\underline{\underline{A}}$ and $\underline{\underline{B}}$ account for the transformation matrices between the involved local reference systems and the expressions of $\underline{J}_{s_A}$ and $\underline{J}_{s_B}$:

$$\underline{\underline{A}} = \underline{\underline{A}}_1 \underline{\underline{A}}_2 \underline{\underline{A}}_3 \underline{\underline{A}}_4 \underline{\underline{A}}_5 \tag{14}$$

with

$$\underline{\underline{A}}_1 = \begin{pmatrix} \cos\beta'\cos\phi & \cos\beta'\sin\phi & -\sin\beta' \\ -\sin\phi & \cos\phi & 0 \end{pmatrix} \tag{15}$$

$$\underline{\underline{A}}_2 = \begin{pmatrix} 1-\sin^2\beta'\cos^2\phi & -\sin\beta'\cos\beta'\cos\phi \\ -\sin^2\beta'\sin\phi\cos\phi & -\sin\beta'\cos\beta'\sin\phi \\ -\sin\beta'\cos\beta'\cos\phi & \sin^2\beta' \end{pmatrix} \tag{16}$$

$$\underline{\underline{A}}_3 = \frac{1}{F(\beta',\phi')} \begin{pmatrix} -\cos\beta' & -\sin\beta'\cos\phi' \\ -\sin\beta'\cos\phi' & \cos\beta' \end{pmatrix} \tag{17}$$

$$\underline{\underline{A}}_4 = \begin{pmatrix} 0 & (1-R_{\perp_A}-T_{\perp_A})\sin\beta'\sin\phi' \\ 1+R_{\parallel_A}-T_{\parallel_A} & 0 \end{pmatrix} \tag{18}$$

$$\underline{\underline{A}}_5 = \frac{1}{F(\beta',\phi')} \begin{pmatrix} \cos\beta'\sin\phi' & \cos\phi' \\ -\cos\phi' & \cos\beta'\sin\phi' \end{pmatrix} \tag{19}$$

wherein $F(\beta',\phi') = \sqrt{1-\sin^2\beta'\sin^2\phi'}$ and

$$\underline{\underline{B}} = \underline{\underline{A}}_1 \underline{\underline{B}}_2 \underline{\underline{B}}_3 \underline{\underline{B}}_4 \underline{\underline{A}}_5 \tag{20}$$

with

$$\underline{\underline{B}}_2 = \begin{pmatrix} 1-\sin^2\beta'\cos^2\phi & -\sin^2\beta'\sin\phi\cos\phi & -\sin\beta'\cos\beta'\cos\phi \\ -\sin^2\beta'\sin\phi\cos\phi & 1-\sin^2\beta'\sin^2\phi & -\sin\beta'\cos\beta'\sin\phi \\ -\sin\beta'\cos\beta'\cos\phi & -\sin\beta'\cos\beta'\sin\phi & \sin^2\beta' \end{pmatrix} \tag{21}$$

$$\underline{\underline{B}}_3 = \frac{1}{F(\beta',\phi')} \begin{pmatrix} -\cos\beta' & -\sin\beta'\cos\phi' \\ 0 & 0 \\ -\sin\beta'\cos\phi' & \cos\beta' \end{pmatrix} \tag{22}$$

$$\underline{\underline{B}}_4 = \begin{pmatrix} 0 & (1-R_{\perp_B}-T_{\perp_B})\sin\beta'\sin\phi' \\ 1+R_{\parallel_B}-T_{\parallel_B} & 0 \end{pmatrix} \tag{23}$$

4 Numerical Tests

Examples are reported in this section to demonstrate the ability of the UAPO solution to counterbalance the GO field jumps. The corresponding figures show GO, UAPO and total fields when P moves on a circular path with radius $\rho = 5\lambda_0$, λ_0 being the free-space wavelength. The junction consists of two co-planar sheets with same thickness ($d = 0.1\lambda_0$), but different electric parameters, i.e., $\varepsilon_{r_A} = 2.5 - j0.25$ and $\varepsilon_{r_B} = 3.7 - j0.16$.

Figures 2 and 3 refer to the output co-polar components when the incidence direction ($\phi' = 45°$) belongs to the first quadrant. The GO field possesses jumps at the reflection and transmission boundaries, whereas the UAPO diffracted field shows its peaks in correspondence of these directions and permits the continuity of the total field. Same comments are valid also when $\phi' = 120°$ (see Figs. 4 and 5).

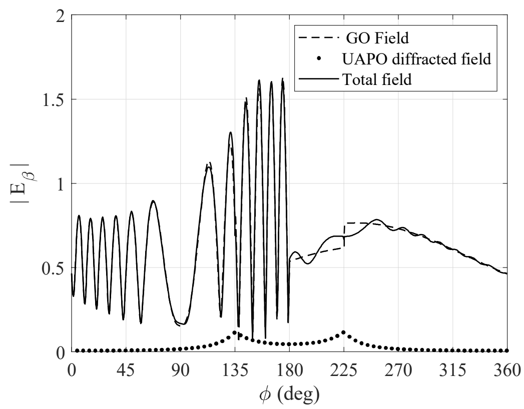

Figure 2. β-component when $E^i_{\beta'} = 1$, $E^i_{\phi'} = 0$ and $\beta' = 60°, \phi' = 45°$.

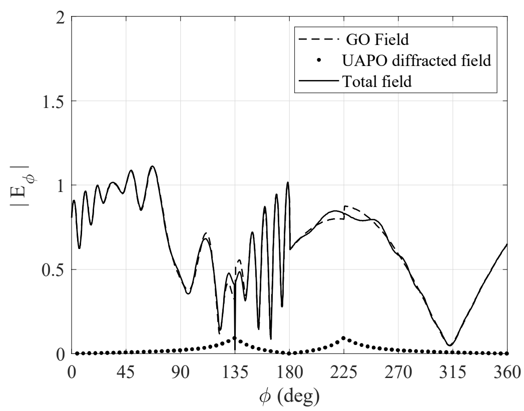

Figure 3. ϕ-component when $E^i_{\beta'} = 0$, $E^i_{\phi'} = 1$ and $\beta' = 60°, \phi' = 45°$.

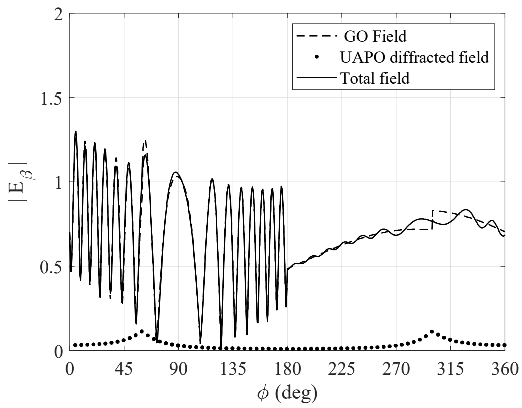

Figure 4. β-component when $E^{i}_{\beta'} = 1$, $E^{i}_{\phi'} = 0$ and $\beta' = 60°, \phi' = 120°$.

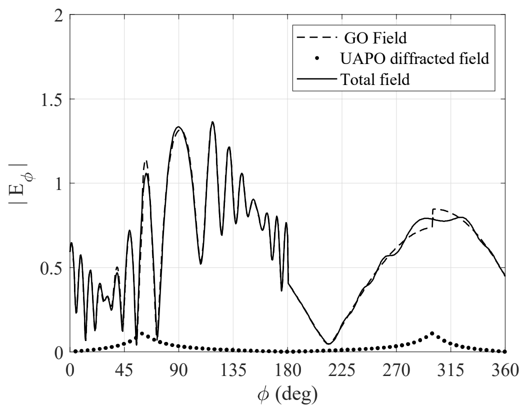

Figure 5. ϕ-component when $E^{i}_{\beta'} = 0$, $E^{i}_{\phi'} = 1$ and $\beta' = 60°, \phi' = 120°$.

5 Conclusions

The plane wave diffraction by a planar junction of co-planar resistive half-planes has been studied by means of the UAPO approach in the UTD framework. The analytic result has been formulated in matrix form when the incidence direction is oblique with respect to the discontinuity. It is compact and easy to manage. Its ability to compensate the jumps of the GO field at the shadow boundaries has been proved.

6 References

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