Nonintrusive Uncertainty Qualification in Multiphysics Modeling

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In complex industrial multi-physics problems, high-precision numerical solving is not always available, since the associated uncertainty problems always exist. Under many circumstances, we hope to evaluate how accurately the mathematical model we have built can describe the real-world physics process and hope to estimate the error in specified outputs of numerical computation reliably. Therefore, uncertainty qualification (UQ) in multi-physics is a challenging and emerging area, owing to the randomness of complicated multi-physics model uncertainty (structural or parametric), which describes the origination, propagation, and interaction of uncertainties from different sources.

Nowadays, some relatively mature methods have been used in demonstrating the process of uncertainty propagating from model inputs to outputs. Among them, the polynomial chaos (PC) method has been increasingly applied to probabilistic UQ over the past decades, which employs spectral expansions over stochastic space for the representation of random variables and uses a combination of different polynomials for probabilistic characterization of uncertain system outputs.

Applying the PC-based UQ approach, one can propagate the PC-represented uncertainty through the PDE governed model by using the Galerkin method, then reformulate the governing equations into equations to compute the PC mode strengths, this approach was designated intrusive, as it introduces reformulated equation system, hence new solver frame and additional computation are indispensable. The advantage of the intrusive approach is that it elaborates outputs’ PC representation by a one-time solution of the reformulated model with great accuracy. However, in the multi-physics process, there usually exist complicated coupled model settings where only a black-box code is available. Therefore the implementation of the intrusive PCUQ method is not feasible. Concerning the features of multi-physics situations, we proposed a non-intrusive method, in which one can propagate uncertainty by numerically evaluating PC mode strength with model outputs employing deterministic or random sampling of the original deterministic model, to maintain original model solving framework by treating complex multi-physics model as a black-box, the computation cost is decreased because only a sequence of deterministic PDE equation needs to solve.

Non-intrusive PCUQ approaches are always challenging with the need to evaluate high-dimensional integrals, this dilemma seems to have significantly deteriorated under the coupling of multi-physics fields since it will induce multistage and multi-scale problems in solving stochastic PDE governed process. To address this, we proposed an approach that keeps the subject of the uncertainty remains localized within each subproblem to reduce the dimension of the stochastic information linking the subproblems. Consequently, the overall computation cost will be further reduced.