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Topology Optimization of Plasmonic Device Using Function Expansion Method and CMA-ES

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Abstract

In this paper, we present a topology optimization method for designing plasmonic devices. We employ the function expansion method to express a device structure in a design region because arbitrary structure can be expressed with relatively few design variables. In addition, we employ CMA-ES for optimizing the design variables.

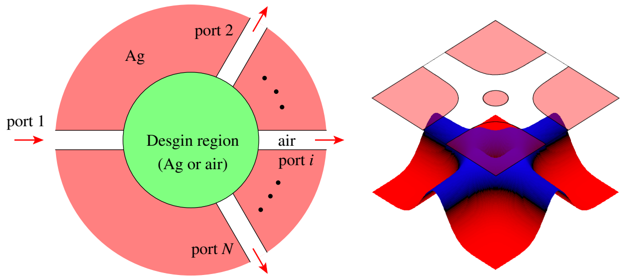
1 Introduction

In order to explore new possibilities of photonic devices, inverse design techniques are intensively studied and various kinds of optimization approaches have been reported [1]. Among these optimization techniques, topology optimizations have the highest design freedom and have the possibility to find out innovative photonic devices beyond human knowledge. In most topology optimizations, a design region is expressed by numerical design variables and the design variables are iteratively updated using numerical simulations.

In our past study on topology optimization [2]-[5], we have developed a new scheme of expressing a structure in the design region, referred to function expansion method. In this approach, the design variables are iteratively updated by the gradient method (GM) based on sensitivity analysis calculated by the adjoint variable method (AVM). GM is an efficient optimization technique for unimodal optimization problems. However, there is a problem that optimized results depend on the initial solution in multimodal optimization problem. In addition, an availability of an efficient sensitivity analysis is required.

Although plasmonic devices are one of promising candidates for improving photonic systems, the sensitivity analysis is generally not so easy in the optimal design of plasmonic devices because metals have a negative relative permittivity. In addition, strong light confinement, which is the preferable nature of plasmonic devices, leads an objective function to be complicated and highly multimodal.

In this paper, in order to develop more robust and efficient optimization method for plasmonic devices, we apply the covariance matrix adaptation evolution strategy (CMA-ES) [6] to our function expansion method.



(a) (b)

Figure 1. Schematic of design problem of plasmonic device. (a) Design model with N input/output ports, (b) Image of representation of refractive index distribution of function expansion method.

2 Topology Optimization

We consider a design problem as shown in Fig. 1. In our optimal design method of photonic devices, a refractive index distribution in the design region is expressed by function expansion method as follows [2]:

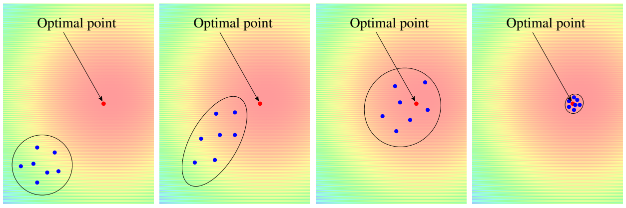
$$n^2(x,y) = n_a^2 + (n_b^2 - n_a^2)H(\xi) \qquad (\xi = w(x,y)) \tag{1}$$

where n_a and n_b are the refractive indices of two materials used in the design region. $H(\xi)$ is the usual Heaviside function used to binarize refractive index distribution. $w(x,y)$ is expressed as follows:

$$w(x,y) = \sum_{i=1}^{N_d} c_i f_i(x,y) \tag{2}$$

where $f_i(x,y)$ $(i = 1,2,\cdots N_d)$ are basis functions and their amplitudes, c_i, are the design variables. Various kinds of basis functions are available and three kinds of basis functions have been used in our previous work [3]. The actual form of basis function will be given when numerical example is shown. Finite element mesh is automatically generated to fit material boundaries when $w(x,y)$ is updated in optimal design [5].

The design variables should be optimized to minimize a given objective function which represents device performance. We employ the CMA-ES in this study. The CMA-ES is a derivative-free method for nonlinear or non-convex continuous optimization problems. In the CMA-ES, new



(a) (b) (c) (d)

Figure 2. A search image of CMA-ES. Search space is updated from (a) to (d).

search points are selected according to a probability distribution expressed by a generalized Gaussian function. The probability distribution expressed by center coordinate and covariance matrix is updated in each optimization iteration based on information of past search results. The search image is shown in Fig. 2. Search step sizes are related to an average distribution of the objective function in the vicinity of search center. The search sizes are shrunk in the directions of convexity and stretched in the other directions.

3 Design Example

We consider an optical diode [4],[7] in which light transmits in the forward direction and the backward transmission is prohibited. The design model of a plasmonic diode is shown in Fig. 3 and the fundamental TM mode incidence is considered. The operation wavelength is assumed to be $\lambda = 1.55$ μm and the relative permittivity of silver is calculated as $\varepsilon_{\mathrm{Ag}} = -103.33 - j8.1302$ based on the Lorentz-Drude model. The relative permittivity of air is assumed to be $\varepsilon_{\mathrm{air}} = 1$. The structural parameters in Fig. 3 are assumed to be $w = W_y = 0.8$ μm, $W_x = 1.5$ μm, $l = d = 0.5$ μm. In order to realize diode operation by using only reciprocal materials, the fundamental mode from port 1 should be converted to the higher-order mode in port 2 and the transmission of the fundamental mode from port 2 to port 1 should be prohibited. Therefore, the objective function to be minimized is given as follows:

$$\text{Minimize } C = C_{\text{forward}} + C_{\text{backward}}, \tag{3}$$

$$C_{\text{forward}} = \sum_{i=0}^{1} \left| S_{11}^{(\mathrm{TM}_i)} \right|^2 + \left| S_{21}^{(\mathrm{TM}_0)} \right|^2 + \left(1 - \left| S_{21}^{(\mathrm{TM}_1)} \right|^2 \right),$$

$$C_{\text{backward}} = \sum_{i=0}^{1} \left| S_{12}^{(\mathrm{TM}_i)} \right|^2 .$$

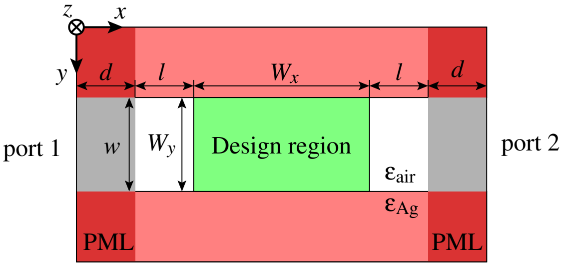


Figure 3. Design problem of a plasmonic optical diode.

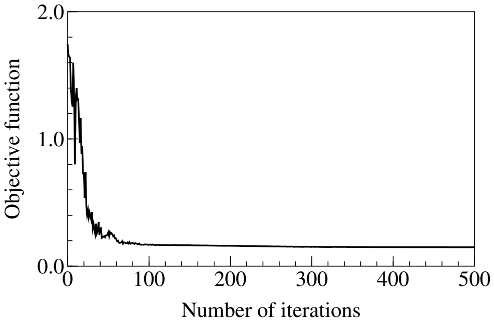
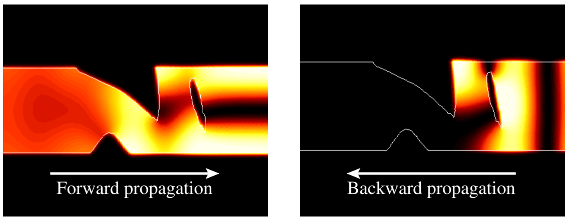


Figure 4. Objective function as a function of iteration step.



(a) (b)

Figure 5. Propagation field in the optimized plasmonic diode. (a) Forward and (b) backward propagation.

where the superscripts of S-parameters denote the transmitted modes. If there is no material loss, the ideal value of the objective function is 0. However, the material loss cannot be ignored, thus, the optimal value of the objective function may be greater than 0. The structure in the design region is expressed by function expansion method and the actual form of $w(x,y)$ is given as follows:

$$w(x,y) = \sum_{i=-N_x}^{N_x-1} \sum_{j=0}^{N_y-1} \left(a_{ij} \cos\theta_{ij} + b_{ij} \sin\theta_{ij} \right), \tag{4}$$

$$\theta_{ij} = \frac{2\pi i}{L_x} x + \frac{2\pi j}{L_y} y, \quad L_x = 1.2W_x, \quad L_y = 1.2W_y$$

where $N_x = 8$, $N_y = 2$.

Figure 4 shows the objective function as a function of design variables. In this optimization, only 28 device structures are evaluated by using the finite element method (FEM) at each iteration step. Therefore, total number of FEM calculation is 14,000. This computational cost is smaller than those in [4], in which several evolutionary approaches, such as GA, PSO, DE, HF, and HFA, are employed. The propagation fields in the optimized plasmonic diode are shown in Fig. 5 for forward and backward propagations. The transmitted power is 0.854 for forward propagation and 8.3×10^{-5} for backward propagation. This result is slightly superior to that of the best result in [4].

4 Conclusion

In this paper, we proposed the topology optimization method combining function expansion method and CMA-ES for the design of plasmonic devices. The function expansion method can express device structures with a relatively small number of design variables and CMA-ES can efficiently optimize design variables without requiring sensitivity analysis. The effectiveness of this approach was shown in the design of plasmonic diode. In our future work, we are considering to apply this method to the design problems of various kinds of plasmonic devices.

5 Acknowledgements

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