



## On Frequency Scaling of Rain Attenuation

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### Abstract

In our paper we deal with the generalized method of frequency scaling. From the attenuation known at one frequency, we determine the rain rate by the inverse procedure of any rain attenuation prediction model, which we then use to estimate the rain attenuation on another frequency. We compare the results on one hand with exact values, on the other hand with the results after the ITU-R frequency scaling method. The method proposed by us seems to be more accurate at the level of instantaneous frequency scaling and also at the level of the attenuation distribution function.

### 1. Introduction

Frequency scaling is a method that can be used to estimate the attenuation on a microwave or mm terrestrial or satellite link at a certain frequency from the knowledge of the attenuation at another frequency. In our treatise we will focus on rain attenuation only because other atmospheric phenomena causes much lower attenuation of radiowaves. In addition to the attenuation scaling (transition) from one frequency to another, the attenuation of some methods can be estimated also for different polarizations and other elevation angles. A number of methods for frequency scaling have been described in the literature and their tests have been described. See, for example, [1],[2],[3] and [4]. To the authors' knowledge, the relatively simple ITU-R method is most recommended, the latest variant of which can be found in [5]. This method has also evolved and has been refined over time.

### 2. Reference Frequency Scaling Method

To test our frequency scaling method we compare our results with the ITU-R P.618-12 technique results [5]. To refresh this recommend method we show the formulas here:

$$A_2 = A_1(\varphi_2/\varphi_1)^{1-H(\varphi_1,\varphi_2,A_1)} \quad (1)$$

$$\varphi(f_i) = \frac{f_i^2}{1+10^{-4}f_i^2}, i = 1,2 \quad (2)$$

$$H(\varphi_1, \varphi_2, A_1) = K \cdot (\varphi_2/\varphi_1)^{0.5} \cdot (\varphi_1 \cdot A_1)^{0.55} \quad (3)$$

where  $A_1$  ( $A_2$ ) is attenuation on frequency  $f_1$  ( $f_2$ ) in dB and  $K=0.00112$ .

### 3. General Frequency Scaling Method Formulation (Novel Technique)

These class of methods is based on inversion of classical rain attenuation prediction methods (models). In the case of model we usually derive rain attenuation from given rain rate  $R$  (first row in Fig.1) while model can be based on effective path length, rain rate link profile estimation, on measurement (empirical models) etc. When we reverse the model (second row in Fig.1), we can estimate rain rate  $R$  from measured (known) attenuation  $A(f_1)$  on frequency  $f_1$ . Knowing rain rate  $R$  and using the model again (third row in Fig.1) the attenuation  $A(f_2)$  on frequency  $f_2$  is estimated. This works as a frequency scaling possibility.

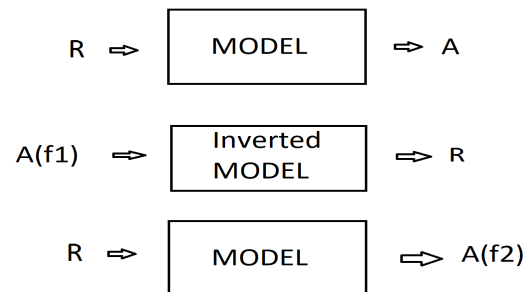


Fig. 1 On general frequency scaling method formulation

Other idea of scaling, i.e. estimation of rain height instead of rain rate is described in [6].

### 4. Selected Attenuation Model for Frequency Scaling

To exercise and to demonstrate our idea we chose the “old” ITU-R rain attenuation prediction method because of its simplicity and quite good accuracy [7]. Many other models could be used, certainly. To remind the mentioned model look at next equations:

$$A_{0.01} = k R_{0.01}^\alpha L_S r_{0.01} \quad (4)$$

$$L_o = 35 \exp(-0.015 R_{0.01}) \quad (5)$$

$$r_{0.01} = \frac{1}{1 + L_G/L_o} \quad (6)$$

where  $k$  and  $\alpha$  are constants dependent on frequency, polarization and elevation angle  $\theta$  (they are tabulated, we recommend [8]),  $R_{0.01}$  is rain rate [mm/h] exceeded for 0.01% of time (in the Czech Republic it is, for instance, 26 mm/h),  $A_{0.01}$  is corresponding attenuation in [dB] exceeded also for 0.01% of time,  $r_{0.01}$  is reduction factor,  $L_s$  is slant path length crossing the rain volume of satellite link (and  $L_G = L_s \cos(\theta)$  while  $\theta$  is satellite link elevation angle),  $L_s = h_R / \cos(\theta)$  where  $h_R$  is rain height, its height is modeled in [9] for instance and very very roughly it is 4 km. For terrestrial links  $\theta = 0^\circ$  and  $L_s$  is the terrestrial link length in such case.

Attenuation values  $A_p$  for other exceedance percentage probabilities  $p$  are estimated from next equation:

$$\frac{A_p}{A_{0.01}} = 0.12 p^{-(0.546 + 0.043 \log p)} \quad (7)$$

## 5. Application of Frequency Scaling General Method - Example

Let us suppose that equation (4) of the model presented in previous Chapter 4 is valid for all exceedance probabilities. Working in the frequency scaling technique we suppose we know attenuation  $A(f_1)$  and we have to derive the corresponding rain rate  $R$  from next equation:

$$A(f_1) = k R^\alpha L_s \frac{1}{1 + L_G/[35 \exp(-0.015 R)]} \quad (8)$$

This equation is transcendent one and therefore a numerical solution is required. We tried to use Matlab standard functions “fzero” or “solve” but these functions did not lead to right results in some cases. As we recognized, these functions are extremely dependent on initial solution estimation. Therefore we decided to find solution of  $R$  in a cycle for rain rates from 0.2 to 240 mm/h searching for minimum of next equation:

$$k R^\alpha L_s \frac{1}{1 + L_G/[35 \exp(-0.015 R)]} - A(f_1) = \min \quad (9)$$

while we prescribed the solution accuracy of searched rain rate to be 0.2 mm/h.

Having find the rain rate  $R$  corresponding to given attenuation  $A(f_1)$  from previous equation, we can then compute corresponding attenuation  $A(f_2)$  on frequency  $f_2$ .

## 6. First Results

For first test of presented novel frequency scaling method described step by step in Chapters 3, 4, and 5 we proposed a rain rate ascending sequence

$R = [1 \ 2 \ 3 \ 4 \ 5 \ 7 \ 10 \ 12 \ 15 \ \dots \ 75 \ 80 \ 90 \ 100]$  [mm/h] and for each value of rain rate  $R$  the corresponding attenuations  $A(f_1)$  and  $A(f_2)$  were computed while just this value of  $A(f_2)$  is taken as the true one for tests of frequency scaling methods. For illustration in our tests we used frequencies of the Aldo Paraboni Q/V Communications and Propagation experiment as the Technology Demonstration Payloads carried by the satellite Alphasat, while  $f_1 = 19.7$  GHz and  $f_2 = 2 \cdot f_1 = 39.4$  GHz.

From computed  $A_1(f_1)$  the value of  $A_2(f_2)$  was estimated using method from chapter XX and reference method from Chapter 2. Next figure 2 is showing the comparison of both methods:

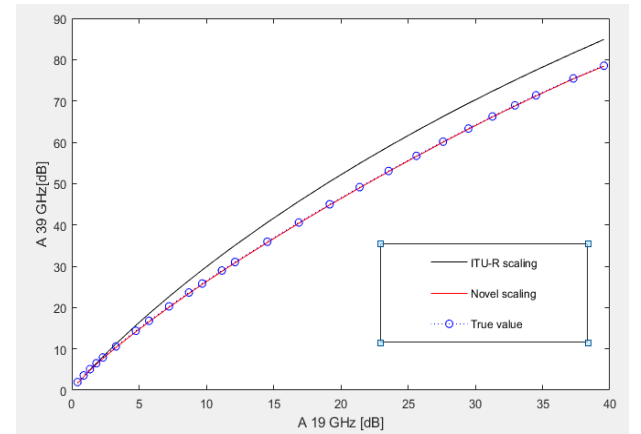


Fig.2 Comparison of novel method of frequency scaling with ITU-R method [5] and “true” values

One can see that novel frequency scaling method after Chapter 5 approaches the reference values much more accurately than the ITU-R method.

## 7. Test of Frequency Scaling Novel Method through Attenuation CCDF Measurement

To test our frequency scaling “novel” method we used also our 3 year satellite link attenuation measurement at the Institute of Atmospheric Physics in Prague (in the form of CCDF, 2016-2018) while CCDF is the Complementary Cumulative Distribution Function. From attenuation values on frequency  $f_1$  and on certain exceedance probability level we estimated the CCDF attenuation values on frequency  $f_2$  by two methods:

1. From measured attenuation CCDF and using the interpolation we estimated  $A_{0.01}$  on frequency  $f_1$ . From  $A_{0.01}$  we estimated  $R_{0.01}$  using equation (9). Knowing  $R_{0.01}$  we computed attenuation CCDF on frequency  $f_2$  through equations (4) and (7). It is in red in Fig.3 while measured CCDF on frequency  $f_2$  is in blue color. In Fig. 3 it is called “method 1”

Found values:

$$A_{0.01}(f_1) = 15.8 \text{ dB}$$

$R_{001} = 32.5$  mm/h using equation (9)  
 $A_{001}(f_2) = 38.3$  dB using equation (4)

By the support of these found values we computed the attenuation CCDF on frequency  $f_2 = 39.4$  GHz using equations (4) and (7) again.

2. We suppose that

$$\frac{A(p, f_1)}{A(p, f_2)} = \frac{A_{001}(f_1)}{A_{001}(f_2)} = p_0 \quad (10)$$

where  $p$  is the percentage exceedance probability.  $A(p, f_2)$  is then computed from measured  $A(p, f_1)$  using equation (10). In our example we found that

$$p_0 = 2.44 \quad (11)$$

The curve labeled “method 2” in Fig. 3 corresponds to these results.

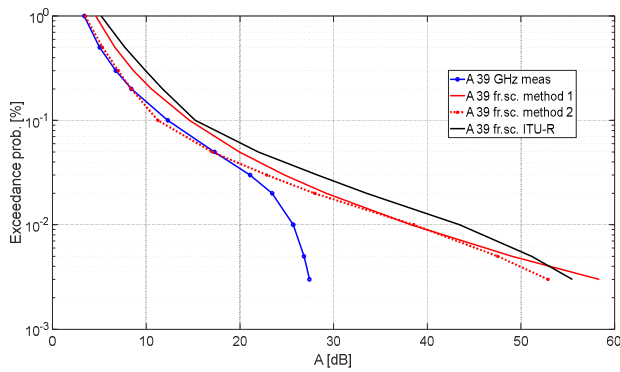


Fig.3 Comparison of attenuation CCDF measured (blue) on  $f_2 = 39.4$  GHz with its estimations from CCDF measured on  $f_1 = 19.4$  GHz (red-method 1 and dotted red – method 2, black – ITU-R method)

From comparisons in Fig.3 we see that the ITU-R method rather failed, method 2 is very accurate for attenuation below 20 dB and exceedance percentage probabilities above 0.03%. The accuracy of method 1 is between method 2 and the ITU-R frequency scaling method. We are showing detail of Fig.3 in Fig.4 with logarithmic attenuation scale. We added there also measured CCDF on frequency  $f_1$  (blue curve).

Remark: The blue curve’s “falling down” for measured attenuations above 20 dB (or for exceedance probabilities above 0.03%) in Fig. 3 is caused by the limited and changing receiver’s power dynamic range and satellite space elliptical motion.

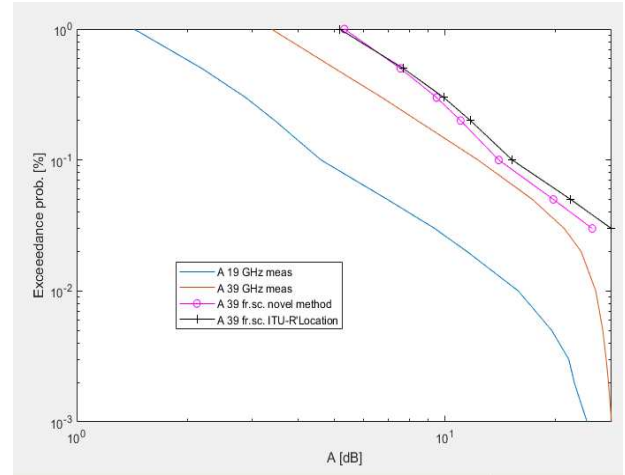


Fig.4 Detail of Fig.3. The measured attenuation CCDF on frequency  $f_1$  is in blue while the measured CCDF on frequency  $f_2$  is in brown. Black curve corresponds to the ITU-R frequency scaling method while magenta color with circles represents the attenuation estimation on  $f_2$  after method 1.

## 8. Instantaneous Frequency Scaling

Let us suppose that method in Chapter 5 is valid for all considered exceedance probabilities. And not only for statistics but also for instantaneous values. In the statistics it should be the “statistical” or “probable” value approaching more or less the true.

We analyzed one attenuation event derived from our Alphasat satellite attenuation link measurement in Prague. The results are shown in Fig.5. It is a scatterplot where circles represent the corresponding couples of measured attenuation values (averaged within 10 second interval) on both frequencies  $f_1$  and  $f_2$ . Red color is the “scaled” value through frequency scaling (our novel method), blue is the ITU-R method (Chapter 2).

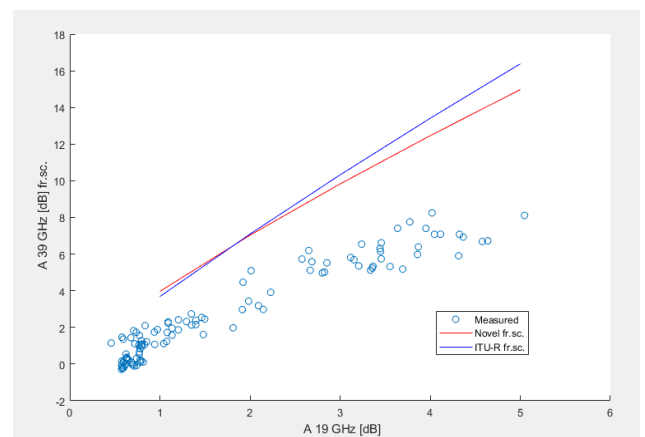


Fig.5 Example of instantaneous frequency scaling from Alphasat satellite link attenuation measurement in Prague (one random rain event). For description see text.

## 9. Conclusion

These are the first attempts to apply new frequency scaling approaches. It will be necessary to process more data gradually to test presented methods, but we have them available. Students from University of Pardubice are also involved in this research. Two students are working on here presented method from Chapter 5, but they are not applying novel approaches using “old” rain attenuation prediction ITU-R model [7], which we used for its simplicity in this study, but to the new ITU-R model that is currently in force [5] and is generally recommended. Also classical “Assis-Einloft” rain attenuation model ([10], [11]) will be tested within our novel approach. Even according to the already obsolete method for predicting rain attenuation, the novel method seems to be more accurate than the ITU method for frequency scaling [5]. We also plan to test the fit of our procedures with exact values (TRUE values), either mathematically modeled or experimentally determined through the RMSE method. Our research will continue in this area, at both University of Pardubice and at IAP Prague (Institute of Atmospheric Physics).

## 10. Acknowledgements

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