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A Superconducting Quantum Interference Device Model for the Design and Optimization of Quantum-Limited Microwave Amplifiers in Harmonic Balance Simulators

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Abstract

We present a dc superconducting quantum interference device (dc-SQUID) model for the design and optimization of quantum-limited parametric amplifiers. These devices are critical for superconducting quantum computing and do not currently have comprehensive models that designers can use to simulate their circuits in standard circuit simulators. Our model has been implemented and tested in the Advanced Design Systems (ADS) simulation software and validated against analytical predictions.

1. Introduction

The superconducting quantum bit (qubit) is one of the most prominent candidates for implementing large-scale, fault-tolerant quantum computers. Superconducting Josephson parametric amplifiers (JPAs) are critical for ultra-low noise superconducting qubit readout and there currently are no comprehensive electronic design automation (EDA) models for these devices. Robust microwave modeling and accurate characterization of superconducting devices and circuits [1-2] will be key for optimizing and scaling future superconducting quantum computation systems. We propose a dc superconducting quantum interference device (dc-SQUID) model compatible with standard circuit simulators that can be used in the design and optimization of JPA devices.

Modeling approaches for SQUIDs and JPAs have been previously reported [1, 4-5]. Compared to prior art, the key benefits of our model include, 1) Our model can be used in commercial harmonic balance circuit simulators. 2) The model poses no limitation to the type of signals in the JPA, which can be useful to explore more complex operation regimes. 3) Our model can account for non-linear effects in the amplifier. 4) Most importantly, our model can seamlessly be integrated with built-in components available in commercial circuit simulators to build more complex circuits, which can be readily simulated with powerful time- or frequency-domain solvers.

We implemented our dc-SQUID model in Keysight advanced design system (ADS) software, used the model to design a simple JPA device, and validated our simulations against analytical predictions and found excellent agreement. The next step is to validate our model against measurements and exploit new quantum-limited amplification topologies for improved performance (e.g., in terms of bandwidth and dynamic range). This kind of models can be of interest to other SQUID-based circuits and can be useful for quantum engineers and designers.

3. Parametric Amplification Model

The readout of a superconducting qubit requires ultra-low-noise amplification of exceedingly small signals reflected from a resonant cavity coupled to the qubit. This is usually achieved via a multi-stage amplification chain comprising a superconducting JPA with near-quantum-limited performance followed by semiconductor cryogenic and room-temperature amplifiers [6]. JPAs typically employ dc-SQUIDs to transfer energy from a pump signal (which acts as the power source for the amplification) to the signal of interest without dissipating power. The core component of many JPA devices, the dc-SQUID consists of two Josephson junctions embedded in a superconducting loop as in Figure 1a. Parametric amplification can be performed using a flux-driven dc-SQUID like that in Figure 1b, where a transformer is used to couple an external flux to the SQUID to modulate its inductance.

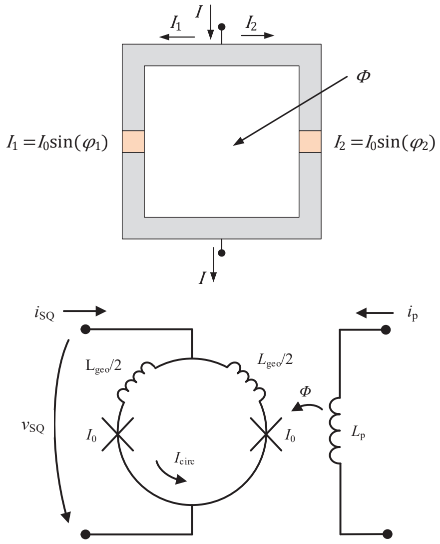


Figure 1. a) Illustration of dc-SQUID consisting of two Josephson junctions (orange) embedded in a superconducting loop (grey) threaded by an external magnetic flux. b) Circuit diagram of a flux-driven dc-SQUID.

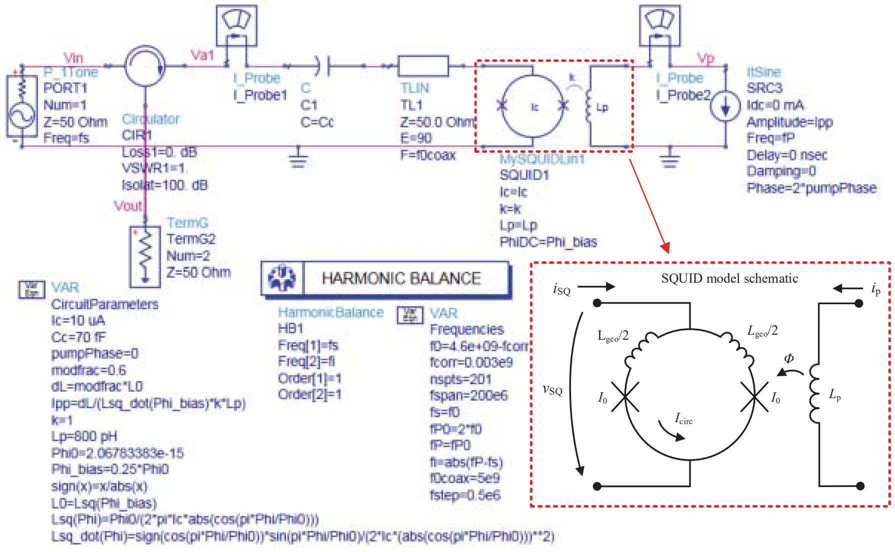
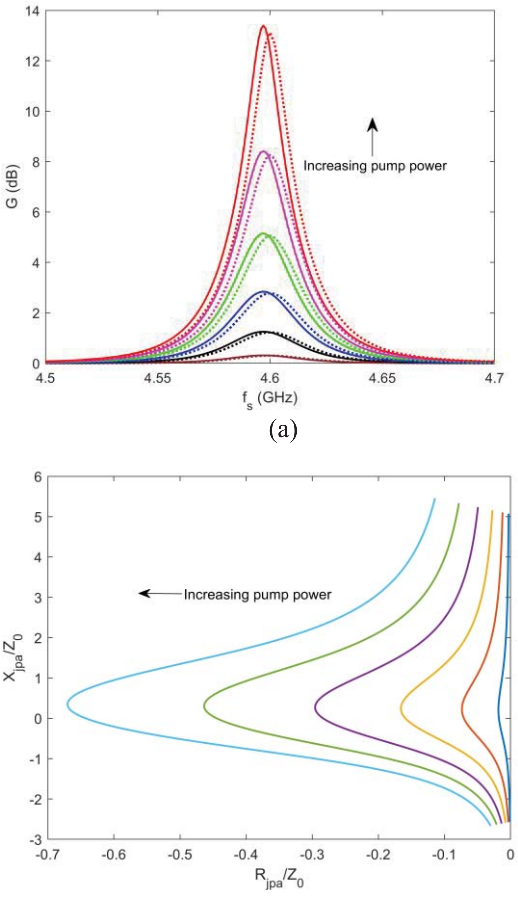


Figure 2. ADS schematic of JPA circuit. The inset shows details of the flux-driven dc-SQUID.

Briefly, our dc-SQUID model is based on a Volterra series expansion of the SQUID's nonlinear Josephson inductance around a quiescent flux bias, and it can be seamlessly integrated with existing components in standard circuit simulators. We implemented this model in Keysight's ADS software, then used it to design a simple JPA and evaluated our design using the ADS harmonic balance solver. Figure 2 depicts the ADS schematic of a JPA circuit incorporating the proposed model, where the flux-driven dc-SQUID is attached to a quarter-wavelength transmission line resonator to form a parametrically driven oscillator [4]. This JPA configuration operates as a negative-resistance amplifier and a circulator is used to separate the incoming signal from the amplified output.

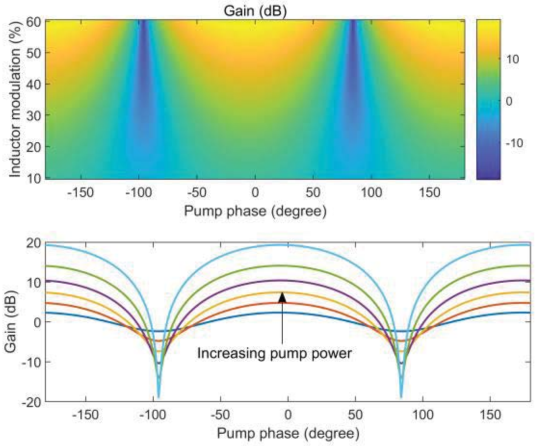
4. Preliminary Results and Discussion

Figure 3 presents preliminary results of a 4.6 GHz JPA design. the gain obtained with the proposed dc-SQUID model and with an analytical model is compared in Figure 3a as a function of detuning and pump strength. Figure 3b presents the JPA impedance as a function of pump strength and detuning showing its operation as a negative-resistance amplifier device. For optimum gain, the device should ideally be biased for zero reactance and maximum absolute resistance at the resonant frequency. The JPA degenerate gain as a function of pump amplitude and phase is depicted in Figure 3c, showing the phase-sensitive interference pattern typical of degenerate operation (i.e., for $f_p = 2f_s = 2f_0$). Figure 4 presents the gain as a function of signal and pump frequencies for a 6 GHz JPA design. At high pump power levels, the amplifier presents multi-peaked gain as observed in Figure 4.



(a)

(b)



(a)

Figure 3. a) Gain as a function of detuning and pump strength for a 4.6 GHz JPA design (proposed model/dashed lines vs. analytical prediction/solid lines). b) JPA impedance as a function of pump strength and detuning. c) JPA degenerate gain as a function of pump amplitude and phase, exhibiting phase-sensitive interference. Note that this design has not been optimized for maximum gain.

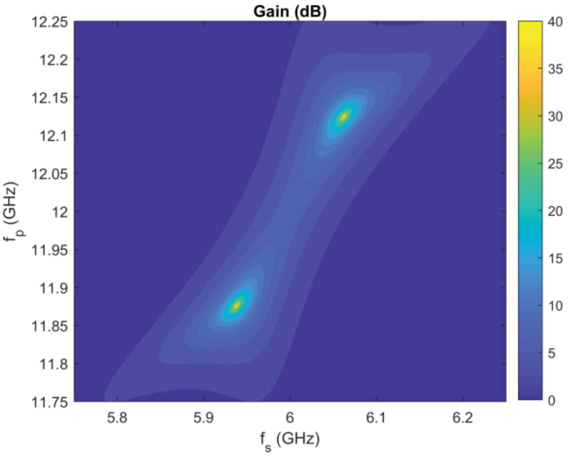


Figure 4. Gain as a function of signal and pump frequencies for a 6 GHz JPA design.

6. Acknowledgements

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