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Informed truncation for grid-based low-rank approximation of $\mathcal{H}$-matrix blocks

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The hierarchical-, or $\mathcal{H}$-matrix approach [1] is often used to accelerate iterative and direct electromagnetic integral-equation solvers, by representing off-diagonal blocks of the method of moments matrix using their low-rank approximations (LRAs). The computational savings rely on a compressed block's rank, $\mathcal{R}$, being significantly lower than its number of rows and columns . For oscillatory-kernel integral equations, the rank deficiency depends strongly on the geometric configuration of the source and observer domains corresponding to the matrix block and the prescribed accuracy [2]; thus, $\mathcal{R}$ must be revealed rather than assumed. While many LRA algorithms exist, the compression stage for $\mathcal{H}$-matrix solvers remains a computational bottleneck. Among the more promising algorithms for its removal are specialized techniques that utilize efficient physics-based field representations.

One such approach is that of non-uniform spherical sampling of the phase-and-amplitude compensated field components [2],[3]. In [2], the components are sampled on proxy surfaces surrounding the observer domain, and a multilevel process reveals $\mathcal{R}$ via the truncation, using a threshold $\tau_{\mathcal{G}}$ corresponding to the prescribed accuracy, of the singular value decomposition (SVD) of smaller grid-interaction matrices. The progression up the multilevel tree is done through interpolation, phase-restoration, and concatenation of child blocks. In the process, a rank-$\mathcal{R}$ orthonormal basis for the block's domain is approximated and can be later complemented to form the LRA, via $\mathcal{R}$ fast matrix-vector multiplications. Here, it is shown that, despite its efficiency, demonstrated in [2] for sets of simple co-oriented dipole sources and observers, the LRA's accuracy deteriorates when vector basis/testing functions, such as the Rao-Wilton-Glisson (RWG) functions [4], are used, especially when a high-accuracy LRA is desired. The deterioration is because the grid-interaction matrices' domains, approximated according to the threshold $\tau_{\mathcal{G}}$, only partially overlap with that of the original matrix block.

This article presents an algorithm that finds the SVD truncation threshold $\tau_{\mathcal{G}}$ in [2] to guarantee that the top-level grid matrix's domain sufficiently overlaps with that of the original matrix block. To this end, a "misrepresentation metric", which quantifies how accurately the rank-$\mathcal{R}^{\mathcal{G}}$ domain of the grid-interaction matrix approximates the block's domain, is computed. This requires the norms of both the block itself and its "residual", defined as the matrix block applied to the orthogonal complement of the projector to the grid-interaction matrix's domain. To avoid a computational bottleneck, the misrepresentation metric is approximated by using uniformly at random chosen portions of the block and the grid-interaction matrix. More specifically, first, an appropriate subsample size k is iteratively determined to accurately represent the block's and grid-interaction matrix's spectra up to the rank-$\mathcal{R}$. The size k is incrementally increased until the subsampled block's revealed-rank converges. Then, the SVD of size $k \times k$ portions of the operator and grid-interaction matrix are computed. The results of this smaller SVD are used to rapidly approximate the misrepresentation metric and find a $\tau_{\mathcal{G}}$ that ensures that the metric falls beneath the desired accuracy level. At the conference, it will be numerically demonstrated that (i) this algorithm's costs are negligible compared to those of computing the LRA and (ii) the algorithm can ensure the accuracy of the LRA algorithm in [2] for oscillatory-kernel integral equations discretized with RWG functions.

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