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Modeling of Field-Aligned Currents in the Magnetosphere

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Abstract

It is known that the combined action of convection and pitch-angle diffusion of electrons and protons is responsible for the formation of plasma pressure distribution in the magnetosphere. Plasma pressure, in turn, determines - within the framework of a given magnetic field model - the density of bulk currents in the magnetosphere. With knowledge of the bulk currents as a function of coordinates, we can calculate the field-aligned currents as a divergence of bulk currents. On the other hand, specifying the convection model is equivalent to specifying the electric field model. Since within the approximation of equipotential field lines the electric field is common to the magnetosphere and ionosphere, bulk currents and field-aligned currents in the ionosphere can be formally calculated subject to the condition that ionospheric conductivity is wholly determined by electron precipitation from the magnetosphere. The precipitation intensity is readily inferred from the same magnetospheric model. Direct observations of plasma distribution in the magnetosphere are faced with large difficulties, because pressure must be known everywhere in the plasma sheet at high resolution, which in situ satellites have been unable to provide. Modeling of distribution of plasma pressure is very important, because the data from multisatellite magnetospheric missions for these purposes would be a very expensive project. Therefore there arises a necessity for modeling processes of near-Earth space. A selection and applying of correct initial system of equations are also very important.

1. Introduction

The problem of magnetosphere-ionosphere coupling is very interesting. Magnetospheric Alfvén waves are reflected by the ionosphere. According to [1], the field-aligned currents (FACs) in the subsidiary Alfvén waves serve to close divergent horizontal currents resulting from the non-uniform ionospheric conductivity. According to [15], the inhomogeneity of ionospheric conductivity can lead to FACs, which originate in the ionosphere. Increasing the conductivity gradient (in the direction which is perpendicular to the incident wave field), can increase the ratio of the Hall and Pedersen conductivities, and lowering the conductivity all lead to a larger rotation of the reflected wave field (in the range of from a few up to 40°). According to [13], the polarization charge produced by the Pedersen current divergence has a role to cancel and intensify the ambient background electric field inside and outside the high-conductivity band, respectively. The polarization charge produced by the Hall current divergence has a role to rotate the electric field from the background electric field, which causes a meandering of ionospheric convection flow along the boundary of a high-conductivity band are always perpendicular to each other. The Hall and Pedersen currents never close each other when conductances are homogeneous, but they can do that at the conductivity gradient region. Authors of [14] presented the formulation of the coupling between the ionospheric horizontal currents and FACs via shear Alfven waves, which can describe the formation of a Cowling channel without any a prior parameterization of the secondary (Hall polarization) electric field strength. They showed that the reflected wave can carry FACs that connect to divergent Hall currents. They identified how large the secondary electric field becomes, how efficiently the divergent Hall current is closed within the ionosphere, and how much of the Hall current continues out to the magnetosphere as FACs. Authors concluded that only a small fraction of FACs is connected to Hall currents at conductance gradients. The ionosphere is an ohmic environment, where the electric field and the current are related by the Ohm's law. If the ionospheric current were purely Hall currents, this would not be a dangerous phenomenon since the Hall current is nondivergent and does not deliver a work. In fact, the ionospheric current is combined and always includes the Pedersen component, and the ionosphere is a real energy consumer (sink). It is known that the contents of the magnetic flux tube to be referred to as the plasma tube throughout the text, transfers from one magnetic flux tube to another in the convection process without surplus and deficiency in the case where the field lines of the magnetic flux tube are equipotential ones. This idealization is quite realistic everywhere apart from polar auroras. The combined action of convection and pitch-angle diffusion of electrons and protons is responsible for the formation of gas pressure distribution in the magnetosphere. Plasma pressure, in turn, determines - within the framework of a given magnetic field model - the density of bulk currents in the magnetosphere. With knowledge of the bulk currents as a function of coordinates, we can calculate the field-aligned currents as a divergence of bulk currents. On the other hand, specifying the convection model is equivalent to specifying the electric field model. Since within the approximation of equipotential field lines the electric field is common to the magnetosphere and ionosphere, bulk currents and field-aligned currents in the ionosphere can be formally calculated subject to the condition that ionospheric conductivity is wholly determined by electron precipitation from the magnetosphere. Sun illumination is important for dayside region. The precipitation intensity is readily inferred from our structurally adequate model of the geomagnetosphere. Thus, we have two systems of field-aligned currents. One system is calculated from the model of plasma pressure distribution in the magnetosphere, and the other is inferred from a given model of the electric field and the electroconductivity model calculated from electron precipitation. This brings up the question: How can these two systems of field-aligned currents be reconciled?

2. Formulation of the problem and computational technique

The flux density of precipitating particles is rather sharply localized in the space and produces in the ionosphere a clearly pronounced precipitation oval. Sharp spatially localized regions of increased conductivity in the ionosphere also correspond to such a distribution of precipitation. In the energy range 1-20 keV in which auroral electrons exist, about 30 electron-volt of the flux energy supplied to the ionosphere are expended in producing an electron-ion pair. Hence the ionization rate is $\sim j_\varepsilon/H\delta\varepsilon$ ($\delta\varepsilon$ in ergs), where j_ε is the energy flux density of precipitating electrons (ergs/cm), and the electron density in steady-state conditions:

$$n_e \sim (j_\varepsilon/H\delta\varepsilon\alpha)^{1/2} \quad (2.1)$$

here $\delta\varepsilon$ is expressed in erg cm^{-2}; recombination coefficient (α), in $cm^3\, s^{-1}$; and *H* (the dynamo layer thickness), in cm. Integral conductivity for the Pedersen current is:

$$\Sigma_p = (e^2 n_e/M_i)\int \nu_{in}/(\omega_{iB}^2 + \nu_{in}^2)dz \qquad (2.2)$$

where **e** is the electron charge, $\mathbf{M}_i$ is the ion mass, ω_{iB} is the ion gyrofrequency, and $\boldsymbol{\nu}_{in}$ is the ion-neutral collision frequency. The integral (dz) is taken over the entire thickness of the current-carrying layer, i.e., from 100 to 120 km. Observational data and theoretical estimates show that the scale of the electric field along the latitude is several times larger than the scale of precipitation and, hence, than the scale of the conductivity region.
Since the ionization latitudinal distribution is much more nonuniform in space than the electric field, we will consider that:

$$\partial J_\theta/\partial\theta \approx E_\theta \partial\Sigma/\partial\theta.$$

$$\partial J/\partial\theta = (\partial\Sigma_p/\partial\theta)E_\theta + (\partial E_\theta/\partial\theta)\Sigma_p \sim E_\theta\, \partial\Sigma_p/\partial\theta \quad (2.3)$$

Hence it follows that the electric field configuration is unimportant for the problem of generation of field-aligned currents in the ionosphere, at least for the divergence of those Pedersen currents, which flow along the latitude and produce "curtain" structures. Of importance are the parameters of precipitation, the intensity of which is closely associated with the spatial distribution of the number density of particles in the magnetosphere and, hence, with the pressure relief. For that reason, there must be a correspondence between the picture of field-aligned currents calculated from the gas pressure distribution in the magnetosphere and the picture of field-aligned currents calculated from the distribution of ionization (i.e. precipitation!). It is this reasoning that dictated the formulation of the problem.
Calculations were performed by the formula:

$$j_{\uparrow\uparrow} \approx j_r = [\partial J_\lambda/\partial\lambda + \cos\theta_a J_\theta + \sin\theta_a \partial J/\partial\theta]/r_o \sin\theta_a \qquad (2.4)$$

Since in high latitudes the direction of geomagnetic field lines is close to a radial direction, we identified the field-aligned currents in the ionosphere with radial ones. The error arising in this case in the value of the field-aligned current for the auroral zone is less than 20%. Surface densities of Pedersen currents along the latitude, J_λ, and along the meridian, J_θ, were calculated by standard formulas:

$$J_\lambda = \Sigma_p E_\lambda \quad \text{and} \quad J_\theta = \Sigma_p E_\theta \qquad (2.5)$$

Results of calculations of field-aligned currents "generated" in the ionosphere will be shown in figure.

It should be noted that only the sign of the field-aligned current whose amplitude exceeded some given value, was plotted. For comparison, for the same instants of time, the figure will present the signs of field-aligned currents generated in the magnetosphere.
Calculations were performed by the formula [11], [4], [5]:

$$j_{II} = cB^I \int_0^l \{[\nabla \mathbf{p_g} \times \nabla \mathbf{p_B}]\cdot\mathbf{B}/p_B B^3\} dl \qquad (2.6)$$

where B^I is the magnetic field strength in the ionosphere, the integral is taken over the entire flux tube from the equator to the ionosphere, and p_B is the magnetic pressure. It is clear that the integrand in (2.6) is proportional to the sine of the angle between the contour lines p_g=const and p_B=const. In a dipole approximation, lines of equal magnetic pressure are concentric circles, and isobars follow plasma pressure relief contour lines.

As the plasma tube drifting toward the Earth in a dipole field, its volume decreases in proportion to L^{-4}, and the situation is the reverse for density, while pressure increases in proportion to ~$L^{20/3}$. However, the process of adiabatic compression is attended by the processes of plasma tube depletion due to pitch-angle diffusion into the loss cone. This process is described by the factor ~exp(-∫dt/τ) = exp(-∫dr/V_rτ) = exp(-∫rdυ/V_υτ) [4], [5]. Thus gas pressure has a maximum on each line of convection. In accordance with the equation for p_g [4], [5], we have:

$$p_g = p_g^0 \left(\frac{L_\infty}{L}\right)^{\frac{20}{3}} \exp\left(-\frac{5}{3}\int \frac{dr}{V_r \tau}\right) \qquad (2.7)$$

It is evident that dt =dR/V_R =R_0dλ/V_λ , Δt = ∫dt is the transport time, i.e., the time over which the flux tube will move from the boundary to the given point on the flux line; and V_R and V_λ are the radial and azimuthal components of convection velocity; L is the McIlwain parameter. Thus (2.7) indicates how gas pressure changes when plasma moves along the convection line at a velocity $V = (V_R^2 + V_\lambda^2)^{1/2}$; τ - is a time over which the plasma tube loses 1/e part or the initial number of particles $\tau=\tau_0 L^4$ (τ_0=1s for electrons, τ_0=12-18 s for protons). The radial components of convection velocity is $V_r=V_0L^{9/2}\cos\lambda$. The azimuthal components of convection velocity is $V_\lambda=-2.5V_0L^{9/2}\sin\lambda+V_1L$; V_1 – is a function of initial values of energy of electrons and protons; $V_0=c[E\times B]/B^2$; γ is the adiabatic exponent; p^0_g is an initial pressure at the boundary $L = L_\infty$ for particles with the corresponding energy. The pitch-angle diffusion of particles into the loss cone, together with adiabatic compression of plasma during the convection into the magnetosphere determines the behavior of the contents of the magnetic flux tube, the plasma tube. Gas pressure builds up under the action of an adiabatic compression of plasma drifting in a magnetic field with increasing strength and precipitation-induced losses. In this case the magnetosphere develops a plasma pressure distribution, such as shown in Fig. 1. Input parameters for modeling are the electric field of magnetospheric convection and the magnetic field in the magnetosphere. The function for the electric field variation can be applied from our papers [6-9], taking into account the well-known paper [12].

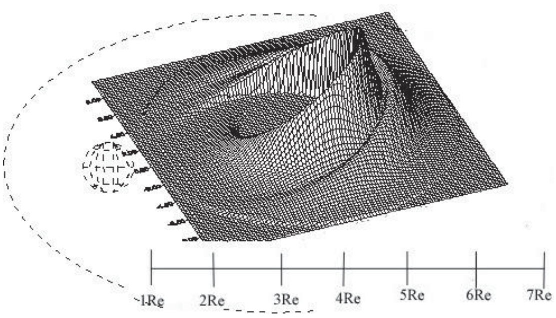

Figure 1. Results of modeling. Plasma pressure distribution. P, [dyn/cm^2] / $L(R_E)$

Our obtained result is also standard. The character of electric field distribution over the dawn–dusk meridian [9] quite corresponds to the classical distribution obtained in [3]. We applied the dipole like magnetic field model, but with special additional parameters for stretching magnetotail. For illustration of results of modeling, we applied the following expressions. The expression (2.7) can be written in the form [7]:

$$p(t`)= G(t)\, A(L) \qquad (2.8)$$

Since in plasma with isotropic pressure (within 3.5 < L < 7, where L is the McIlwain parameter) the plasma pressure relief wholly determines the density of bulk currents for particles with the energy less than 15 keV, then:

$$\mathbf{j}_\perp = c\,[\mathbf{B}\times\nabla p_g]/B^2 \qquad (2.9)$$

where **B** is the magnetic field strength, $\mathbf{p}_g$ is gas pressure, and **c** is the velocity of light.

3. Discussion of results

The problem of compatibility of field-aligned currents generated in the magnetosphere, and of field-aligned currents, which are produced as a result of a spatial inhomogeneity of conductivity (and to a lesser extent, of the electric field), that is, as if they were "generated" in the ionosphere, is part of the problem of ionosphere-magnetosphere coupling [2], [4], [10]. It is clear that in actual fact they are simply parts of one and the same global ionospheric-magnetospheric current system.

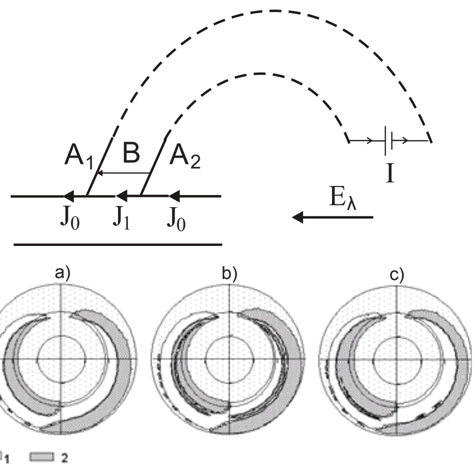

Figure 2. Ionosphere-magnetosphere coupling. Field-aligned currents generated in the magnetosphere: I - zone of inflow currents, 2 - zone of outflow currents; a) t=0 s; b) t=1000 s; c) t=2800

The problem of ionosphere-magnetosphere coupling primarily implies that it is necessary to solve the question as to how the magnetospheric producer of current and power “adjusts itself” to the ionospheric consumer. For a certain special configuration, this problem was solved in [5], [8], [10]. It turned out, firstly, that the ionospheric consumer updates the convection rate and through it the plasma pressure gradient, which determines the density of bulk currents which, in turn, determines the behavior of field-aligned currents through its divergence. Secondly, it turned out that ionospheric and magnetospheric currents are not rigidly linked. Some of the current (and power!) that is not “demanded” by the ionosphere can go into feeding the MHD compressor pumping plasma into the region of increase magnetic pressure - in the earthward direction. For us, the most important issue in this paper is that of ascertaining the direction of the cause-and-effect relationship. Current is primary in the magnetosphere, whereas the electric field is primary in the magnetosphere. Furthermore, the convection system can undergo some adjustment, and together with it the electric field in the ionosphere. But such adjustment is possibly only as corrections of the first approximation to the zero-order approximation. And hence the zero-order approximation, that is, the picture of field-aligned currents obtained essentially for an arbitrary but smooth initial electric field must contain the main elements of the natural system of field-aligned currents which is determine by the distribution of electron precipitation closely associated with the plasma pressure distribution in the magnetosphere.

The results presented in this study induce us to hope that the compatibility of field-aligned currents of magnetospheric and ionospheric origins is feasible. Perhaps the mystery of the substorm lies in the plasma pressure distribution, or more precisely, in the global redistribution of the plasma pressure on the night side of the magnetosphere. Direct observations of plasma distribution in the magnetosphere are faced with large difficulties, because pressure must be known everywhere in the plasma sheet at high resolution, which in situ satellites have been unable to provide. Modeling of distribution of plasma pressure (on ~ 3-12 R_E) is very important, because the data from multisatellite magnetospheric missions for these purposes would be a very expensive project. Therefore there arises a necessity for modeling processes of near-Earth space. A selection and applying of correct initial system of equations are also very important.

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