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A Short-Time Stationarity Test of Radio Signals From the Vela Pulsar Using Polyspectra

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Abstract

Polyphase filterbank data of the Vela pulsar is acquired using the MeerKAT radio telescope and analysed using the second-order frequency-frequency polyspectrum. The lack of components off of the non-stationary manifold indicates that the process is second-order stationary over timescales of 0.8 microseconds. Additionally, the complex process is found to be circular and thus requires only a small subset of the possible definitions of the higher-order moments in its characterisation.

1 Introduction

Pulsar radio signals are extraterrestrial transients which exhibit a number of exotic features. Unfortunately, these signals are faint and undergo adverse channel effects [1], which makes the estimation of many features difficult. Higher-order spectral analysis is a method for analysing non-Gaussian signals which, in theory, offers a high signal-to-noise ratio domain for signals corrupted by Guassian noise [2]. It is believed that radio pulsar signals may contain non-Gaussian components and so recent studies have investigated the application of higher-order spectral analysis on pulsar radio signals [3]. However, the typical computation of the higher-order spectra inherently assumes the stationarity of the signal under operation and the computation of the stationary spectra of non-stationary signals can result in a phenomenon referred to as dimensional-reduction aliasing [4]. The more general polyspectra are better suited for the analysis of non-stationary processes [5]. Additionally, signals channelised by means of a polyphase filterbank are complex signals and in this case there are multiple definitions of the higher-order spectra [6]. In this case it is unclear which definition should be used. In this work, a high time-frequency resolution observation of the Vela pulsar is made using the MeerKAT radio telescope. The data from the polyphase filterbank is tested for wideband stationarity on short timescales and also for intrachannel circularity. The results indicate that pulsar signals are proper, reducing the number of distinct definitions of the higher-order spectra. They are also second order stationary over short time periods. Various interesting features associated with radio pulsars are discussed in Section (2). The approach to higher-order spectra for non-stationary and complex processes is presented in Section (3) and in Section (4) real pulsar data is tested for statistical characteristics that decreases the complexity of future spectral analysis. Finally, the conclusion is provided in Section (5).

2 Radio Pulsars

Radio pulsars are observable as extremely regular periodic fluctuations in the local electromagnetic field strength. They were discovered in 1967 when periodic signals were observed at the output of a radiometer investigating scintillation [7]. The source of the fluctuations was soon after identified as rotating neutron stars [8, 9] and a model of a plasma-filled magnetosphere with open field lines was proposed as the radiation mechanism [10]; the corresponding signal model is amplitude-modulated Gaussian noise [11]. This source-signal model is often referred to as the "lighthouse" model and is in itself quite fascinating. However, pulsar radio signals often display features and behaviours which deviate from it and which have stirred up great curiosity. One such feature is the existence of fluctuations of pulse intensity that exist on timescales significantly smaller than the pulse width and quasi-periodicities within the pulses themselves [12]. Even stranger is the existence of features within individual pulses that account for a large proportion of the pulse energy but exist on the timescale of microseconds and shorter. These features have motivated an alternative model of radio pulsar signal as amplitude-modulated shot noise caused by bunches of coherently radiating particles [13, 14]. Determining whether or not pulsar radio signals are Gaussian, what timescale their most rapid fluctuations occur on and to what extent periodicity exists within individual pulses is made difficult by the fact that individual pulses typically lie well below the instrumental noise floor. Pulsar signals are also afflicted by adverse channel effects such as dispersive smearing and scintillation which smooths out fine features and increases the Gaussianity of non-Gaussian signals. The MeerKAT radio telescope is an incredibly sensitive instrument which has already proven itself to be a powerful tool for pulsar astronomy [15]. Even when using 54 dishes of the MeerKAT to observe Vela, a bright pulsar, the individual pulses are obscured enough to limit the extent to which subtle features are resolvable. This is illustrated in Figure 1. It is therefore common to study the integrated pulse intensity of pulsars instead the single pulse statistics. The normalised intensity profile of Vela as well as the integrated Stokes Q parameter are illustrated in Figure 2. The sweep in the polarisation parameter is typical of pulsars and is the result of a highly linearly polarised source moving through the observer's line of sight along the surface of a rotating sphere [16]. While the average profile provides high fidelity measurement of the average signal behaviour, it does little to uncover the single pulse statistics. Due to their ability to characterise non-Gaussian processes and to suppress Gaussian noise, higher-order spectra provide a potential means of studying the single pulse behaviour of pulsar radio signals.

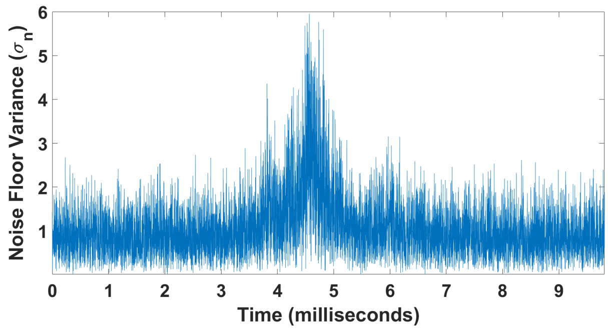

Figure 1. A single pulse from the Vela Pulsar observed with 54 MeerKAT dishes at 1650 MHz with a bandwidth of 835 kHz.

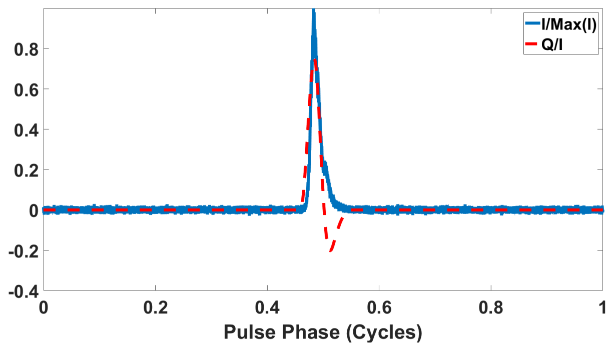

Figure 2. The I and Q Stokes parameters of the observation. The I parameter is normalised by the peak intensity and the Q parameter is normalised by the Stokes I parameter.

3 The Polyspectra of Non-stationary Complex Processes

The higher-order spectra are calculated as the multidimensional Fourier transforms of higher-order moment and cumulant functions. The nth-order moment spectrum of a continuous-time random function $x(t)$ is an $n-1$-dimensional function calculated as,

$$M_x^n(\omega_1,...,\omega_{n-1}) = \int_{\omega_1}\cdots\int_{\omega_{n-1}} m(\tau_1,...,\tau_{n-1})e^{-j\omega_1\tau_1}...e^{-j\omega_{n-1}\tau_{n-1}} d\omega_1...d\omega_{n-1}, \quad (1)$$

with the n_{th} order moment function defined as,

$$m_x^n(\tau_1,...,\tau_{n-1}) = E\{x(t)x(t-\tau_1)...x(t-\tau_{n-1})\}. \quad (2)$$

The τ variables are independent time lags and E is the expectation operator. The cumulant spectrum is defined similarly. These multidimensional functions are rich in information regarding the statistical nature of the transformed signal. Statistical tests such as a test for Gaussianity, a test for linearity [17] and a test for aliasing [18] make use of the higher-order spectra and provide useful tools for hypothesis testing on signals with unknown statistics. An equivalent definition of the moment spectrum in (1) is given in terms of the Fourier transform of process realisations $F(\omega)$,

$$M_x^n(\omega_1,...,\omega_{n-1}) = E\{F^*(\omega_1+...+\omega_{n-1})\prod_{k=1}^{n-1}F(\omega_k)\}, \quad (3)$$

which is the correlation between the product of $n-1$ Fourier components and the Fourier component at the corresponding sum frequency. This estimate of the higher-order spectra is often used due to the simplicity and efficiency of computation of the Fourier components using the Fast Fourier Transform (FFT) algorithm. It also provides intuition regarding the nature of the higher-order spectra and assists in their interpretation. However, the sum frequency in (3) is constrained and so there are only $n-1$ free variables in the function, even though the nth order moment function is in general a function of n independent variables. This is because the definitions of the spectra in (1) and (3) are the stationary moment spectra and are not adequate for the analysis of non-stationary processes, though they can be used for deterministic transient analysis. The generalised expansion of higher-order spectra analysis to the non-stationary case begins by expressing a continuous-time random process using the Fourier-Stieltjes integral,

$$x(t) = \int_{\omega=-\infty}^{\infty} e^{-j\omega t}dF(\omega), \quad (4)$$

where $dF(\omega)$ is the spectral process which describes $x(t)$. Substituting (4) into (2) and performing some algebra one attains that

$$m_x^n(t,\tau_1,...,\tau_{n-1}) = \int_{\omega}\cdots\int_{\omega_{n-1}} e^{-j(\omega+...+\omega_{n-1})t}e^{-j(\omega_1\tau_1+...+\omega_{n-1}\tau_{n-1})} .E\{dF(\omega)\prod_{k=1}^{n-1}dF(\omega_k)\}. \quad (5)$$

Now define the variable ν as

$$\nu = \omega + \sum_{k=1}^{n-1}\omega_k, \quad (6)$$

which allows (5) to be rewritten as

$$m_x^n(t,\tau_1,...,\tau_{n-1}) = \int_{\omega}\cdots\int_{\omega_{n-1}} e^{-j\nu t}e^{-j(\omega_1\tau_1+...+\omega_{n-1}\tau_{n-1})} .E\{dF^*(\nu-\sum_{k=1}^{n-1}\omega_k)\prod_{k=1}^{n-1}dF(\omega_k)\}. \quad (7)$$

From (7) we see that the general moment function only becomes the stationary moment function for $\nu = 0$, which results in the constrained argument of the Fourier product in (3). The implication is that in order for a random process to be stationary its spectral process is required to have orthogonal increments. Therefore, in general the Fourier transform of the nth order moment function is itself an n-dimensional function with the $\nu = 0$ hyperplane being what is referred to as the stationary manifold. The spectral process of the non-stationary n_{th} order moment function is thus given as,

$$M_x^n(\omega, \omega_1, ..., \omega_{n-1})d\omega...d\omega_{n-1} = E\{dF^*(\nu - \sum_{k=1}^{n-1} \omega_k) \prod_{k=1}^{n-1} dF(\omega_k)\}, \quad (8)$$

and any non-zero components on $\nu \neq 0$ are due to non-stationarities in the process. The computation of stationary moment spectra of a non-stationary process can yield inaccurate or misleading results. The need for the full number of dimensions in the spectral description is not uncommon. For example, a spectrogram is often required in order to observe the time evolution of power decomposition onto frequency and is the tool commonly used for pulsar analysis. When dealing with complex processes the task of computing the higher-order spectra is less straight forward. For example, knowledge of both the covariance and pseudo-covariance functions of a complex process is required for a full characterisation of the second-order statistics of the process. The pseudo-covariance of a complex continuous-time process is give as,

$$C_x^*(\tau) = E\{x(t)x(t-\tau)\} \quad (9)$$

It differs from the standard covariance function only in terms of the missing complex conjugation within the expectation operator. The issue of multiple moment function definitions grows with the order of the statistic of interest. For the nth order moment function of a complex process there are 2^n definitions depending on the number of complex conjugations and their positioning. Fortunately the concept of the circularity for complex distributions can be extended to complex processes [19]; a circular process is said to be proper and has non-zero moment functions only for definitions with an equal number of conjugations and non-conjugations in the product [4].

4 A Stationarity Test of the Data

The polyphase filterbank used to observe Vela spans the frequency band from 856 MHz to 1712 MHz with 1024 channels. The sampling rate of the digitizer is 1712 MSPS and the dump rate of the filterbank is approximately 0.8 microseconds. The filterbank data is separated into data chunks corresponding to individual pulse outcomes. For each phase of each outcome the second order polyspectrum is computed using (3) but excluding the constraint of the sum frequency. Finally, the polyspectral components are normalised to attain the second order polycoherence function. The polyspectra estimates for each phase are averaged over the pulse outcomes. Figure 3 shows the cyclic polycoherence of the Fourier component at 1367 MHz with a second component at a Frequency deviation of ν. The averaging only takes place over 50 pulse outcomes; it is apparent that the non-stationary region quickly dies away to zero while the stationary region is fully coherent. This trend is observed over the full band of the filterbank and for all phases, infering that the pulsar radio signal is second order stationary over timespans of 0.8 microseconds.

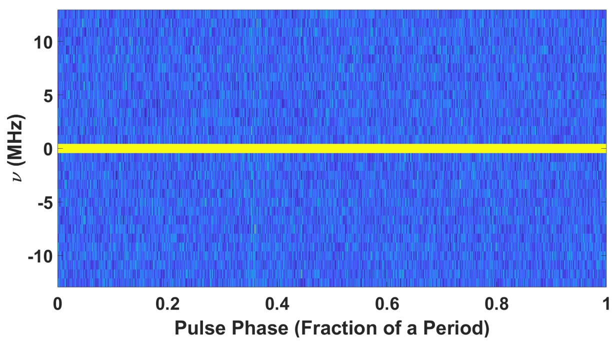

Figure 3. The second-order polycoherence of the Vela filterbank data centered around 1367 MHz.

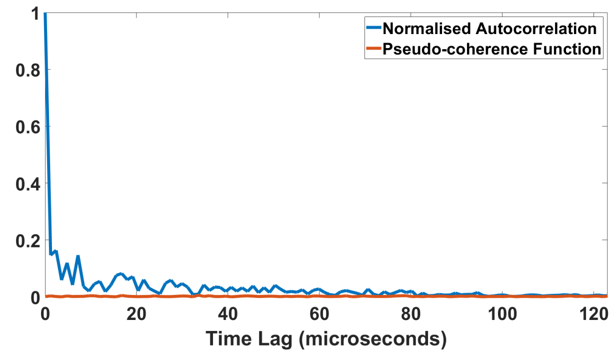

Figure 4. The nomalised autocorrelation function and pseudo-covariance function of the on-pulse Vela data.

A visual inspection of the distribution of the in-pulse signal over the complex plane shows no significant deviation from a circular distribution, which is found to be the case for all phases of the pulse cycle on the band of 1650 MHz. The pseudo-correlation function in (9) is computed for the on-pulse data in this band by means of a pulse-averaged sliding window function which is averaged over 100 pulses. The pseudo-covariance function is normalised to attain the pseudo-coherence function. This function is unity for fully coherent signals and zero for fully incoherent signals. The result is shown in Figure 4 and is on the order of magnitude of 10^{-3} which is a very strong indication of incoherence. The normalised autocorrelation function is also shown where the large component at zero lag indicates that the data contains features which are unresolved by the sampling rate.

5 Conclusion

The higher-order spectra provide rich information on a signal under transformation and are fundamentally linked to correlations of the spectral process. However, these spectra are often only calculated over the subdomain known as the stationary manifold and are less powerful than the full polyspectra when analysing non-stationary signals. Additionally the computation of the spectra of complex processes is made difficult through the existance of multiple definitions. Pulsar radio signals are extraterrestrial transients which have been known to display non-Gaussian behaviour and fluctuations over short time scales. In this work, an observation of the Vela pulsar has been performed using the MeerKAT, the data has been tested for short-time stationarity using the second order polyspectrum and is found to be second order stationary over a time span of 0.8 microseconds. Additionally, the data is found to be circular, decreasing the number of definitions of its higher order spectra. This new information allows for simpler and better informed calculation of the higher-order spectra in future work.

6 Acknowledgements

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