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Numerical implementation and properties of potential based integral equations

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The conventional formulation of scattering typically involves invoking the equivalence theorem, i.e., defining equivalent currents on the manifold, using Green's theorems and direct relations between fields on either side of the manifold to derive the requisite integral equations. Pretty much all the well known examples fall under this description, say the electric field integrals equations or the Müller and so on. All of these formulations suffer from both low-frequency and dense mesh breakdown, and there has been extensive work in developing methods to overcome these bottlenecks. These include preconditioned operators [1, 2], Debye sources [3] and more recently, decoupled potential [5] and decoupled field formulations [4]. The latter two are based on indirect formulations, in that the trace quantities defined on the boundary do not need to be related to physical quantities on the surface. As an aside, it is possible to recast them as direct formulations.

Our focus in this paper is examination of potential based integral equations (and by extension dual field integral equations). To date these methods have largely been subjected to analytic analysis or using Nystr$\acute{}$om methods [4]. As is evident from these papers these equations have a number of benefits. Discrete systems are well behaved at low frequency and immune from dense mesh breakdown. The principal challenge with these equations is the number of operators that are involved; the complexity and cost of these operators pose a significant bottleneck, especially when applied to tessellated structures. On these flat tesselated objects, these range from the nature of singular integrals that need to be evaluated. Furthermore, as shown in [5, 4], at high frequency, the condition number of these equations grows, much like that of the Müller equation and the Calderón preconditioned equations. Its is this set of problems that we seek to solve. Specifically, we will (a) develop integration rules so as to evaluate the all requisite operators, (b) develop fast methods so as to enable analysis of large structures, and (c) develop preconditioners so as to mollify the behavior these equations at high frequencies. For the latter, we will perform the analysis on both tessellated structures and spheres (using canonical basis sets) across a range of frequencies.

References

[1] G. C. Hsiao and R. E. Kleinman, "Mathematical foundations for error estimation in numerical solutions of integral equations in electromagnetics," *IEEE Transactions on Antennas and Propagation*, **45**, pp. 316-328, 1997.

[2] S. B. Adrian, A. Dely, D. Consoli, A. Merlini, and F. P. Andriulli, "Electromagnetic Integral Equations: Insights in Conditioning and Preconditioning," *IEEE Open Journal of Antennas and Propagation*, 2021.

[3] C. L. Epstein and L. Greengard, "Debye sources and the numerical solution of the time harmonic Maxwell equations," *Communications on Pure and Applied Mathematics*, **63**, pp. 413-463, 2010.

[4] F. Vico, L. Greengard and M. Ferrando, "Decoupled field integral equations for electromagnetic scattering from homogeneous penetrable obstacles," *Communications in Partial Differential Equations*, **43**, pp. 159–184, 2018.

[5] J. Li, X. Fu and B. Shanker, "Decoupled Potential Integral Equations for Electromagnetic Scattering From Dielectric Objects, " *IEEE Transactions on Antennas and Propagation*, **67**, pp. 1729–1739, 2019.