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Study on the Effect of using Different Weighting Techniques in a Time Scale Algorithm to Generate an Ensemble Time

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Abstract

It is important to maintain a traceable source of timing information as it is used in many applications. National timing laboratories of different countries are responsible to generate and maintain precision timing based on atomic and/or optical standards. Generally, an ensemble of clocks is used with a time scale algorithm that ensures a stable time compared to a single atomic clock. Time scale algorithms vary based on the techniques used to assign weights to the participating clocks. Two of such methods are compared in this paper. For an ensemble consisting of both Hydrogen-Maser and Cesium atomic clocks, it is found that the results of a fixed weight assignment method are comparable to the results of the existing AT1 method.

1. Introduction

Precision timing information has a myriad of applications including financial time stamping, data encryption and security, and maintenance of stable communication system [1]. International Bureau of Weights and Measurement (BIPM) realizes and disseminates the international reference for precision timing called as Co-ordinated Universal Time (UTC). Physical realizations of UTC – named as UTC(k) are maintained in national metrology institutes or laboratories, which contribute to UTC by sending their clock data to the BIPM in regular intervals (every 5 Julian days) [2]. There are more than 80 participating laboratories which maintains its UTC(k) using atomic clocks. With recent advancements, few laboratories have switched to optical clocks to maintain their respective UTC(k).

Generally, timing laboratories use Hydrogen-Masers and/or Cesium atomic clocks to maintain UTC(k); NIST (National Institute of Standards and Technology) uses more than 20 atomic clocks (2/3 of the atomic clocks are Hydrogen-Masers, and about 1/3 are Cesium beam clocks) to maintain UTC(NIST) [3]. When several atomic clocks are involved, generally a time scale algorithm is used to realize an ensemble time. In simple terms an ensemble time is a weighted average of the participating atomic clocks which is stable than the individual atomic clocks. There are several time scale algorithms like Algos, AT1 and Kalam-filter based algorithm [4 - 5] that are adopted by different

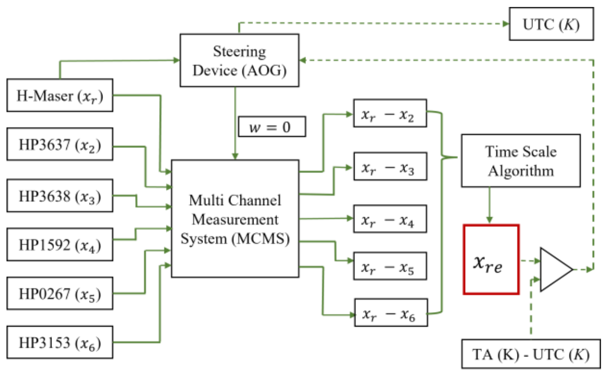

Figure 1. Simplified system diagram showing the signal flow between atomic clocks, Multi-Channel Measurement System (MCMS) and time scale algorithm. The parts in dashed lines are not realized for

laboratories. These algorithms differ on the way they assign the weights to the participating atomic clocks. Some use variances of the prediction error to determine the weights and some use Kalman filter to optimize the weight assignment. In this paper, we implement a time scale algorithm using a fixed weight based on Allan Variance of the atomic clocks and evaluate the results by comparing to the results from the AT1 algorithm.

2. Methodology

In this section, we will show the system diagram, briefly review the time scale algorithm, and discuss the metrics that will be used in the evaluation purpose.

2.1 System Description

Fig. 1 shows a general system diagram, which are adopted by different national timing institutes to generate an ensemble Atomic Time (TA). The system generally consists of multiple atomic clocks (which might include both Cesium and Hydrogen-Masers or only one type of atomic clocks), a Multi-Channel Measurement System (MCMS), and a time scale algorithm. Out of the given number of atomic clocks, generally the most stable one is chosen as a reference clock (x_r). As shown by Fig. 1, a multi-channel measurement system measures the phase difference between the designated reference clock and any clock 'j' of the ensemble ($x_r - x_j$). The multi-channel measurement system measures the differences in a fixed interval of time, every 'τ' seconds. These phase differences are then passed to a time scale algorithm which gives different weights, 'W_j' to the individual clocks and computes an ensemble time. The result of the time scale algorithm is reported as a difference of an individual clock w.r.t the ensemble time (x_{je}). Fig. 1 shows the results in terms of the difference between the reference clock and the ensemble time (x_{re}). This information is then used in a steering device like Auxiliary Output Generator (AOG) to correct the phase and frequency information of the input signal. The input to the AOG is generally the reference clock signal. The AOG then generates the physical signal, UTC(k). For this paper, the system is implemented and discussed until the generation of the ensemble time (x_{re}) only.

2.2 Timescale Algorithm

Here we briefly describe the time scale algorithm in a step wise step fashion using the following mathematical expressions.

Step1: Estimate the time difference (or phase difference) of clock 'j' w.r.t the ensemble time at epoch 'k'. Here y_{je} and d_{je} are the frequency difference and frequency aging factor respectively. The aging parameter is generally 0 for Cesium clocks and is in the order of $10^{-21}\,s^{-1}$ for Hydrogen-Masers [6].

$$\widehat{x_{je}}(k) = x_{je}(k-1) + y_{je}(k-1)\tau + 0.5 d_{je}(k-1)\tau^2 \quad (1)$$

Step 2: Record the time difference of clock 'j' w.r.t the reference clock 'r',

$$x_{rj}(k) = x_r(k) - x_j(k) \quad (2)$$

Step 3: Predict the time difference of reference clock 'r' w.r.t the ensemble using model of clock 'j' (eq. 1),

$$\widehat{x_{re}^{j}}(k) = \widehat{x_{je}}(k) + x_{rj}(k) \quad (3)$$

Step 4: Assign weights 'W' to the participating clocks and estimate the time difference between reference clock 'r' and the ensemble.

$$\widehat{x_{re}}(k) = \sum_{j=1}^{N} W_j(k)\, \widehat{x_{re}^{j}}(k) \quad (4)$$

$$x_{re}(k) = \widehat{x_{re}}(k) \quad (5)$$

Step 5: Update the time difference of clock 'j' w.r.t the ensemble.

$$x_{je}(k) = x_{re}(k) - x_{rj}(k) \quad (6)$$

Step 6: Calculate frequency of clock 'j' w.r.t the ensemble,

$$f_{je}(k) = \{x_{je}(k) - x_{je}(k-1)\}/\tau \quad (7)$$

Step 7: Update the frequency of clock 'j' w.r.t the ensemble,

$$y_{je}(k) = y_{je}(k-1) + \frac{1}{1+w_y}\left(f_{je}(k) - y_{je}(k-1)\right) + d_{je}(k-1)\tau \quad (8)$$

Where, $w_y = \frac{T}{\tau}$, the time constant, T is 4 days for Cesium standards and 10 days for high performance devices [6]. After the steps are completed, the same process is followed again from step 1 for next time epoch ($k+1$).

Here, the main difference between different time scale algorithms is based on how they estimate the weights for the respective atomic clocks. For AT1 algorithm, the weights are determined based on the estimated prediction error. In a recent paper [7], it is mentioned that if an ensemble consists of both Cesium clocks and Hydrogen-Masers, this method of weights assignment based on the errors can be biased towards the stable clock. Thus, the paper suggests on assigning weights based on the stability of the clocks. In this paper, we compute the weights based on stability of the individual clocks and compare the results to that obtained from the AT1 algorithm.

2.3 Allan Deviation

Allan Deviation (σ_y) is a commonly adapted method to express the stability of atomic clocks. In this paper, we use this metric to evaluate the results of the time scale algorithms. Allan Deviation is basically the square root of Allan Variance (σ_y^2). Eq. (9) shows the computation of Allan Variance given the frequency measurement data (y_i) of an oscillator. If phase measurement data (x_i) is given, eq. 7 can be used first for the conversion.

$$\sigma_y^2(\mathrm{m}\tau) = \frac{1}{2m^2(M-2m+1)} \sum_{j=1}^{M-2m+1} \{\sum_{i=j}^{j+m-1} [y_{i+m} - y_i]\}^2 \quad (9)$$

The Allan Variance is calculated with respect to an averaging window. Here, τ is the nominal time step between any two consecutive measurement data and 'm' is any integer multiple of 'τ' to find the stability at the desired time period. 'M' is the total sample size. Higher the value of Allan Deviation, lower is the stability of the oscillator.

3. Database and Data Processing

The data used in this paper are from time and frequency lab of National Metrology Centre (NMC), Singapore. NMC is responsible for maintaining UTC(SG) and it participates in maintaining UTC by sending its atomic clock data to BIPM. For this paper, the ensemble consists of 6 clocks (1 Hydrogen-Maser and 5 Cesium clocks). Fig. 1 shows the names used for these clocks. Hydrogen-Maser is taken as the reference clock and phase differences of rest of the clocks are measured against Hydrogen-Maser using the MCMS. The time difference (τ) between each measurement data is 720 seconds.

A total of 83 days' data (approximately 10,000 datapoints) is used for the evaluation of the results. The starting point is MJD 57737 (or 15th Dec 2016). MATLAB is used as the processing software to implement the algorithm and generate the results.

4. Results & Discussion

We run the algorithm (eq. 1 – 8) using different weights assignment methods and present the results. The results are compared and discussed in the following sub sections.

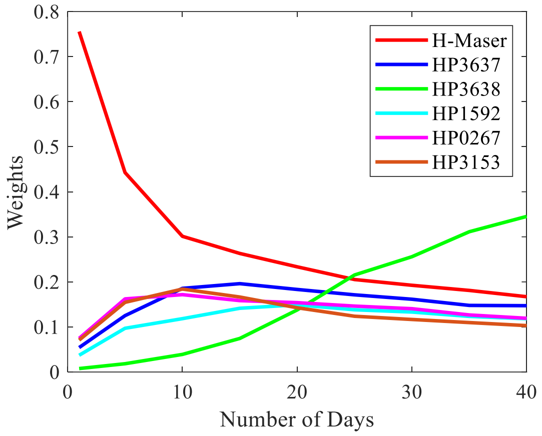

Figure 2. Weights assigned to different clocks based on the Allan Variance of the clocks. [Sum of the weights is equal to 1.]

4.1 Weights Determination

For **AT1 algorithm**, the weight to an individual clock is assigned based on the average prediction error of the respective clock over the previous cycle. The optimum weight of each clock is proportional to the inverse of the variance of the prediction error. This method becomes unstable if one of the clocks is more stable than the others. Therefore, the maximum weight that any clock in the ensemble can have, is limited to 0.3 [6]. Using these criteria, the weights are determined and results from the AT1 algorithm are shown in the next section.

For **Fixed Weight** method, the stability of the clocks is first studied, and a fixed weight is assigned to each of the clocks in the ensemble. The stability is determined based on the Allan Variance of each of the clocks (eq. 9). The sum of the total weights at a time is 1. Fig. 2 shows the weights assigned to each of the clocks in the ensemble w.r.t the averaging window (6 clocks in the ensemble as shown by Fig. 1). Different colors in the plot denote the individual clocks present in the ensemble. In general, it can be observed that the weights assigned to the Hydrogen-Maser is high for a lower averaging window and it decreases gradually. The weights assigned to the Cesium clocks are smaller in the beginning and it increases gradually with the averaging window. This pattern of weights follows the fact that the Hydrogen-Masers have good short-term stability and Cesium clocks have higher long-term stability.

4.2 Allan Deviation Plots

Here we present the Allan Deviation plots to evaluate the results obtained when using the AT1 algorithm and when assigning fixed weights to the clocks to generate the ensemble time. Fig. 3 (a) and (b) show these results for fixed weight and for AT1 algorithm respectively. For both Fig. 3 (a) and (b), x-axis shows the averaging window (in seconds) and y-axis show the Allan Deviation values. Different colors indicate the atomic clocks that are used in the ensemble. Allan Deviation values are calculated using eq. 9 taking frequency output from the algorithm.

The Allan Deviation values of Hydrogen-Maser is constantly lower compared to the values of Cesium clocks in both the figures, especially for a lower averaging window. The specific values are compared in Table 1. We can see that the 1 days' average Allan Deviation values are better than 10 days' Allan Deviation values for Hydrogen-Maser for both Fixed weight method and AT1 respectively. Whereas, the 10 days' Allan Deviation values are higher in case of Cesium clocks.

The Allan Deviation plots for all the atomic clocks follow a similar trend when a fixed weight method or the AT1 algorithm is used. For the Hydrogen-Maser, with a days' averaging period, a stability value of $6.4 \cdot 10^{-15}$ is realized when using the fixed weight method and a value of $5.8 \cdot 10^{-15}$ is realized when implementing the AT1 algorithm. Similarly, the results for Cesium clocks can be compared from Table 1. There is a very less difference between the two methods as shown by the reported values. Therefore, in case of an ensemble where there are both Hydrogen-Masers and Cesium atomic clocks, using a fixed weight assignment method has similar stability as compared to the AT1 algorithm, which assigns weights based on variance of the prediction error. The advantage of the fixed weight allocation method is that it is easy to implement and computationally efficient. It would be interesting to see the comparison of the results for the case when only one type of atomic clocks is involved in the ensemble.

Table 1. Allan Deviation values for different atomic clocks when implementing fixed weight method and AT1 algorithm for different averaing period.

Atomic Clocks	**Allan Deviation**			
	1 Days' Avg (10^{-14})		10 Days' Avg (10^{-14})	
	Fixed Wt.	AT1	Fixed Wt.	AT1
H-Maser	**0.64**	**0.58**	**0.74**	**1.20**
HP3637	1.94	2.32	0.63	0.69
HP3638	5.94	5.96	2.72	2.82
HP1592	2.59	2.49	1.09	0.96
HP0267	1.62	1.97	0.69	0.63
HP3153	1.92	2.13	0.63	0.54

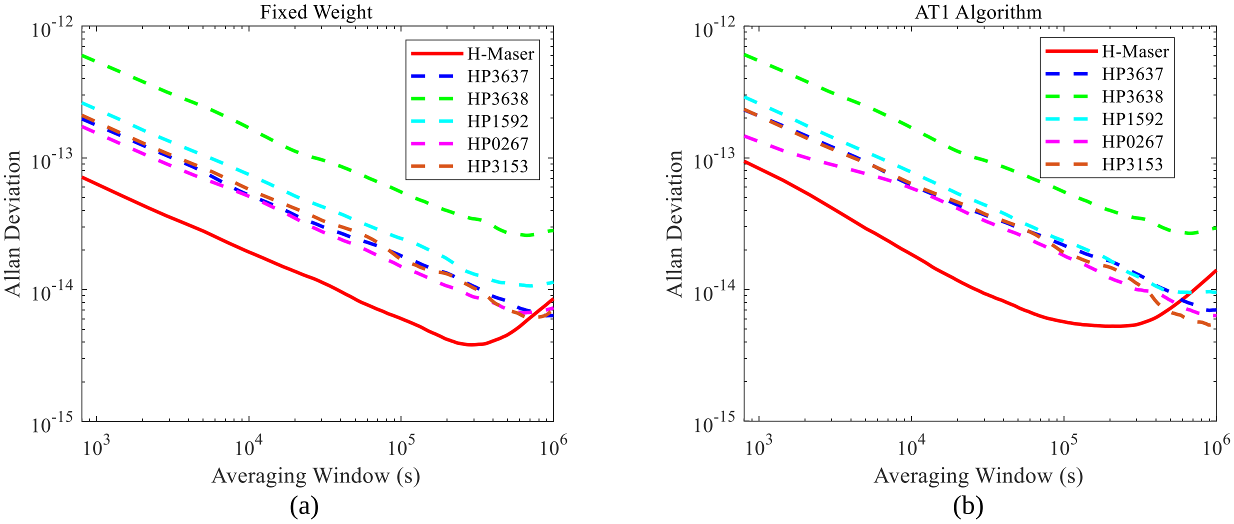

Figure 3. Allan Deviation plots when using (a) fixed weight allocation method and (b) AT1 algorithm. Different colors in the plot indicate the individual atomic clock used in the ensemble.

There can be other methods of determining the weights to calculate the ensemble time. Also, it depends on other factors like sample size considered to calculate the deviation values, chosen averaging window, number of atomic clocks in the ensemble, and types of atomic clocks (either all the same atomic clocks or mixed atomic clocks). Important things to consider while evaluating time scale algorithms are the computational efficiency of the algorithm and the required precision of the results.

5. Conclusion

In this paper, we implemented a time scale algorithm for an ensemble of clocks containing 1 Hydrogen Maser and 5 Cesium clocks. The algorithm used two different methods of assigning the weights. For an ensemble with both Hydrogen Maser and Cesium atomic clocks, using fixed weight method has similar stability as compared to results of AT1 algorithm.

The next step is to use the ensemble time i.e., x_{je} to steer an auxiliary output generator to obtain UTC(k) as shown by the system diagram in Fig. 1. Once the UTC(k) is realized, the values are sent to BIPM every 5 Julian days for computation of UTC. BIPM later publishes the UTC(k)-UTC values online. Currently, our research group is working on the prediction of UTC(k)-UTC ahead of time such that the UTC(k) can be corrected with resolution of better than 5 Julian days.

6. Acknowledgements

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