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An Efficient Iterative Scheme for HEMP Simulation with Consideration of Self-consistency

Ning Dong, Yan-zhao Xie, *Senior Member, IEEE*.
School of Electrical Engineering, Xi'an Jiaotong University

Abstract

Recent researches about the spatial distribution and statistical results of the high-altitude nuclear electromagnetic pulse (HEMP) put forwards requirements for the efficiency of the simulation. However, the simulation of HEMP with the consideration of self-consistency was complex and time-consuming including the interaction between the electrons and the electromagnetic (EM) fields. An efficient iterative scheme was proposed in this paper, the electrons and the EM fields were decoupled and simulated separately. The self-consistent effect was neglected in the first iteration and then taken into account by considering results of the EM fields in each iteration as the independent excitation field for electrons in the next iteration. The parallel computation strategy was practiced for the simulation of Compton electrons and air conductivity with the decoupled algorithm to further improve the efficiency up to tens times faster. The existence of the self-consistent effect was concluded to clearly weaken the Compton current and reduce the EM fields of HEMP.

1. Introduction

The generation of early-time high-altitude nuclear electromagnetic pulse (HEMP) has been widely studied and concluded to be a multiple physical processe caused by the prompt gamma radiation output of the high-altitude nuclear explosion[1]-[6]. The simulation of HEMP depends on the modeling of the high-energy Compton electrons produced by prompt gamma photons and low-energy secondery electrons produced by Compton electrons. Fast-moving Compton recoil electrons constitute Compton current and become the source of the HEMP. Secondary electrons act to attenuates the generation and propagation of HEMP. Meanwhile, Compton recoil electrons and secondary electrons are affected by the background electromagnetic (EM) fields. The generation of HEMP can be seen as a self-consistent process between electrons and the EM fields.

The self-consistent effect mainly referred to the interactions between the Compton recoil electrons and the EM fields. It should be noted that secondary electron mobility is also related to the EM fields in a different way[7].

Self-consistency play a significance role in the HEMP simulation. Literature reports that it not only shortens the duration time also reduces the peak EM fields amplitude significantly[8]. However, the self-consistent calculation methods share common features of higher complexity and more inefficiency than the non-self-consistent method. This self-consistent effect There was always a trade-off between accuracy and efficiency. Meanwhile, the researches focus on the spatial distribution and statistical results of the HEMP put forwards requirements for the simulation efficiency.

In this study, an efficient HEMP simulation method based on an iterative scheme was proposed to deal with the self-consistent effect between the EM fields and Compton electrons, and the interaction between EM fields and secondary electrons. Within the iterative scheme, the particles and EM fields were solved separately at first, assuming they were weakly coupled. Then the missing self-consistent interactions were corrected during the iterative process. The iterative simulation scheme can greatly improve the efficiency of the HEMP simulation.

2. Methodology

2.1. Self-consisitency Physical Model

A full computational HEMP model should include simulation of the Compton recoil and the secondary electrons and solution of the EM fields. Key physical parameters and their interaction relationship in HEMP model are illustrated in Fig. 2. In addition, when the self-consistent influence represented by the arrow with dashed lines was ignored, it degenerated into a non-self-consistent HEMP simulation.

2.2. Iterative Scheme for HEMP Simulation

An iterative scheme was proposed here to simulate HEMP with the consideration of self-consistency. In this method, the electrons and the EM fields were first solved separately, assuming the particle behaviour and EM fields were weakly coupled. Then, the missing coupling self-consistent interactions were corrected by the iterative process. The time-consumption of the proposed method was supposed to be lower than the established method. Parallel simulation strategy and SOR iteration method were utilized to further improve the efficiency.

The Compton electrons were calculated as test particles at first. Test particles mean the change of EM fields caused by the electrons was ignored during electron simulations, only the geomagnetic field was considered.

It should be noted that, when self-consistency was taken into consideration, the EM fields exerted on the Compton electrons were varied with both time and position. The Compton electrons generated at different times were affected by different EM fields. Therefore, the velocity of electrons generated at different times should be simulated differently. An auxiliary

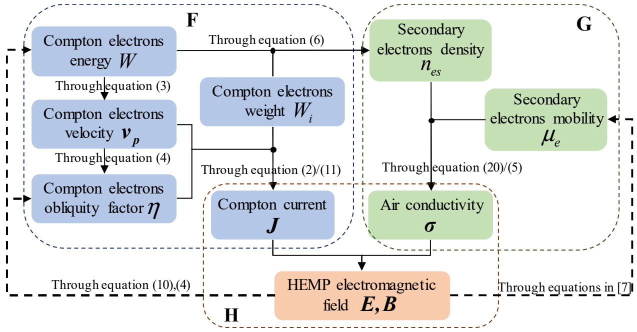

Fig. 2. Relationship between physical parameters. The equation before and after the slash are used in the case with and without the consideration of self-consistency, respectively.

variable u was thus introduced to describe the generation moment of electrons, described as

$$u = \tau - \tau' + (r - r')/c \tag{1}$$

The momentum equation in a retarded time system was rewritten as

$$\begin{aligned} m\gamma \frac{d\boldsymbol{v}_p(\tau')}{d\tau} &= -e\boldsymbol{E}(u+\tau'-(r-r')/c) \\ &\quad - e\boldsymbol{v}_p(\tau') \times (\boldsymbol{B}_e + \boldsymbol{B}(u+\tau'-(r-r')/c)) - g(W)\frac{\boldsymbol{v}_p}{|\boldsymbol{v}_p|} \end{aligned} \tag{2}$$

where $$(r - r')/c = \int_0^{\tau'} v_{p,r}(\tau')d\tau'$$

Therefore, the integral equation of Compton curren was altered as

$$\boldsymbol{J}_C(\tau) = \int_W -eW_i \int_0^{\tau} f(u) \frac{\boldsymbol{v}_p(\tau')}{\eta(\tau')} \frac{du}{1 - v_{p,r}(\tau')/c} \tag{3}$$

With this new equation, the Compton current considering the existence of EM fields can be simulated.

The simulation for the Compton current was cast in a compact operator as

$$\boldsymbol{J} = \mathbf{F}\ (\boldsymbol{E}, \boldsymbol{B}) \tag{4}$$

where operator $\mathbf{F}$ represents the above algebraic operations in Eq. (3) for each spatial finite element in the calculation area. The Compton current vectors in the time domain were obtained with the location EM fields as input.

As analyzed in Section II. B, the build-up of air density relied on the energy loss of the Compton current and local electric field. The secondary electrons were simulated as a consequence of Compton recoil electrons. The density of secondary electrons was simulated from the dynamic process including generation and vanishing.

The mobility of secondary electrons did not exist since the electric field equaled zero. In the first iteration, the air conductivity was calculated using a simplified air conductivity model [9].

When the interaction between EM fields and electrons was taken into consideration in the later iterations, the equilibrium ohmic model was used to simulate the sir conductivity.

The calculation of air conductivity was cast in a compact operator as

$$\boldsymbol{\sigma} = \mathbf{G}(\boldsymbol{J}, \boldsymbol{E}) \tag{5}$$

The electric field differential equation was solved based on the finite difference expression along radial direction for the transverse component and along time direction for the radial component, as

$$\begin{aligned} \boldsymbol{E}_{t,j,n+1} &= \frac{4 - c\mu_0 \Delta r \sigma_{j,n+1/2} - 2\Delta r / r_{n+1/2}}{4 + c\mu_0 \Delta r \sigma_{j,n+1/2} + 2\Delta r / r_{n+1/2}} \boldsymbol{E}_{t,j,n} \\ &\quad - \frac{4c\mu_0 \Delta r}{4 + c\mu_0 \Delta r \sigma_{j,n+1/2} + 2\Delta r / r_{n+1/2}} \boldsymbol{J}_{Ct,j,n+1/2} \end{aligned} \tag{6}$$

$$\begin{aligned} \boldsymbol{E}_{r,j+1,n} &= \frac{2 - \Delta t c^2 \mu_0 \sigma_{j+1/2,n}}{2 + \Delta t c^2 \mu_0 \sigma_{j+1/2,n}} \boldsymbol{E}_{r,j,n} \\ &\quad - \frac{2\Delta t c^2 \mu_0}{2 + \Delta t c^2 \mu_0 \sigma_{j+1/2,n}} \boldsymbol{J}_{Cr,j+1/2,n} \end{aligned} \tag{7}$$

where n and j refer to distance and time indices, respectively. It was the same for the solution of the magnetic field.

The solution of the EM fields differential equation was cast in a compact operator as

$$\boldsymbol{E}, \boldsymbol{B} = \mathbf{H}\ (\boldsymbol{J}, \boldsymbol{\sigma}) \tag{8}$$

Collected these functions, the HEMP was predicted using the following equation system

$$\begin{cases} \boldsymbol{J} = \mathbf{F}\ (\boldsymbol{E}, \boldsymbol{B}) \\ \boldsymbol{\sigma} = \mathbf{G}(\boldsymbol{J}, \boldsymbol{E}) \\ \boldsymbol{E}, \boldsymbol{B} = \mathbf{H}\ (\boldsymbol{J}, \boldsymbol{\sigma}) \end{cases} \tag{9}$$

Notably, the equation system only showed the relationship between the given variables and it did not mean that only the variables in the bracket were required in the HEMP simulation.

The solution of this system was supposed to satisfy all three

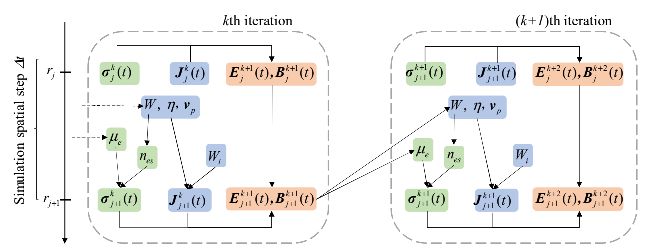

Fig. 1. Graphical depiction of the computation order of the HEMP physical parameters in the proposed iteration scheme with the consideration of self-consistency.

functions at the same time. Such that the self-consistent HEMP solver can be achieved by the iterative scheme. A fixed-point iterative scheme was built to solve the HEMP equation system. The evaluation of the Compton current, conductivity, and EM fields were carried out sequentially, as

$$\begin{cases} \boldsymbol{J}^k = \mathrm{F}\ (\boldsymbol{E}^k, \boldsymbol{B}^k) \\ \boldsymbol{\sigma}^k = \mathrm{G}(\boldsymbol{J}^k, \boldsymbol{E}^k) \\ \boldsymbol{E}^{k+1}, \boldsymbol{B}^{k+1} = \mathrm{H}(\boldsymbol{J}^k, \boldsymbol{\sigma}^k) \end{cases}, k = 1, 2, ... \tag{10}$$

with k referring to the iteration time. The iterative scheme carried out a sequence of calculations until the results converged to the correct solution.

The major steps of the iterative scheme were listed as follows:

1) In the first iteration, self-consistency was ignored. With only the geomagnetic field considered, The initial condition was given as

$$\boldsymbol{E}^{(1)}=0, \boldsymbol{B}^{(1)}=\boldsymbol{B}_e \quad (11)$$

Here, the simplified air conductivity model was used to obtain the initial value of $\boldsymbol{\sigma}$ instead of the equilibrium ohmic model. This can effectively speed up iterations.

2) Applying Compton current $\boldsymbol{J}^1$ and air conductivity $\boldsymbol{\sigma}^1$, the EM fields $\boldsymbol{E}^2$ and $\boldsymbol{B}^2$ were solved.

3) The iterations were continued to correct the missing interaction between electrons and EM fields and to consider the self-consistency between the electrons and the EM fields. The results of the EM fields were taken into the new simulation loop.

The Compton current $\boldsymbol{J}^k$ and air conductivity $\boldsymbol{\sigma}^k$ were simulated considering the existence of EM fields $\boldsymbol{E}^k$ and $\boldsymbol{B}^k$. The newly generated EM fields affected both electron velocities and obliquity factors. However, the weight and initial situation of Compton recoil electrons had nothing to do with the EM fields, which were consistent with the result in the first iteration. The density of secondary electrons was determined by kinetic energy loss and needed to be updated. The air conductivity $\boldsymbol{\sigma}^k$ was obtained using the equilibrium ohmic model.

4) Applying the Compton current $\boldsymbol{J}^k$ and air conductivity $\boldsymbol{\sigma}^k$, the EM fields $\boldsymbol{E}^{k+1}$ and $\boldsymbol{B}^{k+1}$ were obtained as the result of the *k*th iteration.

5) After the second iteration loop, the error of the EM fields was calculated for the whole simulation region between two iterations. Steps *3)* and *4)* were repeated until the error of the EM fields was less than a given error.

The numerical results of $\boldsymbol{J}^K$, $\boldsymbol{\sigma}^K$, $\boldsymbol{E}^{K+1}$ and $\boldsymbol{B}^{K+1}$ of the last iteration were considered as an expected solution for the HEMP simulation equation system in Eq. (9), which were regarded as a self-consistent solution for HEMP of the proposed iterative scheme

The order of the HEMP physical parameters computation in the iterative scheme was shown in Fig. 1. Compared to the graphical depiction for EXEMP, the simulation of electrons between spatial finite elements was decoupled. The simulation of the Compton current and air conductivity can be executed independently for each spatial finite element. This algorithm in the proposed iterative scheme made parallel execution possible.

2.3. Stability and Convergence Analysis

To achieve stability and convergence of the time domain difference solution of HEMP, the discretization of space and time should be constrained by the Courant-Friedrichs-Lewy (CFL) stability condition $2c\Delta\tau/\Delta r \le 1$. The time step size should be less than or equal to the time required for the EM wave passing through one spatial step.

Dispersion caused by the discretization of Maxwell's equations was avoided by limiting the size of the spatial step by wavelength, it should satisfy the condition $\Delta r < \lambda/10$. The upper frequency of HEMP was considered as 100 MHz [12]. Therefore, the time step which satisfied the CFL condition was set at $\Delta t = 0.1\text{ns}$, and the spatial step size was set at $\Delta r = 0.1$ m . In fact, the simulation results showed that the spatial step could be enlarged and still achieve a good result.

Convergence analysis of the iterative algorithm was divided into two parts: the convergence of the Compton current and convergence of air conductivity. The EM fields converged naturally as a result.

The convergence of Compton current was given qualitatively, with Compton current opposite to the direction of electron movement and electron velocity opposite to the EM fields. Therefore, the newly generated EM fields weakened the electron velocity and reduced the current. In later iterations, when the Compton current decreased, the EM fields decreased and the weakening effect of the EM fields on electron motion also decreased, which increased the Compton current in turn. The reverse was also the same. The Compton current tended to be balanced during iterations, as the changes of EM fields and current were mutually restricted.

Air conductivity was also affected by the electric field, but there were different rules. Its convergence was shown quantitatively. A large electric field led to low air conductivity that made the electric field even larger. The difference of the electric fields between two iterations was obtained as

$$\begin{aligned}\boldsymbol{E}_{r,n}^{k+1}-\boldsymbol{E}_{r,n}^{k} &= \boldsymbol{E}_{r,n-1}^{k+1}-\boldsymbol{E}_{r,n-1}^{k} \\ &\quad -\Delta\tau c^2\mu_0 eN_{es}\left(\mu(\boldsymbol{E}_{r,n}^{k})\boldsymbol{E}_{r,n}^{k}-\mu(\boldsymbol{E}_{r,n}^{k-1})\boldsymbol{E}_{r,n}^{k-1}\right) \\ &\le \sum_{m=1}^{n}\left|\Delta\tau c^2\mu_0 eN_{es}\left(\mu(\boldsymbol{E}_{r,m}^{k})\boldsymbol{E}_{r,m}^{k}-\mu(\boldsymbol{E}_{r,m}^{k-1})\boldsymbol{E}_{r,m}^{k-1}\right)\right|\end{aligned} \quad (12)$$

Let $\phi(E)=\mu(E)E$ and the inequality was obtained as

$$\boldsymbol{E}_{r,n}^{k+1}-\boldsymbol{E}_{r,n}^{k} \le \tau c^2\mu_0 eN_{es}\left|\boldsymbol{\phi}'_{\boldsymbol{max}}(\boldsymbol{E}_{r,m}^{k}-\boldsymbol{E}_{r,m}^{k-1})\right|$$

For the early-time component of HEMP, lasting for less than 1 μs, there was the inequality

$$\begin{aligned}&\tau c^2\mu_0 eN_{es}\left|\phi'_{max}\right|<1 \\ &\sigma_{r,n}^{k+1}-\sigma_{r,n}^{k}<\left|\sigma_{r,n}^{k}-\sigma_{r,n}^{k-1}\right|\end{aligned} \quad (13)$$

The convergence of air conductivity within the iteration was thus shown.

When the current and conductivity converged through iterations, the EM fields of HEMP converged as a consequence.

3. Numerical Validation

Several cases were simulated to validate the accuracy, robustness, and efficiency of the proposed iterative method.

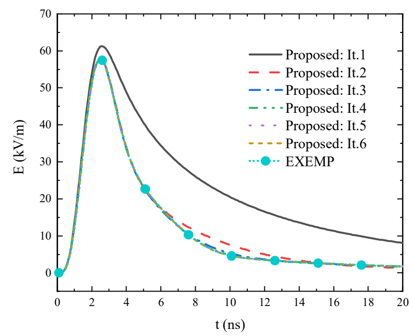

Fig. 3. Numerical results of the electric field(on the ground) time-domain waveform, simulated by the EXEMP method and the proposed iterative scheme.

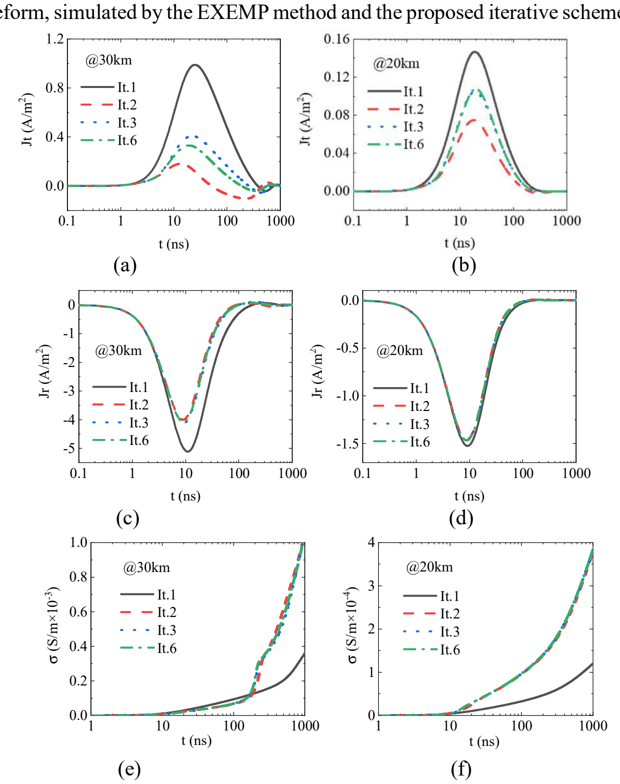

Fig. 4 Numerical results simulated by the proposed iterative scheme of transverse Compton current at heights of 30 and 20 km (a and b), radial Compton current at heights of 30 and 20 km (c and d), and air conductivity at heights of 30 and 20 km (e and f), the total prompt gamma dose was 3 kT.

3.1. Case 1: validation of the proposed iterative scheme

TABLE I

THE COMPUTATI TIME OF DIFFERENT CODE FOTHE A SAONME CASE WITH AND WITHOUT THE CONSIDERATION OF SELF-CONSISTENCY (UNIT: MINUTE)

Method	EXEMP	Proposed iterative scheme		
Number of processors	1	1	20	56
Non-self-consistent	1.7	1.7	1.1	1.0
Self-consistent	193.0	88.9	5.9	3.2

In the first case, the HEMP generated by a high-altitude nuclear explosion was simulated, with the burst at 100 km. The total prompt gamma dose was 3 kT, with the gamma photons monoenergetic and 1.6 MeV in energy. The value of the geomagnetic field was simplified as uniform in space and the field strength set at 50 μT. The atmospheric density was described by the exponential model. The value of this case was determined with reference to previous studies [10][11].

The time-domain waveforms of the total electric field, the Compton current, and air conduction were simulated with consideration of self-consistency by the proposed iterative scheme and the EXEMP method. The numerical results of the electric field time-domain waveform on the earth surface were shown in Fig. 3. After six times iterations, the convergence of the proposed iterative method matched well. The self-consistent effect had more impact on the wave tail, which was consistent with the historical conclusion. Ignoring the self-consistent effect in simulations may lead to higher amplitude and longer duration of HEMP.

3.2. Case 2: parallel computing optimization and efficiency analysis

In this case, the parallel computing strategy was offered to further improve the efficiency of the proposed iterative scheme.

For both methods, the calculation time increased with the consideration of consistency. Compared with the non-iterative EXEMP method, the computational efficiency is greatly improved up to 60 times with error of less than 1% by the proposed iterative scheme.

4. Conclusions

This paper developed an efficient iterative scheme for HEMP simulation by considering the self-consistent phenomenon in the HEMP generation process. The convergence of the iterative method was analyzed qualitatively and quantitatively. The proposed iterative scheme was accelerated by a parallel computing strategy. Compared with the non-iteration method, the proposed method improve the efficiency up to tens times faster.The proposed iterative scheme provided an efficient approach for transferring the non-self-consistent simulation into a efficiency self-consistent simulation.

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