



Quantitative phase and absorption contrast imaging

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Extended Abstract

Phase retrieval in its most general form is the problem of reconstructing a complex valued function from phaseless information of some transform of that function. This problem arises in various fields such as X-ray crystallography, electron microscopy, coherent diffractive imaging and astronomy. The mathematical and computational of these problems has a long history and a variety of different algorithms has been proposed in the literature. Gerchberg and Saxton [4], and later Fienup [3], introduced a scheme based on iterative projections which is simple to implement and proved to be very flexible in practice. These methods are fast but, due to the absence of convexity, they do not always converge to the true solution unless good prior information about the sought object is available. In recent years, Chai *et al.* [2] and Candés *et al.* [1] proposed an algorithm that lifts the phase retrieval problem to a higher dimension to make it convex. This type of algorithm solves a low-rank matrix linear system using nuclear norm minimization whose result always converges to the true solution, even without prior information of the object and independently of its complexity. However, when lifting the problem to a higher dimension, the dimension increases quadratically and the solution becomes infeasible if the scale of the problem is large.

Instead we vectorize the matrix linear system to write it as a linear vector equation, and use a *Noise Collector* [5] to reduce its dimensionality dramatically, thus being able to solve large problems as in [6]. The reduction of dimensionality is carried out in two steps mimicking the forms in which waves travel. In the first step of the proposed algorithm, we assume that the intensities add incoherently, and treat the coherent contribution to the data as a modeling error which is absorbed by a *Noise Collector* [5]. The *Noise Collector* is able to absorb this modeling error efficiently, even if it is large, provided the sought object is simple enough to be sparsely represented in a given basis. The second step takes into account the coherent contribution to the data. This step is crucial for phase contrast imaging that visualizes transparent, or semi-transparent, objects which are otherwise invisible.

In this way, the algorithm merges coherent and incoherent imaging, as it considers the coherent and incoherent contributions to the data sequentially. As a by-product, the proposed algorithm can also be used for phase retrieval with partially coherent observations without any modification and without loss of resolution, which is important for X-ray phase contrast imaging as fully coherent X-ray sources are hard to produce. Only the signal-to-noise ratio (SNR) of the created images is affected, as expected, when partially coherent observations are used. If the observations are fully incoherent, then the transparent objects cannot be recovered.

References

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