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Number of Sources Detection and AoA Estimation of a Wireless Transmitter in Multipath Channels

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Abstract

Detecting the number of sources from which the signal of a single wireless transmitter has originated, as well as estimating its angle-of-arrival (AoA), are critical parameters for a plethora of remote sensing applications (e.g. military, localization, radio-astronomy, etc.). A rather interesting observation is that these two problems are inter-twined since the most popular solution is based on eigenvalue decomposition (EGD) of the correlation matrix of the received signal. The Akaike information criterio (AIC) has been used in conjunction with EGD for detecting the number of sources or the number of AoAs (the later is a pre-requisite for AoA estimation). In this work we present an algorithm for a two-step application of AIC so that we can detect simultaneously the number of sources and AoAs for a MIMO transmitter that operates in a multipath environment. Results indicate the good performance of the proposed scheme for different multipath topologies and system configurations.

1 Introduction

Wireless digitally modulated signals always convey information beyond the data. If we limit the scope of our discussion to one transmitter, information that can be conveyed includes the number of independent sources that constitute the wireless signal, and the angle-of-arrival (AoA) of the signal at a receiver (Rx) of interest. It is interesting that the two problems are intertwined: A pre-requisite for solving the AoA estimation problem is the knowledge of the number of AoAs. Under certain cirumstances determining the number of involved wireless sources coincides with the number of AoAs which means that solving the first problem provides an input to solve the second. As we will see the later is not always the case which means that the number of sources and the number of AoAs must be detected independently. The question that concerns this paper is how to detect and estimate these two parameters when the signal goes thought a scattering environment that causes multipath.

The problem of detecting the number of simultaneously transmitting single-antenna wireless sources by using a linear combination of their signals was described by Kailath in [1], where a *non-parametric* information-theoretic approach was developed. The method uses the eigenvalues of the sample covariance matrix (without assuming any particular structure) for cacluating the Akaike Information Criterio (AIC) or the minimum description length (MDL) for model selection. The same approach, and the same AIC metric, has been used in Multiple-Input Multiple Output (MIMO) systems for determining the number of antennas of a single MIMO transmitter [2], which is essentially the equivalent problem with the one in [1]. Regarding the second problem, estimating the AoA can be accomplished in several different ways all of which use the received signal vector at an array of antennas (Eve in Fig. 1). One of the most popular class of techniques, referred to as *subspace methods*, exploit the structure of the received signal covariance matrix and offer very high angular resolution. The multiple signal classification (MUSIC) [3] algorithm, and ESPRIT [4], belong to this class of techniques. Besides subspace methods, techniques like the Bartlett and Capon/MVDR beamformers can also be used at the cost of lower angular resolution [5].

In this paper, under the assumption of a receiver that consists of a uniform linear array (ULA), and a Rayleigh multipath fading channel, we propose an algorithm for jointly estimating the number of antennas and the AoA of a single multi-source wireless transmitter. The main observation that drives this paper, and the associated algorithm, is that in multipath channels the received signal can reveal the number of antennas before we remove the impact of multipath, and the number AoAs afterwards. A concrete and extendable algorithm that exploits this observation is proposed.

2 System Model

The system model in Fig. 1 consists of a multi-source transmitter and potentially several scatterers (just one is illustrated in Fig. 1). Its receiver is not presented since it is irrelevant to our study. The data modulated signal has bandwidth B Hz and is assumed to be narrow-band, that is $B \ll f_c$ where f_c is the carrier frequency. The model also includes a ULA, which not part of the nominal communication system, hence an unauthorized receiver (URx). Our subsequent discussions on the signal models and the estimation algorithms concern the ULA.

Baseband Model: The ULA at the URx consists of N_{Rx} el-

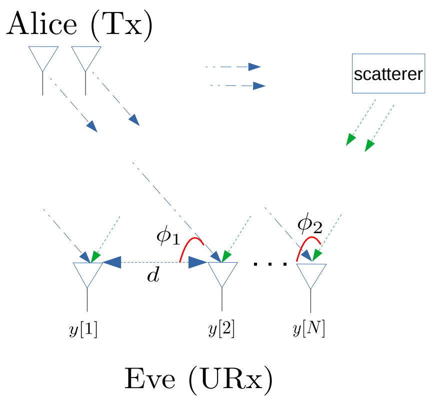


Figure 1. The wireless system with a multi-source transmitter and a ULA deployed at an unauthorized Rx.

ements spaced d meters apart. In a real-life setting, the URx is usually far away from the Tx so that the impinging waves at the ULA can be approximated with specular plane waves (arriving in parallel). We also assume a static Rayleigh flat fading channel, with complex fading coefficient h. Hence, for one path between the Tx and the URx the overall complex baseband channel gain is $h\exp(j\eta(\phi,\theta))$, where ϕ,θ are the AoA and AoD respectively. Here, without loosing generality we consider only the AoA. In the model we separate the two elements of the baseband channel gain into the Rayleigh complex gain and the steering vector of the ULA as described next. First, the baseband modulated signal of the multi-source transmitter is captured in the $N_{\text{Tx}} \times 1$ vector $\mathbf{s}$ while in our model it is replicated M times and packed in the $MN_{\text{Tx}} \times 1$ vector $\mathbf{x} = [\mathbf{s}^T \ldots \mathbf{s}^T]^T$, to capture the signal transmitted across the M paths[1]. The data in $\mathbf{s}$ are the data across the sources and are all assumed to be uncorrelated. Note that the $MN_{\text{Tx}} \times MN_{\text{Tx}}$ covariance matrix $\mathbf{C_x}=\mathbb{E}[\mathbf{x}\mathbf{x}^H]$ has rank N_{Tx}, i.e. it is not full rank. Hence, the received signal vector at the ULA is

$$\mathbf{y} = \mathbf{A}\mathbf{H}_{\text{iid}}\mathbf{x} + \mathbf{w}, \tag{1}$$

where $\mathbf{H}_{\text{iid}}$ ($MN_{\text{Tx}} \times MN_{\text{Tx}}$) is a block diagonal matrix that contains the iid samples of the Rayleigh fading channel:

$$\mathbf{H}_{\text{iid}} = \text{diag}(\underbrace{h_1 \ldots h_1}_{N_{\text{Tx}} \text{ copies}} \ldots h_M \ldots h_M)$$

$\mathbf{w}$ is the AWGN vector. $\mathbf{A}$ is the unknown $N_{\text{Rx}} \times MN_{\text{Tx}}$ steering matrix. Each column of $\mathbf{A}$ contains the steering vector that captures the phase difference between the received signal at each ULA element that originates from the i-th AoA:

$$\mathbf{a}^T(\phi_i) = [1 \;\; e^{j2\pi f_c \frac{d\cos\phi_i}{c}} \;\; \ldots \;\; e^{j2\pi f_c \frac{(N_{\text{Rx}}-1)d\cos\phi_i}{c}}] \tag{2}$$

In this model $d\cos(\phi_i)/c$ is the additional time required for the RF signal to travel between two antenna elements of the ULA (Fig. 1 clearly illustrates the geometry). Consequently, if we assume we have M AoAs this matrix is:

$$\mathbf{A} = \begin{bmatrix} 1 & .. & 1 \\ e^{j2\pi f_c \frac{d\cos\phi_1}{c}} & .. & e^{j2\pi f_c \frac{d\cos\phi_M}{c}} \\ \ldots & \ldots & \ldots \\ \underbrace{e^{j2\pi f_c \frac{(N_{\text{Rx}}-1)d\cos\phi_1}{c}}}_{N_{\text{Tx}} \text{ copies}} & .. & e^{j2\pi f_c \frac{(N_{\text{Rx}}-1)d\cos\phi_M}{c}} \end{bmatrix} \tag{3}$$

It is evident that if we have more than one sources at a single transmitter i then the column vector $\mathbf{a}(\phi_i)$ is repeated accordingly in this matrix.

3 Number of Antennas and AoAs Estimation

3.1 The AIC/MDL Metrics

AIC in an i.i.d. Channel: If there is no spatial correlation across the antennas of the ULA at the Rx, then the model in (1) is simplified and the result is the well-known i.i.d. MIMO channel:

$$\mathbf{y} = \mathbf{H}_{\text{iid}}\mathbf{x} + \mathbf{w} \tag{4}$$

For $\mathbf{H}_{\text{iid}}$ it is rank($\mathbf{H}_{\text{iid}}$)=min($N_{\text{Rx}}, MN_{\text{Tx}}$) because the number of rows in $\mathbf{H}_{\text{iid}}$ is now N_{Rx}. The $N_{\text{Rx}} \times N_{\text{Rx}}$ *signal* covariance matrix is $\mathbf{C_s}$=$\mathbf{H}_{\text{iid}}\mathbf{C_x}\mathbf{H}_{\text{iid}}^H$. It will also be rank($\mathbf{C_s}$)=min($N_{\text{Rx}}, N_{\text{Tx}}$). Hence, if $N_{\text{Rx}} \geq N_{\text{Tx}}$ the rank gives us the number of simultaneously transmitting sources [2]. This is the basic principle that was used for solving the problem of finding the number of single antenna sources described by Kailath [1]. A practical concern is that we only have access to $\widehat{\mathbf{C}}_\mathbf{y}$[2] and not to $\mathbf{C_s}$. This is a problem because the eigenvalues of the covariance matrix include the ones from the noise subspace. In [1] the authors considered this impact of AWGN by using the eigenvalues of $\widehat{\mathbf{C}}_\mathbf{y}$ as a definitive metric. So if l_i indicates the i-th eigenvalue of the covariance matrix, the AIC is calculated as:

$$\text{AIC}(m) = -2(N_{\text{Rx}} - m)T\log\left(\frac{\prod_{i=m+1}^{N_{\text{Rx}}} l_i^{1/(N_{\text{Rx}}-m)}}{\frac{1}{N_{\text{Rx}}-m}\sum_{i=m+1}^{N_{\text{Rx}}} l_i}\right) + 2m(2N_{\text{Rx}} - m) \tag{5}$$

The estimated number of sources is given as the value of m that minimizes the AIC, i.e. Hence, for the iid channel the AIC metric can be used for estimating the number of sources when $N_{\text{Rx}} \geq N_{\text{Tx}}$.

For our model the *signal* covariance matrix is $\mathbf{A}\mathbf{H}_{\text{iid}}\mathbf{C_x}\mathbf{H}_{\text{iid}}^H\mathbf{A}^H$. Also rank($\mathbf{C_s}$)=min($N_{\text{Rx}}, N_{\text{Tx}}, M$). *Hence, if $N_{Rx} \geq M > N_{Tx}$ the rank gives us the number of simultaneously transmitting sources.*

[1] Note that were are not interested in decoding data, hence all modulated source signals are packed in the same vector $\mathbf{x}$.

[2] The covariance matrix is estimated as $\widehat{\mathbf{C}}_\mathbf{y}$ from the data with an unbiased estimator.

3.2 Spatial Smoothing

Spatial smoothing algorithms [5] separate the ULA into L subarrays. With spatial smoothing the objective is to restore the rank of $\mathbf{C_s}$. The smoothed estimate for L subarrays is:

$$\bar{\mathbf{C}}_{\mathbf{y}}^{(L)} = \mathbf{A}\mathbf{H}_{\text{iid}}\bar{\mathbf{C}}_{\mathbf{x}}^{(L)}\mathbf{H}_{\text{iid}}^{H}\mathbf{A}^{H} + \sigma^2\mathbf{I} \tag{6}$$

This means that even if smoothing restores the rank of the source covariance matrix to MN_{Tx}, the rank M of $\mathbf{A}$ is not affected. *This is central to the second step of the algorithm, which proposes to calculate AIC after spatial smoothing since we can recover M as the number of AoAs.*

4 AoA Estimation with MUSIC

A ULA Rx can calculate the AoA of wireless signals that are linearly superposed by exploiting the difference in the AoA of the signals at different antennas spaced at known locations (Fig. 1). Subspace processing methods like the MUSIC algorithm have been used in the literature for extracting the AoA from different types of wireless signals [3, 6]. Essential information for MUSIC to work is the knowledge of the number of AoAs which is something that can be accomplished with AIC.

Although the basic step of MUSIC is to perform Eigenvalue Decomposition (EVD) on $\widehat{\mathbf{C}}_{\mathbf{y}}$, we delve a little deeper into it next. The covariance matrix of the received signal $\mathbf{y}$ is:

$$\begin{aligned}\mathbf{C_y} &= \mathbb{E}[(\mathbf{y}-\mathbb{E}[\mathbf{y}])(\mathbf{y}-\mathbb{E}[\mathbf{y}])^H] = \mathbf{A}\mathbf{H}_{\text{iid}}\mathbf{C_x}\mathbf{H}_{\text{iid}}^{H}\mathbf{A}^{H} + C_{\mathbf{w}}\\ &= \mathbf{A}\mathbf{H}_{\text{iid}}\mathbf{C_x}\mathbf{H}_{\text{iid}}^{H}\mathbf{A}^{H} + \sigma^2\mathbf{I}\end{aligned} \tag{7}$$

The covariance matrix of the *signal component* is $\mathbf{C_s} = \mathbf{A}\mathbf{H}_{\text{iid}}\mathbf{C_x}\mathbf{H}_{\text{iid}}^{H}\mathbf{A}^{H}$. For MUSIC we want $M < N_{\text{Rx}}$ which makes $\mathbf{C_s}$ singular, i.e. non-invertible:

$$\det(\mathbf{A}\mathbf{H}_{\text{iid}}\mathbf{C_x}\mathbf{H}_{\text{iid}}^{H}\mathbf{A}^{H}) = \det(\mathbf{C_y} - \sigma^2\mathbf{I}) = 0$$

From linear algebra we know that for a matrix $\mathbf{C_s}$ there are $\dim(\mathbf{C_s}) - \text{rank}(\mathbf{C_s})$ vectors that satisfy:

$$\mathbf{A}\mathbf{H}_{\text{iid}}\mathbf{C_x}\mathbf{H}_{\text{iid}}^{H}\mathbf{A}^{H}\mathbf{q_m} = 0, \tag{8}$$

i.e. these vectors are the solution set of the previous linear system. But this also means that $\mathbf{q_m}$ is an eigenvector of $\mathbf{A}\mathbf{H}_{\text{iid}}\mathbf{C_x}\mathbf{H}_{\text{iid}}^{H}\mathbf{A}^{H}$ for the zero-eigenvalue (i.e., $\mathbf{A}\mathbf{H}_{\text{iid}}\mathbf{C_x}\mathbf{H}_{\text{iid}}^{H}\mathbf{A}^{H}\mathbf{q_m} = 0 \times \mathbf{q_m}$). Furthermore from (7), (8):

$$\mathbf{A}\mathbf{H}_{\text{iid}}\mathbf{C_x}\mathbf{H}_{\text{iid}}^{H}\mathbf{A}^{H}\mathbf{q_m} = (\mathbf{C_y} - \sigma^2\mathbf{I})\mathbf{q}_m = 0$$

Hence, the zero-value eigenvectors $\mathbf{q_m}$ are also eigenvectors of $\mathbf{C_y}$ and they all have the same eigenvalue σ^2 (this is the noise subspace). Regarding the remaining non-zero eigenvalue eigenvectors of $\mathbf{C_s}$ let us assume that they satisfy $\mathbf{A}\mathbf{H}_{\text{iid}}\mathbf{C_x}\mathbf{H}_{\text{iid}}^{H}\mathbf{A}^{H}\mathbf{q_m} = \lambda_m\mathbf{q_m}$. To calculate all the eigenvectors of $\mathbf{C_y}$ we proceed based on the last expression:

$$\mathbf{A}\mathbf{H}_{\text{iid}}\mathbf{C_x}\mathbf{H}_{\text{iid}}^{H}\mathbf{A}^{H}\mathbf{q_m} = \lambda_m\mathbf{q_m} \Rightarrow \tag{9}$$

$$\mathbf{A}\mathbf{H}_{\text{iid}}\mathbf{C_x}\mathbf{H}_{\text{iid}}^{H}\mathbf{A}^{H}\mathbf{q_m} + \sigma^2\mathbf{I}\mathbf{q}_m = \lambda_m\mathbf{q_m} + \sigma^2\mathbf{I}\mathbf{q}_m \overset{(7)}{\Rightarrow}$$

$$\mathbf{C_y}\mathbf{q_m} = (\lambda_m + \sigma^2\mathbf{I})\mathbf{q}_m \tag{10}$$

The last derivation indicates that matrix $\mathbf{C_s}$ shares all its non-zero-eigenvalue eigenvectors $\mathbf{q_m}$ (as captured by (9)), with the ones of $\mathbf{C_y}$ while their eigenvalues differ by σ^2.

Next, we perform EVD of $\widehat{\mathbf{C}}_{\mathbf{y}}$[3] from which we obtain the two categories of eigenvectors $\mathbf{q_m}$ for the signal and noise sub-spaces that we discussed in the last paragraph. Recall that we assume that there are M AoAs we want to resolve, so the matrices that contain the eigenvectors are $\mathbf{Q}_1 = [\mathbf{q}_1, \ldots, \mathbf{q}_M]$, while the eigenvectors for the zero-value eigenvalues are contained in $\mathbf{Q}_2 = [\mathbf{q}_{M+1}, \ldots, \mathbf{q}_{N_{\text{Rx}}}]$. So $\mathbf{Q}_2$ is a space spanned by the zero-value eigenvectors.

The basic observation is that the noise sub-space is orthogonal to signal space, i.e. $\mathbf{a}^H(\phi)\mathbf{Q}_2 = 0$. This allows us to calculate the MUSIC pseudo-spectrum:

$$P_{\text{MUSIC}}(\phi) = \frac{1}{\mathbf{a}^H(\phi)\mathbf{Q}_2^H\mathbf{Q}_2\mathbf{a}(\phi)} \tag{11}$$

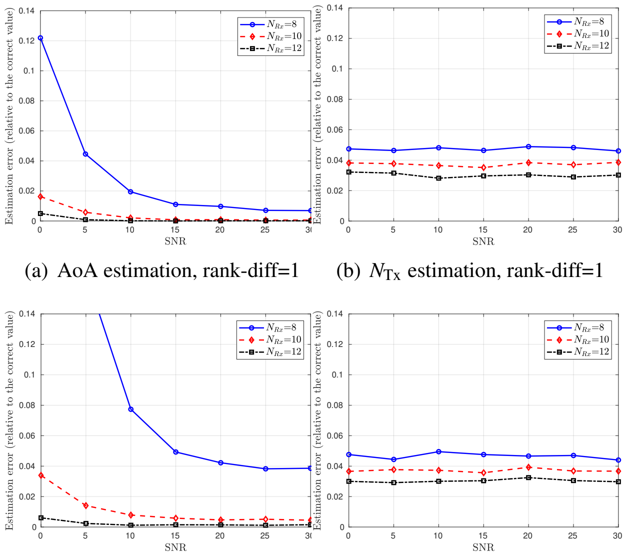
The peaks in $P_{\text{MUSIC}}(\phi)$ contain the AoAs. A more efficient method is the ESPRIT algorithm that is still based in the same fundamental steps [4].

5 Discussion & Algorithm

Based on the presentation of the AIC metric and the MUSIC algorithm we can reach our first conclusion for the signal model in (1). It is already clear that *the number of simultaneously transmitting sources is not always equal to the rank of* $\mathbf{C_s}$ *since this matrix is now affected by the number of AoAs in* $\mathbf{A}$. More precisely, $\text{rank}(\mathbf{A}\mathbf{H}_{\text{iid}}\mathbf{C_x}\mathbf{H}_{\text{iid}}^{H}\mathbf{A}^{H})$ is affected mainly by two matrices (since $\mathbf{H}_{\text{iid}}$ has dimensions $MN_{\text{Tx}} \times MN_{\text{Tx}}$ and is full rank). Clearly, the rank of $\mathbf{C_s}$ depends on the number of uncorrelated source signals in the vector $\mathbf{x}$ and the number of AoAs in $\mathbf{A}$. Example: If I have a Tx with 2 sources, over 4 paths (including LOS), and each source is transmitting an independent bitstream, then N_{Tx}=2, $\text{rank}(\mathbf{C_x})$=2, and $\text{rank}(\mathbf{A}\mathbf{H}_{\text{iid}}\mathbf{C_x}\mathbf{H}_{\text{iid}}^{H}\mathbf{A}^{H})$=2 because $\mathbf{A}$ contains 4 linearly independent columns. That is $\text{rank}(\mathbf{C_s})=\min(\text{rank}(\mathbf{A}), \text{rank}(\mathbf{C_x}))$. In this example before spatial smoothing AIC will give the result 2 since it calculates $\min(\text{rank}(\mathbf{A}), \text{rank}(\mathbf{C_x}))$. But even if this is the incorrect input for MUSIC, the estimation of the number of sources is correct. *Consequently, AIC estimation provides the number of AoAs and sources only when we have more paths than sources, and in the opposite case it provides only the number of AoAs.*

The proposed algorithm is summarized next. During the first steps the covariance matrix is estimated from the data snapshot, and then the number of sources is estimated with the AIC metric as $\widehat{N}_{\text{tmp}}$ according to (5). $\widehat{N}_{\text{tmp}}$ is a temporary result since the number of AoAs must also be estimated so that the algorithm determines whether it can decide on the

[3]Note that as a covariance matrix $\mathbf{C_x}$ is Hermitian $\Rightarrow$ $\mathbf{C_s}$ is also a covariance matrix, hence Hermitian, since $\mathbf{s}$ is the result of linear processing of $\mathbf{x}$. For uncorrelated sources $\mathbf{C_x}$ is diagonal, full rank, and consequently $\mathbf{A}\mathbf{C_x}\mathbf{A}^H$ is already a valid diagonalization of $\mathbf{C_s}$.



(a) AoA estimation, rank-diff=1 (b) N_{Tx} estimation, rank-diff=1

(c) AoA estimation, rank-diff=2 (d) N_{Tx} estimation, rank-diff=2

Figure 2. Estimation results.

number of sources (as discussed in the final part of 3.1). Next, spatial smoothing is performed. For spatial smoothing to work the length of the sub-array L must be higher than the number of correlated signals which in our case is equal to the number of paths M (including LOS). But M is unknown at this stage, so L is set to the maximum value that it can take for an array of such size which is $N_{\mathrm{Rx}} - 2$. Next we recalculate AIC with the smoothed covariance matrix for estimating $\widehat{M}$, and finally calculate the MUSIC pseudo-spectrum that provides the actual AoAs in the $M \times 1$ vector $\hat{\boldsymbol{\theta}}$. The algorithm reaches a conclusion for the number of sources only when the condition $\widehat{N}_{\mathrm{tmp}} < \widehat{M}$ is satisfied.

6 Simulation Results

To evaluate the performance of our algorithm we configured a ULA with critically spaced antennas, i.e. d=$\lambda/2$ (a necessary requirement for MUSIC), and we considered different number of ULA antenna elements N_{Rx}. The transmitted signals of B=1MHz were BPSK and modulated a 5GHz carrier. We also assumed we had access to 10 snapshots of $\mathbf{y}$ for estimating the covariance matrix (in practice this means 10 samples of the same symbol and same complex channel gain h). The objective of our evaluation is to estimate the number of sources and AoAs when the signal experiences scatterers at different random locations that cause different AoAs because of multipath. Hence, for each specific ULA SNR we tested our joint estimator against 100 different transmitters at different AoAs and equipped with different N_{Tx}. The transmitters across each simulation run were uniformly and randomly distributed so that the AoAs are also uniformly distributed. In the y axis we plot the estimation error relative to the correct value of the parameter of interest versus different receiver SNR in x axis.

In the first set of results we considered that the actual number of paths in the randomly generated topologies is on average 1 more than the number of used sources in each topology e.g., 3 paths 2 sources, 5 paths 4 sources, etc. This allows us to evaluate how the performance of the algorithm depends on the rank of $\mathbf{C_s}$. Our results in Fig. 2(a) indicate that for increasing SNR and number of antenna elements at the ULA the AoA estimate with MUSIC is improved as expected. Regarding the number of sources estimation error illustrated in Fig. 2(b) SNR does not have significant impact. But note that the performance is already very good for low SNR. The reason for this is that AIC is robust to higher noise power (lower SNR) as the metric in (5) indicates (eigenvalues of the noise subspace are easily excluded).

Next, we examine a rank difference of 2, e.g. real values M=4 and N_{TX}=2, M=5 and N_{Tx}=3, etc. Now MUSIC performs worse but for higher N_{Rx} the implications of larger difference between the number of paths/AoAs and N_{TX} is not so important. The reason for the worse performance is purely because of the performance of the smoothing algorithm, since the AIC performance remains the same as Fig. 2(d) indicates. Smoothing cannot de-correlate the signals so effectively leading to worse performance for MUSIC, but as we said this can be alleviated with more antenna elements at the ULA.

7 Conclusions

Estimating the number of sources and number of AoAs is not trivial with methods like AIC since the result correct only under certain channel conditions. In this paper we identified certain multipath scenarios ($M > N_{\mathrm{Tx}}$) where we can apply the AIC metric twice allowing us thus to solve the joint problem of number of sources and AoA estimation with high accuracy.

References

[1] M. Wax and T. Kailath, "Detection of signals by information theoretic criteria," *IEEE Trans. on ASSP*, vol. 33, no. 2, pp. 387–392, 1985.

[2] O. Somekh *et al.*, "Detecting the number of transmit antennas with unauthorized or cognitive receivers in mimo systems," in *MILCOM 2007*.

[3] R. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Transactions on Antennas and Propagation*, vol. 34, no. 3, pp. 276–280, 1986.

[4] R. Roy and T. Kailath, "Esprit-estimation of signal parameters via rotational invariance techniques," *IEEE Trans. on ASSP*, vol. 37, no. 7, pp. 984–995, 1989.

[5] H. Van Trees, *Optimum Array Processing*. Wiley-Interscience, 2002.

[6] M. Arash *et al.*, "Localization efficiency in massive mimo systems," Tech. Rep., 2020.