



## High-resolution, quantitative signal subspace imaging for synthetic aperture radar

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### Extended Abstract

Synthetic aperture imaging is used in many applications such as ultrasonic non-destructive testing, mine detection, surveillance, and radar imaging. The main idea behind synthetic aperture imaging is that a single transmitter/receiver is used to probe an unknown region by emitting known pulses into the medium and recording the time-dependent responses as it moves along a given path. Fourier transforming these time-dependent measurements yields their corresponding frequency responses. In this work we focus our attention on the synthetic aperture radar imaging problem. However, the methodology used here can be directly applied to other related problems.

We consider synthetic aperture radar imaging of a region containing point-like targets. To image the targets, we introduce a modification and generalization of MUSIC (multiple signal classification) for SAR imaging. MUSIC is a sampling method that has been widely used in several imaging applications [1, 3, 4]. Because signal subspace imaging methods do not work on the SAR measurements directly, we rearrange the frequency-dependent data at one measurement location as a matrix using the Prony method [5]. Then we form a block-diagonal matrix with the set of Prony matrices from all spatial locations on the flight path. An image of the reflectivity on the ground is then formed using a signal subspace method applied to this block-diagonal matrix. This signal subspace method is a generalization of MUSIC that projects the illuminating vector for each point  $y$  in the imaging region on both the noise and signal subspaces [2]. The noise subspace provides high spatial resolution and the signal subspace provides quantitative information about the targets. The result of combining these two subspaces is a high-resolution quantitative imaging method. The relative balance between the noise and signal subspaces depends on the noise level in the data which is controlled through a user-defined regularization parameter,  $\varepsilon$ .

There are two main results in this paper. The first main result is the resolution analysis for this modified and generalized MUSIC method that shows an enhancement in resolution compared to classical SAR imaging by a factor  $\sqrt{\varepsilon}$ . Namely we obtain a cross-range resolution of  $O(\sqrt{\varepsilon}(c/B)(L/a))$  and a range resolution of  $O(\sqrt{\varepsilon}(c/B)(L/R))$ . Here  $c$  denotes the speed of the waves,  $B$  denotes the bandwidth,  $a$  denotes the synthetic array aperture,  $L$  denotes the distance from the center of the flight path to the center of the imaging region, and  $R$  denotes the range offset. The second main result is the stability analysis of the method to random perturbations of the travel times. This analysis shows that the method provides stable reconstructions when  $\varepsilon$  is chosen to satisfy  $\sigma^2 \ll \varepsilon < 1$  with  $\sigma^2$  denoting the maximum variance of the random perturbations of the travel times. Our numerical simulations are in agreement with these theoretical findings.

### References

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