



Control of Scattering Singularities by Means of Complex Time Delay

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Control of wave scattering in arbitrary complex systems is a goal of modern science and technology, ranging from telecommunications to wireless power transfer to mesoscopic transport. We utilize the scattering matrix formalism to relate a vector of incoming waves amplitudes and phases to the resulting outgoing waves after passing through the complex system. Lately, there has been renewed interest in the properties of the scattering matrix in the complex frequency plane.[1] This landscape is decorated with the poles and zeros of the scattering matrix, most of which lie off of the real frequency axis. Identifying the locations of these features gives tremendous insight into the scattering properties of the system, and the movement of these features in the complex plane as the system is perturbed is also of great fundamental and practical interest. Knowledge of pole/zero information has practical application in the design of microwave circuits, transmission through mesoscopic structures, and the creation of embedded eigenstates, among many other examples. Perturbing a given system and bringing a scattering zero to the real axis enables coherent perfect absorption of all excitations incident on the scattering system. Engineering the collision of zeros and poles to create new types of scattering singularities is also of interest for applications such as sensing.

We have introduced a complex generalization of the Wigner-Smith time delay τ_W for subunitary scattering systems.[2] We utilize theoretical expressions for complex time delay as a function of excitation energy, uniform and nonuniform loss, and coupling and find very good agreement between theory and experimental data taken on microwave graphs containing an electronically variable lumped-loss element. We find that the time delay and the determinant of the scattering matrix share a common feature in that the resonant behavior in $\text{Re}[\tau_W]$ and $\text{Im}[\tau_W]$ serves as a reliable indicator of the condition for coherent perfect absorption (CPA). The use of complex time delay provides a means to identify the poles and zeros of the scattering matrix from experimental data. We have also established an approach to achieving CPA at an arbitrary frequency in complex scattering systems.

As an application, we study the statistical properties of τ_W for a subunitary wave-chaotic scattering system. We demonstrated that the mean value of the $\text{Re}[\tau_W]$ distribution function for a system with uniform absorption strength η is equal to the fraction of scattering matrix poles with imaginary parts exceeding η . This was tested experimentally with an ensemble of microwave graphs with either one or two scattering channels and variable uniform attenuation.[3] As another application, we identify the poles and zeros of the scattering matrix of a simple quantum graph by means of systematic measurement and analysis of Wigner, transmission, and reflection complex time delays. We examine the ring graph because it displays both shape and Feshbach resonances, the latter of which arises from an embedded eigenstate on the real frequency axis. Our analysis provides a first-principles understanding of sharp resonant scattering features, and associated large time delay, in a variety of practical devices, including photonic microring resonators, microwave ring resonators, and mesoscopic ring-shaped conductor devices. We also provide a prescription for maximizing the real part of all complex time delays.

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