Energy Efficiency Optimization for Radar-Communication Coexistence with Statistical CSIT

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Abstract

In this paper, we investigate maximizing the energy efficiency (EE) of a single-cell massive multiple-input multiple-output (MIMO) communication system coexisting with a surveillance radar with statistical channel state information at the transmitter (CSIT). EE is maximized under the constraint of the received signal-to-disturbance ratio (SDR) on the radar side. Since the problem involves coupled variables, alternating optimization is adopted to decompose the original problem into communication and radar optimization. The communication optimization problem can be simplified via the proposed power allocation algorithms. For the radar optimization, there exists a closed-form solution for the radar transmit power. Simulation results verify the performance of the proposed algorithm.

1 Introduction

As the communication spectrum gradually expands to millimeter waves and terahertz bands, which coincides with the traditional sensing spectrum, using radar-communication coexistence (RCC) for spectrum sharing is quite important [1]. In order to increase spectral efficiency and reduce energy consumption, optimizing EE seems to be a meaningful choice. Most of the existing research on EE optimization of RCC uses instantaneous channel state information (CSI). However, in practice, it is difficult to acquire the instantaneous channel state information at the transmitter (CSIT), especially in the massive multiple-input multiple-output (MIMO) downlink [2]. Note that obtaining statistical CSI that changes relatively slowly from the BS is generally not a challenging task [3]. Therefore, the statistical CSI seems to be a better choice.

This paper approaches the maximization of EE with statistical CSI in the scene of RCC. We assume there exists a surveillance radar and a massive MIMO system. The objective function EE is maximized while the received signal-to-disturbance ratio (SDR) is used to guarantee the performance of the radar. To reduce the computational complexity, the problem then is divided into communication and radar sides. The radar problem has a closed-form solution for radar transmit power. By applying an iterative power allocation method, the problem of the communication side can be handled in low complexity. Numerical results validate the feasibility of the proposed method.

2 System Model

We consider an RCC scene including a surveillance radar and a single-cell downlink massive MIMO communication system on the same frequency bandwidth. We assume an \( M_t \)-antenna BS transmits signals to \( K \) users each with \( M_k \) antennas for the communication module. The signal for user \( k \) is \( x_k \), and the signal emitted by BS is \( x = \sum_k x_k \in \mathbb{C}^{M_t \times 1} \). We assume that \( x_k \) and \( x_{k'} \) are independent for \( k \neq k' \). The covariance matrix of transmit signal for the \( k \)-th user is \( \mathbf{Q}_k = \mathbb{E}\{x_k x_k^*\} \in \mathbb{C}^{M_t \times M_t} \). For the surveillance radar, it emits a modulated pulse with pulse repetition time \( T \) and average transmit power \( P_r \), and the number of unambiguous range cells is \( N = 1 \). We use the sequence \( q = |q(0)|^T \in \mathbb{C} \) whose length is denoted by \( L \) to modulate the pulse, and we normalize \( q \) as \( ||q||^2 = 1 \) [4].

The communication signal received by the \( k \)-th user is written as

\[
\mathbf{r}_k = \mathbf{H}_k \mathbf{x} + \sqrt{P_r} q \mathbf{a} + \mathbf{w}_k \in \mathbb{C}^{M_t \times 1},
\]

where \( \mathbf{H}_k \in \mathbb{C}^{M_t \times M_t} \) is the communication channel matrix from the BS to user \( k \), \( \mathbf{a} \sim \mathcal{N}(0_{M_t}, \sigma_a^2 \mathbf{I}_{M_t}) \) is a vector representing the amplitude of the radar echoes that receive antennas received, \( \mathbf{w}_k \sim \mathcal{N}(0_{M_t}, \sigma_w^2 \mathbf{I}_{M_t}) \) represents the received Gaussian noise vector of the \( k \)-th user.

The jointly correlated Rayleigh fading model is adopted to describe the channel model [5], and then the channel matrix from the BS to the \( k \)-th user \( \mathbf{H}_k \) can be defined as

\[
\mathbf{H}_k = \mathbf{U}_k \mathbf{G}_k \mathbf{V}_k^H,
\]

where \( \mathbf{U}_k \in \mathbb{C}^{M_t \times M_t}, \mathbf{V}_k \in \mathbb{C}^{M_k \times M_k} \) are deterministic unitary matrices, representing the eigenvectors of the receive correlation matrix and the BS correlation matrix of \( \mathbf{H}_k \), respectively. \( \mathbf{G}_k \) is referred to as the beam domain channel matrix [6]. The statistical CSI of \( \mathbf{G}_k \) is defined as

\[
\mathbf{\Omega}_k = \mathbb{E}\{\mathbf{G}_k \odot \mathbf{G}_k^*\} \in \mathbb{R}^{M_t \times M_t}.
\]

For massive MIMO channels, as \( M_t \to \infty \), \( \mathbf{V}_k \) in (2) can be well approximated as [7]

\[
\mathbf{V}_k \approx_{M_t \to \infty} \mathbf{V}.
\]
where \( V \) is irrelevant to the users’ locations and is dependent on the antenna array geometry of BS [7].

We consider \( w'_k \) as a worst-case design [8] Gaussian noise
\[
 w'_k = \sum_{i \neq k} H_i x_i + \sqrt{P_t} q \alpha + w_k. \tag{5}
\]
The covariance of \( w'_k \) is written as
\[
P_k = \sum_{i \neq k} \mathbb{E}\{H_i Q_i H_i^H \} + P_t |q|^2 \sigma_{a_i}^2 I_{M_k} + \sigma_{u_i}^2 I_{M_k} \in \mathbb{C}^{M_k \times M_k}. \tag{6}
\]

The ergodic rate of user \( k \) can be written as [9]
\[
 \mathbf{R}_k = \mathbb{E}\{ \log \det(\mathbf{P}_k + H_k Q_k H_k^H) \} - \log \det(\mathbf{P}_k)
 = \mathbb{E}\{ \log \det(\mathbf{P}_k + G_k V^H Q_k V G_k^H) \} - \log \det(\mathbf{P}_k), \tag{7}
\]
where the second equality comes from expanding \( H_k \) according to formulae (2), (4) and later employing Sylvester’s determinant identity, i.e., \( \det(\mathbf{I} + \mathbf{MN}) = \det(\mathbf{I} + \mathbf{NM}) \). Moreover, \( \mathbf{P}_k \) in (7) is defined as
\[
 \mathbf{P}_k = P_t |q|^2 \sigma_{a_i}^2 I_{M_k} + \sigma_{u_i}^2 I_{M_k} + \sum_{i \neq k} \mathbb{E}\{G_k V^H Q_i V G_k^H\} = \mathbf{Y}_i \mathbf{Q} \mathbf{Y}_i^H. \tag{8}
\]

Considering the independently distributed properties of \( G_k \) elements, \( \mathbf{Y}_i(X) \) can be proved to be diagonal [2]. The diagonal elements of \( \mathbf{Y}_i(X) \) are written as
\[
 [\mathbf{Y}_i(X)]_{m,m} = \text{tr}\left\{ \text{diag}\left\{ (\mathbf{Q}_i)_{m,:} \right\} \mathbf{X} \right\}. \tag{9}
\]

The radar receiver simultaneously forms \( V \) orthogonal beams, so the number of azimuth bins is \( V \). Note that there is only one range bin since \( N = L = 1 \). Therefore, while omitting the range bin, the resolution cell is \( j \in \mathcal{X} \) where \( \mathcal{X} \subseteq \{1 \ldots V\} \) [4]. Assume that in the \( v \)-th azimuth bin, there is a point target with delay \( T_r \). The radar signal received in the \( v \)-th beam can be written as
\[
 y_j = \sqrt{P_t} g_v q + \sum_{\tau = 0}^{\infty} \mathbf{P}_v \mathbf{x}_\tau + \sqrt{P_t} \gamma_v + u_v, \in \mathbb{C}, \tag{10}
\]
where \( g_v \in \mathbb{C}, \gamma_v \in \mathbb{C} \) denote the echo amplitudes of the point target and radar clutters in the \( v \)-th azimuth bin with zero means and variances \( \sigma_{g_v}^2 \) and \( \sigma_{\gamma_v}^2 \), respectively; \( \mathbf{P}_v = [\beta_{1,v}, \ldots, \beta_{M_v,v}]^T \), where \( \beta_{m,v} \in \mathbb{C} \) denotes the amplitude of communication signal from antenna \( m \) in the \( v \)-th azimuth bin with zero mean and variance \( \sigma_{\beta_{m,v}}^2 \); \( s \) denotes the echoes that separated by delay \( sT \); \( \mathbf{x}_\tau = [x_{1,v}, \ldots, x_{M_v,v}]^T \), where \( x_{m,v} \) is the communication signal transmitted by antenna \( m \); \( u_v \in \mathbb{C} \) represents the received noise with zero mean and variance \( \sigma_{u_v}^2 \).

Then the received SDR at resolution cell \( v \) is written as [4]
\[
 \text{SDR}_v = \frac{P_t \sigma_{g_v}^2 |q|^2}{P_t \sigma_{\gamma_v}^2 |q|^2 + C + \sigma_{u_v}^2}, \tag{11}
\]
where \( C \) is given by
\[
 C = \mathbb{E}\left\{ \left| \sum_{d = 0}^{\infty} \mathbf{P}_v^T x_d \right| \right\}^2 = \text{tr}\left\{ \begin{bmatrix} \sigma_{\beta_{m,v}}^2 & 0 & \cdots \\ 0 & \sigma_{\beta_{m,v}}^2 & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \mathbf{Q} \right\}. \tag{12}
\]
Moreover, \( \mathbf{Q} \) in (12) denotes the sum of \( \mathbf{Q}_k, k = 1, \ldots, K \).

In order to calculate EE, we consider a three-part power consumption model [2], i.e.,
\[
 P_{\text{tot}} = \eta \sum_k \text{tr}(\mathbf{Q}_k) + M_t P_d + P_t, \tag{13}
\]
where the scale factor \( \eta \) denotes the transmit amplifier inefficiency, \( \sum_k \text{tr}(\mathbf{Q}_k) \) denotes the whole power budget, \( P_d \) represents the dynamic power loss of each antenna, \( P_t \) denotes the static circuit power consumption.

We define \( \rho_v \) as the minimum required SDR in resolution bin \( v \). Then, the optimization problem is defined as
\[
 \max_{\rho_v \in \mathbb{R}} \frac{\sum_k \mathbf{R}_k}{\eta \sum_k \text{tr}(\mathbf{Q}_k) + M_t P_d + P_t} \quad \text{s.t.} \quad Q_k \geq 0, \forall k, \sum_{k=1}^{K} \text{tr}(\mathbf{Q}_k) \leq P_{r,\text{max}}, \quad \text{SDR}_v(\mathbf{Q}, P_r) \geq \rho_v, \forall v \in \mathcal{X}, \quad 0 \leq P_r \leq P_{r,\text{max}}. \tag{14}
\]

### 3 Proposed Approach

Problem (14) involves coupled variables, which is quite challenging, so the alternating optimization is employed to disassemble the problem into some reduced complexity sub-problems.

The sub-problem on the radar side is written as
\[
 \max_{P_r \in \mathbb{R}} \text{EE}(\mathbf{Q}, P_r) \quad \text{s.t.} \quad 0 \leq P_r \leq P_{r,\text{max}}, \quad \text{SDR}_v(\mathbf{Q}, P_r) \geq \rho_v, \forall v \in \mathcal{X}. \tag{15}
\]

From (11), constraint \( v \) can be written as
\[
 P_r \geq \frac{\rho_v (C + \sigma_{\gamma_v}^2)}{\sigma_{\gamma_v}^2 |q|^2 - \rho_v \sigma_{\beta_{m,v}}^2 |q|^2}. \tag{16}
\]

From the expression of EE, we can see that EE is strictly decreasing as \( P_r \) increases, and choose the minimum \( P_r \) that meets the radar constraints. We have
\[
 P_r = \max_{v \in \mathcal{X}} \frac{\rho_v (C + \sigma_{\gamma_v}^2)}{\sigma_{\gamma_v}^2 |q|^2 - \rho_v \sigma_{\beta_{m,v}}^2 |q|^2}. \tag{17}
\]
The sub-problem on the communication side is written as
\[ \mathcal{P} : \max_{\mathbf{Q}_1, \ldots, \mathbf{Q}_K} \mathbb{E} \] 
subject to \[ \sum_{k=1}^{K} \text{tr}(\mathbf{Q}_k) \leq P_{\text{c, max}}, \] \[ \mathbf{Q}_k \succeq \mathbf{0}, \forall k. \] \tag{18} 

In order to work out the problem \( \mathcal{P} \) more easily, we can apply eigenvalue decomposition to disassemble the covariance matrix of signals for the \( k \)-th user into \( \mathbf{Z}_k = \mathbf{\Psi}_k \mathbf{A}_k \mathbf{\Psi}_k^H \). The eigenmatrix \( \mathbf{\Psi}_k \) denotes the subspace where the transmit signal lies in. The elements of the matrix \( \mathbf{A}_k \) denote the power corresponding to each dimension/direction of the subspace for the transmit signals [2]. Taking the characteristics of the massive MIMO into consideration, we can identify \( \mathbf{\Psi}_k \) in the proposition below.

**Proposition 1:** The optimal eigenmatrix \( \mathbf{\Psi}_k \) in problem \( \mathcal{P} \) is given by \( \mathbf{V} \) in (4), i.e.,
\[ \mathbf{Q}_k^{\text{optimal}} = \mathbf{V} \mathbf{\Lambda}_k \mathbf{V}^H. \] \tag{19} 

The proof of proposition 1 can be obtained using a similar approach in [2], and we omit it in this paper. Problem \( \mathcal{P} \) then can be simplified as a power allocation problem. The problem is written as
\[ \mathcal{P}_1 : \max_{\mathbf{\Lambda}} \frac{\Sigma_k (r_k^{+}(\mathbf{A}) - r_k^{-}(\mathbf{A}))}{\eta \Sigma_k \text{tr}(\mathbf{A}_k) + M_t P_d + P_t} \] 
subject to \[ \Sigma_k \text{tr}(\mathbf{A}_k) \leq P_{\text{c, max}}, \] \[ \mathbf{\Lambda}_k \succeq \mathbf{0}, \forall k. \] \tag{20} 

where we have
\[ \mathbf{\Lambda} = \{A_1, \ldots, A_k\}, \]
\[ r_k^{+}(\mathbf{A}) = \mathbb{E}\{\log(\det(\mathbf{P}_k(\mathbf{A}) + \mathbf{G}_k \mathbf{A}_k \mathbf{G}_k^H))\}, \] \tag{21} 
\[ r_k^{-}(\mathbf{A}) = \log(\det(\mathbf{P}_k(\mathbf{A}))). \]

Note that while calculating problem \( \mathcal{P}_1 \), computing the expectation values is quite resource-consuming. We exploit the deterministic equivalent of the expectation to replace the original one. Then, \( r_k^{+}(\mathbf{A}) \) in (20) can be recast as
\[ r_k^{+}(\mathbf{A}) = \log(\det(\mathbf{E}_k + \mathbf{F}_k(\mathbf{A}))) + \log(\det(\mathbf{I}_{M_t} + \mathbf{Z}_k \mathbf{A}_k)) - \text{tr}(\mathbf{I}_{M_t} - \mathbf{\Phi}_k^{-1}). \] \tag{22} 

Similarly to (9), it is shown that the diagonal elements of \( \mathbf{\Gamma}_k(\mathbf{X}) \) are given by
\[ \mathbf{[\Gamma}_k(\mathbf{X})]_{l,l} = \text{tr}\{\mathbf{diag}\{(\mathbf{\Omega}_l)|_{l,l}\} \mathbf{X}\}. \] \tag{24} 

After using the deterministic equivalent (DE) method, we can find that \( r_k^{+}(\mathbf{A}) \) and \( r_k^{-}(\mathbf{A}) \) are both concave over \( \mathbf{A} \), which leads to a non-convex numerator, so directly using traditional fractional programming methods may exhibit an exponential complexity [2]. Therefore, we resort to the majorization-minimization (MM) algorithm. We can find the first-order Taylor expansion of \( r_k^{+}(\mathbf{A}) \) denoting by \( \Delta r_k^{+}(\mathbf{A}) \). Note that \( r_k^{+}(\mathbf{A}) \leq \Delta r_k^{+}(\mathbf{A}) \). After replacing \( r_k^{+}(\mathbf{A}) \) with \( \Delta r_k^{+}(\mathbf{A}) \), the numerator of the objective function is lower-bounded by a concave term. Then, problem can be reformulated as
\[ \mathcal{P}_2^{(l)} : \max_{\mathbf{A}} \frac{\Sigma_k (r_k^{+}(\mathbf{A}) - \Delta r_k^{+}(\mathbf{A}))}{\eta \Sigma_k \text{tr}(\mathbf{A}_k) + M_t P_d + P_t} \] 
subject to \[ \Sigma_k \text{tr}(\mathbf{A}_k) \leq P_{\text{c, max}}, \] \[ \mathbf{\Lambda}_k \succeq \mathbf{0}, \forall k. \] \tag{25} 

where
\[ \Delta r_k^{+}(\mathbf{A}) = r_k^{+}(\mathbf{A}^{(l)}) + \text{tr}(\mathbf{\Theta}_k^{(l)}(\mathbf{A}_k - \mathbf{A}_k^{(l)})) \] \tag{26} 
\[ \mathbf{\Theta}_k^{(l)} = \{\mathbf{\Theta}_k^{(l)}\}_{l,k}. \]

In (26), \( l \) represents the iteration times. In addition, \( \mathbf{\Theta}_k^{(l)} \), the derivative of \( \frac{\partial}{\partial \mathbf{A}_k} \Sigma_k r_k^{+}(\mathbf{A}^{(l)}) \), is written as
\[ \mathbf{\Theta}_k^{(l)} = \sum_{k' \neq k} \Sigma_{l' \neq l} \frac{\mathbf{R}_k^{l,l'}}{P_t |q|^2 \sigma_n^2 + \Sigma_{k'} \text{tr}(\mathbf{A}_k)} \] \tag{27} 
where \( \mathbf{R}_k^{l,l'} = \text{diag}(\mathbf{\omega}_k^{l,l'}) \) and \( \mathbf{\omega}_k^{l,l'} \) denotes the \( t \)-th row of \( \mathbf{\Omega}_k \). \( \mathbf{\Theta}_k^{(l)} = \sum_{k' \neq k} |\mathbf{A}_k^{(l)}| \). Note that \( \mathbf{\Theta}_k^{(l)} \) is diagonal and the \( n \)-th diagonal elements are given by
\[ [(\mathbf{\Theta}_k^{(l)})]_{n,n} = \sum_{k' \neq k} \sum_{l' \neq l} \frac{|\mathbf{\Omega}_k^{l,l'}|}{P_t |q|^2 \sigma_n^2 + \Sigma_{k'} \text{tr}(\mathbf{A}_k^{(l)})} \] \tag{28} 

Since the objective of \( \mathcal{P}_2^{(l)} \) is a concave-convex fractional function, we resort to Dinkelbach’s algorithm to work out the problem. The problem can be reformulated as
\[ \mathcal{P}_3^{(l)} : \max_{\mathbf{A}} \Sigma_k (r_k^{+}(\mathbf{A}) - \Delta r_k^{+}(\mathbf{A})) - \lambda^{(l)} \] 
\[ \text{subject to} \] 
\[ \Sigma_k \text{tr}(\mathbf{A}_k) \leq P_{\text{c, max}}, \] \[ \mathbf{\Lambda}_k \succeq \mathbf{0}, \forall k. \] \tag{29} 

Problem \( \mathcal{P}_3^{(l)} \) later can be tackled through traditional convex optimization techniques.
4 Numerical Results

We adopt the QuadDiGa channel model [10] with a suburban macro cell scenario. The BS has \( M_t = 128 \) antennas, with \( \eta = 5 \), \( P_0 = 30 \) dBm, \( P_i = 40 \) dBm, and \( P_{\text{max}} = 100 \) mW. We assume there are \( K = 8 \) users with \( M_k = 4 \) antennas each. The variance of the noise is \( \sigma_n^2 = 15 \) dBm. We assume the radar pulses are modulated by a Barker code with length \( L = 1 \), and the maximum average radar power is \( P_{\text{max}} = 25 \) W. The variance of the noise is \( \sigma_n^2 = 15 \) dBm. We set \( \sigma_{\nu v}^2 = 6.34 \times 10^{-2}, \forall \nu v \in \mathcal{X} \) and \( \sigma_{\nu v}^2 = 6.34 \times 10^{-3}, \forall \nu v \in \mathcal{Y} \). Moreover, we assume \( \rho_j = \rho, \forall j \in \mathcal{Y} \). For the mutual interference, we set \( \sigma_{\nu v}^2 = \sigma_{\nu v}^2, \forall \nu v \in \mathcal{Y} \).

Figure 1 compares the EE performance in non-interfering, joint and disjoint cases versus the minimum required SDR (\( \sigma^2 = 25 \) dBm, \( P_{\text{max}} = 100 \) mW)

![Figure 1. EE performance in non-interfering, joint and disjoint cases versus the minimum required SDR (\( \sigma^2 = 25 \) dBm, \( P_{\text{max}} = 100 \) mW)](image)

Figure 1 compares the EE performance in non-interfering, joint and disjoint cases versus the minimum required SDR. From the figure, we can see that not all \( \rho \)'s are feasible. Actually through calculation and simulations, the objective function has a feasible solution only if \( \rho \leq 8.5 \) dB when \( \sigma^2 = 25 \) dBm. If \( \rho \) exceeds that range, we can not have a \( P_i \) that satisfies both the SDR constraint and \( P_t \) constraint. The curve corresponding to the non-interfering case is zero slope since in that case EE optimization only depends on the communication system. While for the disjoint design, the solution to maximize SDR is \( P_t = P_{\text{max}} \) so, for different \( \rho \)'s, \( P_t \) remains a constant which leads to the same EE.

5 Conclusion

We investigated the precoding design of a massive MIMO system with statistical CSIT and the transmit power design of a surveillance radar based on EE maximization in the RCC system. Since the optimization problem involved coupled variables, we exploited alternating optimization to divide the problem into precoding design and radar transmission power design problems. The optimal solution of radar transmit power could be expressed in a closed form. The precoding design problem could be simplified as a power allocation problem, which could be reformulated into a convex problem. Simulation results showed the feasibility and of the proposed algorithm.

References


