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# Method of Characteristic Modes for Modeling of Multimodal Scattering by Perfectly Conducting Bodies in Quantum Optics

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## Abstract

We propose a numerical technique for modeling the quantum multimode light scattering by a perfectly conducting body. Using the novel quantization approach, we give the quantum adaptation of the characteristic mode technique. Calculating the first- and the second-order field correlation functions by this method, we demonstrate how scattering affects quantum-statistical features of the field. As an example, we consider scattering of the two single-photon Gaussian beams on a cylinder with a circular cross-section. We expect that this method will be useful for designing quantum-optical devices.

## 1. Introduction

The recent progress in quantum information, quantum computing, and various types of quantum technologies make it timely to develop theoretical and numerical methods for the analysis of realistic quantum optical elements, such as quantum circuits [1], quantum transmission lines [2], quantum antennas [3]. The quantum-optical concepts were extended to rather low frequencies, looking promising for new applications of microwave photons in quantum informatics, quantum radars [4], and metrology. In this process, the basic elements of these devices became multi-modal involving the scattering and diffraction.

In contrast to the classical electrodynamics, the Maxwell equations for the quantum EM-field are reformulated in terms of the field operators. The conventional type of such reformulation is based on the decomposition of the field into plane waves while exchanging the classical weight coefficients by the pairs of creation-annihilation operators. While appealing for its simplicity, the plane wave basis is not always efficient for the physical analysis and numerical modeling of complicated multi-mode processes. In this talk we propose the highly efficient method of characteristic modes [5] for the scattering of quantum EM-field.

## 2. Quantization of characteristic modes

As a representative case, we consider a two-dimensional problem of scattering by an infinitely long perfectly conducting cylinder with arbitrary cross section. The impinging field is polarized along the axis of the cylinder. Let us introduce the basis functions in terms of the characteristic modes of the scatterer

$$f_{nk}(\boldsymbol{\rho}) = \frac{1}{2}\left(S_{nn}(k)E_{nk}(\boldsymbol{\rho}) + E^*_{nk}(\boldsymbol{\rho})\right) \qquad (1)$$

where $E_{nk}(\boldsymbol{\rho})$ is the $n$th characteristic field with the given wavenumber $k = \omega_k c$ , the quantities $S_{nn}(k)$ are the diagonal elements of the scattering matrix, which is unitary and diagonal in the chosen basis [5].

As usual [6], the general field operators are decomposed into the positive- and negative-frequency components as $\hat{E}(\boldsymbol{\rho}) = \hat{E}^+(\boldsymbol{\rho}) + \hat{E}^-(\boldsymbol{\rho})$ (and the same for the magnetic vector potential $\hat{A}(\boldsymbol{\rho})$). The corresponding operators are expressed as

$$\hat{A}^+(\boldsymbol{\rho},t) = \int_0^\infty \frac{\mathrm{E}_k}{\omega_k} e^{-i\omega_k t} \sum_n \hat{c}_{nk} f_{nk}(\boldsymbol{\rho})\, dk, \qquad (2)$$

$$\hat{E}^-(\boldsymbol{\rho},t) = -i\int_0^\infty \mathrm{E}^*_k e^{i\omega_k t} \sum_n \hat{c}^+_{nk} f^*_{nk}(\boldsymbol{\rho})\, dk, \qquad (3)$$

with bosonic commutation relations for creation-annihilation operators $\left[\hat{c}_{nk}, \hat{c}^+_{n'k'}\right] = \delta_{nn'}\delta(k-k')$ (all other pairs of operators commute), $\delta_{nn'}$ and $\delta(k-k')$ being Kronecker symbol and Dirac delta-function, respectively, and $\mathrm{E}_k = \sqrt{\hbar\omega_k c/\pi}$ - the normalization coefficient. The operators (2), and (3) in the far zone satisfy the canonical equal-time commutation relation

$$\left[\hat{A}^+(\boldsymbol{\rho},t), \hat{E}^-(\boldsymbol{\rho}',t)\right] = -\frac{i}{2\varepsilon}\hbar\delta(\boldsymbol{\rho}-\boldsymbol{\rho}') \qquad (4)$$

where $\varepsilon$ is the permittivity of the surrounding space.

In the case of a monochromatic field, we account for all characteristic modes at the same frequency.

## 3. Scattering of two single-photon beams by a circular cylinder

As a simplest application example, we consider scattering of two Gaussian beams by a circular cylinder as depicted in Fig. 1.

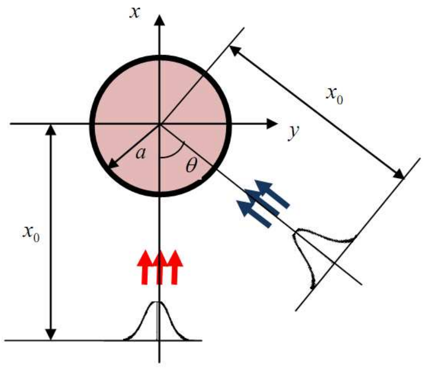


**Figure 1.** Geometry of the scattering of two identical Gaussian beams by the cylinder.

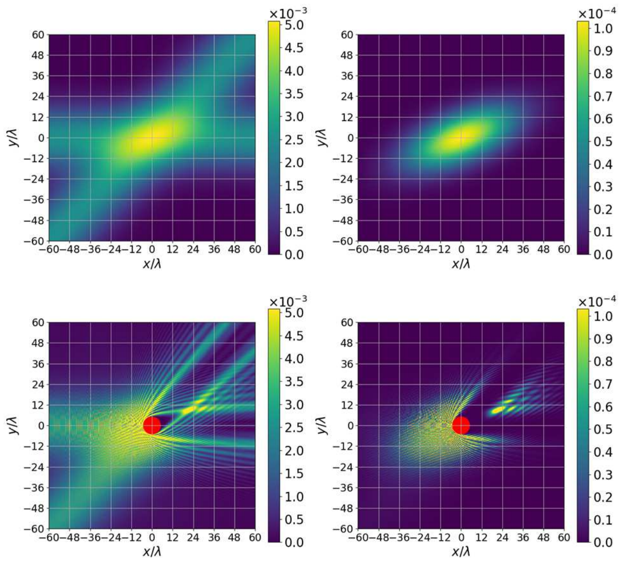


**Figure 2.** Scattering of two single-photon Gaussian beams: (a) and (c) intensity $I(\boldsymbol{\rho})$ in the absence and in the presence of the cylinder (colored red), respectively; (b) and (d) equal-point normalized second-order correlation function $G^{(2)}(\boldsymbol{\rho},\boldsymbol{\rho})$ in the absence and in the presence of the cylinder, respectively; $a = 5\lambda$, $\theta = 45^{\circ}$, $x_0 = 0$, $\beta = 1/25\lambda$ ($\beta = 1/\sqrt{2W}$, where $W$ is the width of the beam in the plane $x = x_0$).

Field intensity of the incident beams is presented in Fig. 2(a), while the total field in the presence of the cylinder is depicted in Fig. 2(c).

Considering the second-order correlation function of the fields with and without the cylinder in Figs. 2(b) and 2(d), respectively, one can see that the scattering dramatically changes the quantum statistical properties of light. Arrow-shaped region behind the cylinder is the one where bright color corresponds to a considerable average number of photons, and relatively high probability of two-photon registration. For two detectors placed in such "bright" regions, one would have high probability of simultaneous registration of photons. Such behavior is associated with the manifestation of the Hong-Ou-Mandel effect.

## 4. Conclusion

In this work we developed a general numerical technique for modeling of the quantum light scattering. The method is based on the characteristic mode technique [6]. The method is universal with respect to the configuration of lossless scatterer, structure of the incident field, and the scatterer dimensions relative to the wavelength. It is promising for various applications in quantum optics.

## 5. Acknowledgements

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## References

[1] S. Haroche, M. Brune and J. M. Raimond, "From cavity to circuit quantum electrodynamics", *Nat. Phys.*, **16,** March 2020, pp. 243-246. doi.org/10.1038/s41567-020-0812-1.

[2] A. Roy, and M. Devoret, "Quantum-limited parametric amplification with Josephson circuits in the regime of pump depletion", *Phys. Rev.* B **98**, 045405, pp. 1-27, July, 2018. doi:10.1103/PhysRevB.98.045405.

[3] G. Ya. Slepyan, S. Vlasenko, and D. Mogilevtsev, "Quantum Antennas", *Adv. Quantum Technol.* 1900120 2020, pp. 1-33. doi: 10.1002/qute.201900120.

[4] I. Peshko, D. Mogilevtsev, I. Karuseichyk, A. Mikhalychev, A.P. Nizovtsev, G. Ya. Slepyan, and A. Boag, "Quantum noise radar: superresolution with quantum antennas by accessing spatiotemporal correlations", *Opt. Exp*, **27**, 29217, September, pp. 1-15, 2019. doi.org/10.1364/OE.27.029217.

[5] Y. Chen, and C.-F. Wang, "Characteristic Modes: Theory and Applications in Antenna Engineering," John Wiley & Sons, Inc. 2015.

[6] M. O. Scully, and M. S. Zubairy, "Quantum Optics," Cambridge University Press, Cambridge, UK 1997.