



## **Solution to the Landau-Lifshitz Equation of Motion for a Relativistic Electron in a Counterpropagating Laser Beam**

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The deceleration of relativistic electrons by intense counterpropagating optical laser beams have produced X-rays and, more recently,  $\gamma$ -rays in the laboratory. Although the detailed theoretical determination of X-ray and  $\gamma$ -ray production by the interaction of high-energy electrons with intense optical lasers may require the inclusion of quantum effects, a hybrid approach that incorporates quantum corrections into a classical solution can predict reasonable results. Because the classical Lorentz-Abraham-Dirac (LAD) equation of motion does not have a closed-form solution to the problem of an electron in a plane wave, nor is it amenable to a numerical solution, especially when many charges are involved, the more readily solvable Landau-Lifshitz (LL) approximation to the LAD equation of motion has become the classical equation of choice for this problem within much of the physics community.

Beginning with a critical examination of the Lorentz-Abraham (LA) classical equation of motion for an extended charge and the closely related Lorentz-Abraham-Dirac (LAD) equation of motion for a mass-renormalized point-charge, the Landau-Lifshitz (LL) approximate solution to the LAD equation of motion is determined for an electron subject to a counterpropagating linearly or circularly polarized plane-wave pulse with an arbitrarily shaped envelope. A convenient three-vector formulation of the LL equation is used to derive closed-form expressions for the velocities and associated powers of the electron directly in terms of the time in the laboratory frame. The three-vector formulation also reveals definitive criteria for the LL solution to be an accurate approximation to the LAD equation of motion and for the LL solution to reduce to the solution of the Lorentz force (LF) equation of motion that ignores radiation reaction. Semi-classical analyses are used to obtain simple conditions for determining the regimes where the quantum effects of either Compton electron scattering by the incident photons or electron recoil produced by the emitted photons is significant. It is proven that the LL approximation becomes an inaccurate solution to the LAD equation of motion only for large enough electron velocities and plane-wave intensities that quantum recoil effects on the electron can greatly alter the classical solution. Comparisons are made with previously published analytical and numerical solutions to the LL equation of motion for the velocity of an electron in a counterpropagating plane wave.