



## Scattering by Wide-Angle Cones: A Uniform GTD Solution

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Scattering by circular cones has an exact conical harmonics solution [1]. This solution has been used recently in [2] to explore the scattering phenomenology of a complex source beam (CSB) diffraction by a cone. The goal of the present research is to construct ray-based solutions that address the physics of tip diffraction explicitly. A solution to this problem exists only in the limit of thin cones [3]. Here we address the complementary case of wide-angle cones, which is relevant, for example, for urban or terrain propagation. Compared to the thin-cone case which is dominated by the tip diffraction, the wide-cone has a rich phenomenology that involves an interplay between the tip diffraction and geometrical optics (GO) reflections.

The problem has been derived recently in [4] by deriving an approximate spectral integral solution for wide-angle cones whose form is much simpler than the exact spectral integral. This solution, however, is invalid in the geometrical optics transition zones. In the present work, we evaluate the approximate spectral integral of [4] uniformly and express the field in the transition zone using  $W_{-3/2}$  (the Weber functions of order -3/2). Far from this zone, this expression reduces to a saddle-point and a singular branch-point contribution, describing, respectively, the tip diffraction and the GO reflections. The new transition function is therefore used only inside the transition zone, whereas far from this zone we switch to GO reflections plus the tip diffracted field of [4]. The accuracy of the new solution is demonstrated in Figure 1 via comparison to the exact conical harmonics solution.

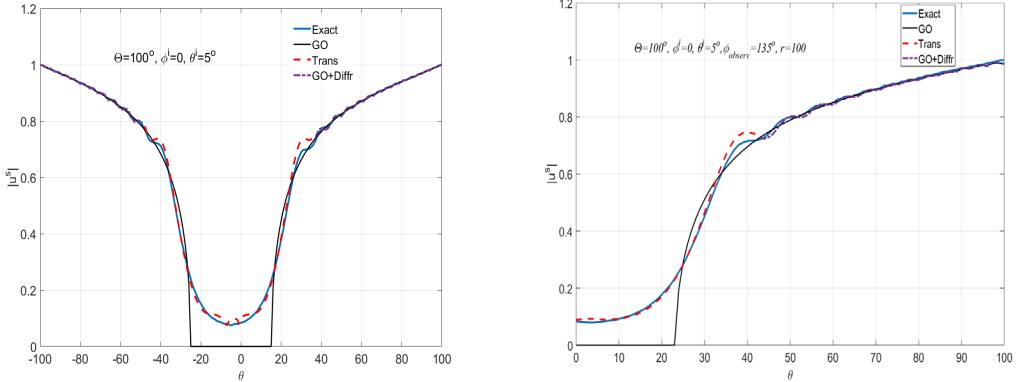


Figure 1. Magnitude of the scattered field as the function of observation angle  $\theta$  at a distance  $kr = 100$  for a cone with an external angle  $\Theta = 100^\circ$  (internal half-apex angle of  $80^\circ$ ) illuminated by an incident plane-wave from  $(\theta^i, \phi^i) = (10^\circ, 0^\circ)$ . Left: Field in the symmetry plane  $\phi = [0, \pi]$ . Right: Field in the plane  $\phi = \frac{3\pi}{4}$ . Solid blue lines: Exact solution. Black lines: GO. Red-dashed and violet-dashed-dotted lines: The new solution in the transition zone (the Weber function) and outside the transition zone (GO + tip diffraction), respectively. The transition from the uniform solution to the GO + tip diffraction solution occurs at  $45^\circ$ .

1. J. Bowman, B.A. Senior, P.L.E. Uslenghi, *Electromagnetic and Acoustic Scattering by Simple Shapes*, NH Amsterdam, 1969.
2. M. Katsav, E. Heyman, and L. Klinkenbusch, “Beam diffraction by the tip of a cone and creeping beam waves: Phenomenology analysis via the generalized complex-source method,” *IEEE Trans. Antennas Propagat.*, **69**, pp. 7830–7845, Nov. 2021.
3. L.B. Felsen, “Plane-wave scattering by small-angle cones,” *IRE Trans Antennas Propagat.*, **5**, pp. 121–129, 1957.
4. M. Katsav and E. Heyman, “Diffraction coefficient of wide-angle cones”, *IEEE Trans. Antennas Propagat.*, **68**, pp. 6777–6786, Sept. 2020.