3rd URSI AT-AP-RASC, Gran Canaria, 29 May – 3 June 2022

Exploiting the NIE model for the inverse obstacle scattering problem in case of strong or metallic scatterers

Martina T. Bevacqua and Tommaso Isernia
Università Mediterranea di Reggio Calabria, DIIES, Reggio di Calabria, Italy
(martina.bevacqua@unirc.it, tommaso.isernia@unirc.it)

Abstract

In this contribution a new method for inverse obstacle scattering problem is proposed starting from a recently introduced smart rewriting of the scattering equations, known as New Integral Equation (NIE) model. This latter involves a redefinition of the contrast function encoding the target properties. Interestingly, in both cases of strong and metallic scatterers, this function turns to be approximatively real, homogenous and approaching the unitary value. These properties are then exploited in the proposed method to accurately retrieve the shape and size of strong and/or metallic targets, as described in the following.

1. Introduction

Inverse obstacle scattering problem consists in retrieving the supports of unknown targets from the scattered fields when they are illuminated from known incident fields [1].

Many methods have been proposed in literature due to the applicative relevance of the problem. For instance, *sampling* methods are based on the sampling of the scenario under investigation over an arbitrary grid of points and the evaluation in each of these points of an indicator. In particular, the indicator assumes different values depending on whether the sampling point is inside or outside the unknow target. These methods include the linear sampling method [2], the orthogonality sampling method [3], the factorization method [4]. They are simple and fast, however, sometimes they do not allow to properly image non-convex targets.

On the other hand, some methods are based on iterative reconstruction algorithms. An example[1] is reported in [5], wherein the contrast source inversion (CSI) method and a binary regularization are adopted. In particular, only the imaging of homogeneous dielectric objects (the so-called binary objects) is dealt with and an approximate contrast value for each object has to be assumed.

In this contribution, we exploit a recent introduced model, known as new integral equation model (NIE) [6], which rewrites the basic equations of inverse scattering problem in order to reduce its non-linearity. This model has been proposed to accurately retrieve both the shape and the electromagnetic properties of the unknown targets. However, such a model is herein exploited for shape reconstruction of both strong and metallic targets. In particular, the proposed approach takes advantages from the redefinition of the function encoding the target properties involved in the NIE model. Interestingly, this function is expected to be well approximated by a binary function which assume unitary value inside the target and zero value outside. Then, unlike [5], there is no need to assume an approximate contrast value for each object and consider just homogenous objects, as briefly explained in the following.

2. Statement of the problem

The inverse obstacle scattering problem can be described by means of two integral equations: the data and the state equations. The first one relates the collected scattered field E_s to the contrast sources W, while the state equation expresses the contrast sources W in term of contrast function χ encoding the target properties. In case of 2D scenario and TM polarized field, the two equations read as follows:

$$E_s(\underline{r}_m, \underline{r}_t) = \int_\Omega G_b(\underline{r}_m, \underline{r}') \, W(\underline{r}', \underline{r}_t) d\underline{r}' \quad (1)$$

$$W(\underline{r}, \underline{r}_t) = \chi(\underline{r}') E_i(\underline{r}, \underline{r}_t) + \chi(\underline{r}') \int_\Omega G_b(\underline{r}, \underline{r}') \, W(\underline{r}') d\underline{r}' \quad (2)$$

where E_i is the incident, $\underline{r}$ scans the investigation domain Ω, $\underline{r}_m$ and $\underline{r}_t$ are respectively the positions of the receiving and transmitting antennas exploited to perform the scattering experiments. G_b is the Green's function pertaining to the background medium having complex permittivity ϵ_b. Finally, $\chi(r) = {\epsilon_s(r)}/{\epsilon_b} - 1$ relates the unknown complex permittivity of the scatterers embedded in the investigation domain Ω, denoted by ϵ_s, to that of the host medium.

[1] With no claim to exhaustivity of the quoted references.

2.1 NIE model

The problem (1)-(2) is unfortunately non-linear, as the contrast sources W also depend on the unknown of the problem χ. Moreover, it is also ill-posed due to the properties of the integral radiation operator in (1).

The NIE model rewrites in a different fashion the standard state equation (2) by extracting the local effect of the induced currents and introducing a new auxiliary unknown R encoding the target properties. This rewriting can somehow alleviate the amount and the effects of non-linearity [6],[7].

In the NIE model the state equation is rewritten as follows:

$$\beta(\underline{r})W(\underline{r},\underline{r}_t) = R(\underline{r})E_i(\underline{r},\underline{r}_t) + R(\underline{r})A_i^{NIE}[\beta(\underline{r}')W(\underline{r}',\underline{r}_t)] \tag{3}$$

wherein:

$$R(\underline{r}) = \frac{\beta(\underline{r})\chi(\underline{r})}{\beta(\underline{r})\chi(\underline{r}) + 1} \tag{3.a}$$

is the modified contrast function and β is such that the denominator of $R(\underline{r})$ is different from zero. In [1] β is assumed to be a real constant value belonging to the interval [0.5, 6]. Finally,

$$A_i^{NIE}[\cdot] = I + \frac{1}{\beta(\underline{r})}\int_{\Omega} G_b(\underline{r},\underline{r}')[\cdot]d\underline{r}' \tag{3.b}$$

With respect to equation (2), the integral internal operator is substituted by A_i^{NIE}, while the induced currents and the function χ have been now replaced by βW and R, respectively.

3. Rationale of the proposed method

The above model has been proposed to both quantitatively and qualitatively retrieve the contrast function χ. In particular, remarkable performance is expected against strong scatterers, as also theoretically explained in [7] by analyzing and comparing the degree of non-linearity with respect to traditional model (1)-(2). However, an increased difficulty may arise in the final mapping of the actual physical contrast χ from the modified contrast function R. Indeed, very large β and/or strong scatterers implies a kind of binary behavior of $R(\underline{r})$, so that one would have to extract a generically varying function from a binary one.

Nevertheless, one can take advantages from the binary behavior of $R(\underline{r})$ to solve the corresponding inverse obstacle scattering problem, which aims at retrieving the support of the unknown targets. Indeed, in case of strong and/or metallic targets, the 'ground truth' function R exactly corresponds to the support of the targets at hand.

Moreover, as $R(\underline{r})$ is expected to be well approximated by a binary function which assume unitary value inside the target and zero value outside, one can enforce such property to alleviate the ill-posedness of the problem. For instance, one can adopt a binary regularization (as discussed in [5]) or enforce sparsity of R in term of step function (as discussed in [8]).

4. Numerical Examples

Performance of the proposed method has been tested within a non-linear regime. In particular, a CSI-like method [8] has been adopted to solve the relevant inverse scattering equations underlying the NIE model, that are equations (1) and (3). Moreover, a sparsity regularization technique has been enforced by considering the unknown contrast profile R to be sparse in terms of the step functions. In particular, the l_1-norm of the discretized version of partial derivative (i.e., the vector containing the finite differences) of the unknown function R has been added as a weighted penalty term to the usual CSI cost functional, as described in [8].

The test target is a sinusoidal lossless circular cylinder with maximum value of complex permittivity equal to 3 and diameter λ_b, beingλ_b the wavelength in the background medium (see figures 1(a)-(b)). The target is embedded in a square domain of side $L = 2\lambda_b$, discretized into 46×46 small cells. The target has been probed by means of 18 receivers and transmitters modelled as line sources located on a circumference of radius $3.75\ \lambda_b$. The scattered field data, simulated by means of a full wave in-house forward solver based on MoM, have been corrupted with a random Gaussian noise with a $\text{SNR} = 30\text{dB}$.

The initial guess of the iterative procedure is reported in Figures 1(c)-(d) and represents the amplitude of back-propagation solution. We consider the amplitude of back-propagation solution as the unknown function R is expected to have a very low imaginary part. Moreover, $\beta = 10$ and the background medium is assumed the free space.

Despite the sinusoidal distribution of χ, the corresponding actual function R is approximatively homogenous and assumes a maximum value equal to 0.95, as shown in Figures 1(e)-(f). This confirms the expected behavior of the auxiliary variable R and validates the rationale underlying the proposed method.

The final reconstruction is shown in Figures 1(g)-(h). As can be seen, this preliminary example confirms the potentiality of the method to accurately retrieve the target support and size, also in case of non-homogenous target.

More details about the proposed approach, as well as some numerical examples also against metallic and non-convex targets, will be given at the conference.

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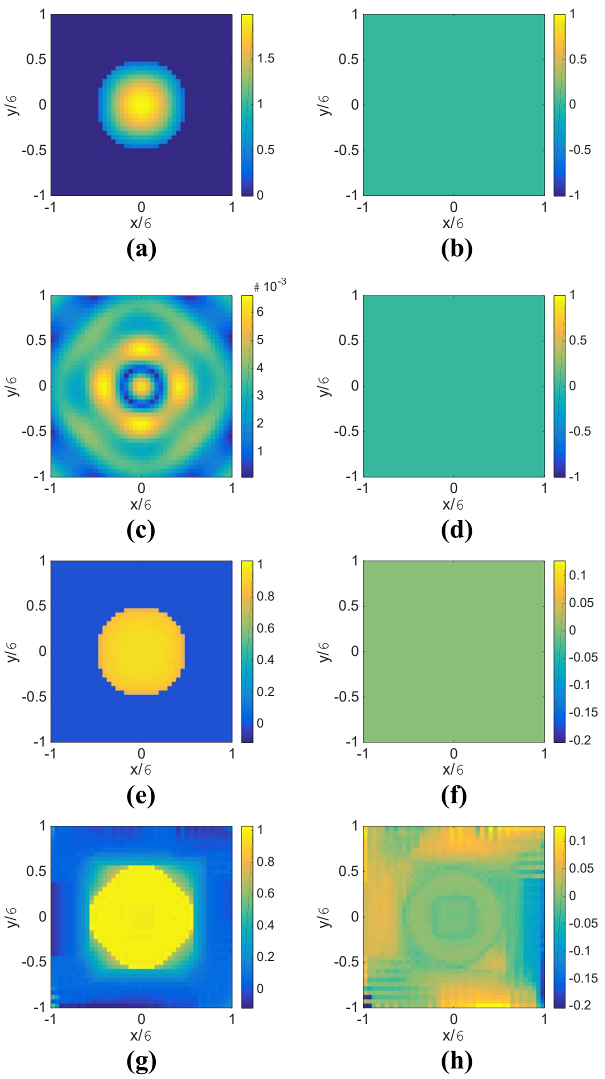

Figure 1. Assessment against a sinusoidal lossless circular cylinder. Real (a) and imaginary (b) parts of the actual χ profile. Real (c) and imaginary (d) parts of the starting guess of the iterative procedure. Real (e) and imaginary (f) parts of the actual R profile. Real (g) and imaginary (h) parts of the retrieved R function.

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