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AoA Estimation of Spatially Correlated MIMO Transmitters in Wireless Passive Radar Applications

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Abstract

In this work we are concerned with the problem of estimating the AoA of MIMO transmitters when their antennas exhibit spatial correlation. The problem is of interest in passive RADAR applications where a receiver that is not part of the nominal communication link wants to estimate the AoA. We explore the conditions under which we we can calculate correctly the number of AoAs, allowing us correct application of the high resolution MUSIC algorithm. Our results in indicate that for spatially correlated channels we must know the degree of spatial correlation before we can calculate the AoAs.

1 Introduction

Modern communication systems may be needed to be capable of both efficient communication and RADAR functionality. This is possible since wireless digitally modulated signals always convey additional information besides the data itself. However, the problem is more challenging in passive systems where a receiver is not part of the nominal communication link and the signal has not been designed to support RADAR. For such a system one type of information that we can extract is the angle-of-arrival (AoA) of the signal at a particular unauthorized receiver (URx).

Estimating the AoA can be accomplished in several different ways all of which use the received signal vector at an array of antennas (Eve in Fig. 1). One of the most popular class of techniques, referred to as *subspace methods*, exploit the structure of the received signal covariance matrix and offer very high angular resolution. The multiple signal classification (MUSIC) [1] algorithm, and ESPRIT [2], belong to this class of techniques. Besides subspace methods, techniques like the Bartlett and Capon/MVDR beamformers can also be used for AoA estimation at the cost of lower angular resolution. It is interesting that a pre-requisite for solving the AoA estimation problem with MUSIC is the knowledge of the number of AoAs.

In this paper, under the assumption of a receiver that consists of a uniform linear array (ULA), and a Rayleigh fading channel, we identify first the conditions when the number of AoAs can be correctly estimated. Next, we show that the situation is more perplexed when a more realistic MIMO channel experiences spatial correlation across the antenna elements. Spatial correlation across antenna elements at the Tx and/or Rx is a typical concern of every practical MIMO system that hampers performance in practical scenarios.

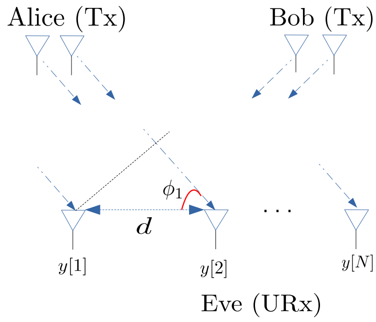


Figure 1. The wireless communication system with MIMO transmitters with a ULA deployed at an unauthorized Rx.

2 System Model

The system model in Fig. 1 consists of a number of multi-antenna transmitters (just two are illustrated in Fig. 1) that may communicate with each other or another receiver which is not presented since it is irrelevant to our study. One or more transmitters, with potentially several antennas, transmit simultaneously a signal of bandwidth B Hz over the same time-frequency slot depending on the channel allocation scheme. The data modulated signals are assumed to be narrow-band, that is $B << f_c$ where f_c is the carrier frequency. The model also includes a ULA, which not part of the nominal communication system, hence an unauthorized receiver (URx), that calculates the number of AoAs with the AIC metric before AoA execution. Therefore, our subsequent discussions on the signal models and the estimation algorithms concern the ULA.

Baseband Model with spatial correlation: The ULA at the URx consists of N_{Rx} elements spaced d meters apart. The impinging waves at the ULA are assumed to be specular plane waves (arriving in parallel). We also assume a static Rayleigh flat fading channel, with complex fading coefficient h. Hence, for one path between the Tx and the URx the overall complex baseband channel gain is $h\exp(j\eta(\phi,\theta))$, where ϕ,θ are the AoA and AoD respectively. Here, without loosing generality we consider only the AoA. In the model we separate the two elements of the baseband channel gain into the Rayleigh complex gain and the steering vector of the ULA as described next. The baseband modulated signals from all the MIMO transmitters in our model are packed in the vector $\mathbf{x}$[1], and are all assumed to be uncorrelated making the covariance matrix $\mathbf{C_x}$=$\mathbb{E}[\mathbf{xx}^H]$ full rank.

We use the Kronecker model to describe the spatially correlated MIMO channel (even though our overall methodology does not preclude the use of other models). With this model if $\mathbf{C}_{\text{Tx}}$ and $\mathbf{C}_{\text{Rx}}$ are the spatial correlation matrices between the antenna elements at the transmitters and the URx, then the channel matrix becomes:

$$\mathbf{H} = \mathbf{C}_{\text{Rx}}^{1/2}\mathbf{H}_{\text{iid}}(\mathbf{C}_{\text{Tx}}^{1/2})^H \quad (1)$$

$\mathbf{H}_{\text{iid}}$ ($N_{\text{x}}\times N_{\text{Tx}}$) contains the iid samples of the Rayleigh fading channel. Since we consider spatial correlation at the Rx only the previous model is simplified to $\mathbf{H} = \mathbf{C}_{\text{Rx}}^{1/2}\mathbf{H}_{\text{iid}}$. Regarding the specific structure of the correlation matrices again we selected a popular representation of them that is also not a necessity. We adopt the clustered spatial correlation model where the correlation matrix can be modeled as the Hadamard product (element-wise multiplication) of matrix $\mathbf{A}$ and a second matrix $\mathbf{B}$, where the later captures the precise spatial correlation [3]:

$$\mathbf{C}_{\text{Rx}}^{1/2} = \mathbf{A}(\phi)\odot\mathbf{B}(\phi,\sigma_\phi) \quad (2)$$

In the above σ_ϕ^2 is the variance of the stochastic spatial correlation model at the Tx [3]. Higher σ_ϕ^2 corresponds to higher spatial correlation. $\mathbf{A}$ is the unknown $N_{\text{Rx}}\times N_{\text{Tx}}$ steering matrix since the transmitted signal $\mathbf{x}$ spans in an N_{Tx}-dimensional space. Each column of $\mathbf{A}$ contains the steering vector that captures the phase difference between the received signal at each element of the ULA that originates from the i-th AoA/source:

$$\mathbf{a}^T(\phi_i) = [1 \;\; e^{j2\pi f_c\frac{d\cos\phi_i}{c}} \;\; \ldots \;\; e^{j2\pi f_c\frac{(N_{\text{Rx}}-1)d\cos\phi_i}{c}}]$$

In this model $d\cos(\phi_i)/c$ is the additional time required for the RF signal to travel between two antenna elements of the ULA (Fig. 1 clearly illustrates the geometry). Consequently, if we assume we have M AoAs this $N_{\text{Rx}}\times N_{\text{Tx}}$ matrix is:

$$\mathbf{A} = \begin{bmatrix} 1 & .. & 1 \\ e^{j2\pi f_c\frac{d\cos\phi_1}{c}} & .. & e^{j2\pi f_c\frac{d\cos\phi_M}{c}} \\ \cdots & \cdots & \cdots \\ e^{j2\pi f_c\frac{(N_{\text{Rx}}-1)d\cos\phi_1}{c}} & .. & e^{j2\pi f_c\frac{(N_{\text{Rx}}-1)d\cos\phi_M}{c}} \end{bmatrix} \quad (3)$$

It is evident that if we have more than one antennas at a single transmitter i then the column vector $\mathbf{a}(\phi_i)$ is repeated accordingly in this matrix.

[1] Note that were are not interested in decoding data, hence all modulated source signals are packed in the same vector $\mathbf{x}$.

Based on the previous discussion the final baseband model becomes:

$$\mathbf{y} = (\mathbf{A}\odot\mathbf{B})\mathbf{H}_{iid}\mathbf{x}+\mathbf{w} \quad (4)$$

where $\mathbf{w}$ is the AWGN vector.

This expression is critical in the overall discussion of this paper since it shows that the beamsteering matrix in now $\mathbf{A}\odot\mathbf{B}$.

3 The AIC metric

If the spatial correlation matrix $\mathbf{B}$ is equal to $\mathbf{I}$, i.e. no spatial correlation across the antennas and no ULA at the Rx, then the model is simplified and we have the well-known i.i.d. MIMO channel $\mathbf{y} = \mathbf{H}_{\text{iid}}\mathbf{x}+\mathbf{w}$. For $\mathbf{H}_{\text{iid}}$ it is rank($\mathbf{H}_{\text{iid}}$)=min(N_{Rx},N_{Tx}). The $N_{\text{Rx}}\times N_{\text{Rx}}$ *signal* covariance matrix is $\mathbf{C_s}$=$\mathbf{H}_{\text{iid}}\mathbf{C_x}\mathbf{H}_{\text{iid}}^H$. It will also be rank($\mathbf{C_s}$)=min($N_{\text{Rx}}$,$N_{\text{Tx}}$). Hence, if $N_{\text{Rx}}\geq N_{\text{Tx}}$ the rank gives us the number of simultaneously transmitting antennas [4]. This is the basic principle that was used for solving the problem of finding the number of single antenna sources described by Kailath [5] with AIC. A practical concern is that we only have access to an estimate of $\mathbf{C_y}$, namely $\widehat{\mathbf{C}}_\mathbf{y}$, and not to $\mathbf{C_s}$. In [5] the authors considered this impact of AWGN in the AIC metric. *Hence, for the iid channel the AIC metric can be used for estimating the number of AoAs when $N_{Rx}\geq N_{Tx}$.* However, the same is not possible for our channel model since we are now concerned about the rank of $(\mathbf{A}\odot\mathbf{B})$ which depends on spatial correlation.

4 AoA Estimation with MUSIC

A ULA Rx can calculate the AoA of wireless signals that are linearly superposed by exploiting the difference in the angle of arrival of the signals at different antennas spaced at known locations (Fig. 1), that is it exploits the beamsteering vector $\mathbf{a}^T(\phi)$. Subspace processing methods like the MUSIC algorithm have been used in the literature for extracting the AoA from different types of wireless signals [**?**, 1, 2, 6] including WiFi. This class of methods are based on the special structure of the covariance matrix of the received signal. Essential information for MUSIC to work is the knowledge of the number of AoAs which is something that can be accomplished with AIC.

Although the basic step of MUSIC is to perform Eigenvalue Decomposition (EVD) on $\widehat{\mathbf{C}}_\mathbf{y}$, we delve a little deeper into it next. At the Rx the covariance matrix of the received signal $\mathbf{y}$ is:

$$\begin{aligned}\mathbf{C_y} &= \mathbb{E}[(\mathbf{y}-\mathbb{E}[\mathbf{y}])(\mathbf{y}-\mathbb{E}[\mathbf{y}])^H] = \mathbf{A}\mathbf{H}_{\text{iid}}\mathbf{C_x}\mathbf{H}_{\text{iid}}^H\mathbf{A}^H + C_\mathbf{w} \\ &= \mathbf{A}\mathbf{H}_{\text{iid}}\mathbf{C_x}\mathbf{H}_{\text{iid}}^H\mathbf{A}^H + C^2\mathbf{I} \end{aligned} \quad (5)$$

The covariance matrix of the *signal component* is $\mathbf{C_s} = \mathbf{A}\mathbf{H}_{\text{iid}}\mathbf{C_x}\mathbf{H}_{\text{iid}}^H\mathbf{A}^H$. For MUSIC we want $M < N_{\text{Rx}}$ which makes $\mathbf{C_s}$ singular, i.e. non-invertible:

$$\det(\mathbf{A}\mathbf{H}_{\text{iid}}\mathbf{C_x}\mathbf{H}_{\text{iid}}^H\mathbf{A}^H) = \det(\mathbf{C_y}-C^2\mathbf{I}) = 0$$

From linear algebra we know that for a matrix $\mathbf{C_s}$ there are $\dim(\mathbf{C_s}) - \text{rank}(\mathbf{C_s})$ vectors that satisfy:

$$\mathbf{A}\mathbf{H}_{\text{iid}}\mathbf{C_x}\mathbf{H}_{\text{iid}}^H\mathbf{A}^H\mathbf{q_m} = 0, \tag{6}$$

i.e. these vectors are the solution set of the previous linear system. But this also means that $\mathbf{q_m}$ is an eigenvector of $\mathbf{A}\mathbf{H}_{\text{iid}}\mathbf{C_x}\mathbf{H}_{\text{iid}}^H\mathbf{A}^H$ for the zero-eigenvalue (i.e., $\mathbf{A}\mathbf{H}_{\text{iid}}\mathbf{C_x}\mathbf{H}_{\text{iid}}^H\mathbf{A}^H\mathbf{q_m} = 0 \times \mathbf{q_m}$). Furthermore from (5), (6):

$$\mathbf{A}\mathbf{H}_{\text{iid}}\mathbf{C_x}\mathbf{H}_{\text{iid}}^H\mathbf{A}^H\mathbf{q_m} = (\mathbf{C_y} - \sigma^2\mathbf{I})\mathbf{q}_m = 0$$

Hence, the zero-value eigenvectors $\mathbf{q_m}$ are also eigenvectors of $\mathbf{C_y}$ and they all have the same eigenvalue σ^2 (this is the noise subspace). Regarding the remaining non-zero eigenvalue eigenvectors of $\mathbf{C_s}$ let us assume that they satisfy $\mathbf{A}\mathbf{H}_{\text{iid}}\mathbf{C_x}\mathbf{H}_{\text{iid}}^H\mathbf{A}^H\mathbf{q_m} = \lambda_m\mathbf{q_m}$. To calculate all the eigenvectors of $\mathbf{C_y}$ we proceed based on the last expression:

$$\mathbf{A}\mathbf{H}_{\text{iid}}\mathbf{C_x}\mathbf{H}_{\text{iid}}^H\mathbf{A}^H\mathbf{q_m} = \lambda_m\mathbf{q_m} \Rightarrow \tag{7}$$

$$\mathbf{A}\mathbf{H}_{\text{iid}}\mathbf{C_x}\mathbf{H}_{\text{iid}}^H\mathbf{A}^H\mathbf{q_m} + \sigma^2\mathbf{I}\mathbf{q}_m = \lambda_m\mathbf{q_m} + \sigma^2\mathbf{I}\mathbf{q}_m \overset{(5)}{\Rightarrow}$$

$$\mathbf{C_y}\mathbf{q_m} = (\lambda_m + \sigma^2\mathbf{I})\mathbf{q}_m \tag{8}$$

The last derivation indicates that matrix $\mathbf{C_s}$ shares all its non-zero-eigenvalue eigenvectors $\mathbf{q_m}$ (as captured by (7)), with the ones of $\mathbf{C_y}$ while their eigenvalues differ by σ^2.

Next, we perform EVD of $\widehat{\mathbf{C}}_{\mathbf{y}}$[2] from which we obtain the two categories of eigenvectors $\mathbf{q_m}$ for the signal and noise sub-spaces that we discussed in the last paragraph. Recall that we assume that there are M AoAs we want to resolve, so the matrices that contain the eigenvectors are $\mathbf{Q}_1$=$[\mathbf{q}_1, ..., \mathbf{q}_M]$, while the eigenvectors for the zero-value eigenvalues are contained in $\mathbf{Q}_2$=$[\mathbf{q}_{M+1}, ..., \mathbf{q}_{N_{\text{Rx}}}]$. So $\mathbf{Q}_2$ is a space spanned by the zero-value eigenvectors.

The basic observation is that the noise sub-space is orthogonal to signal space, i.e. $\mathbf{a}^H(\phi)\mathbf{Q}_2$=0. This allows us to calculate the MUSIC pseudo-spectrum:

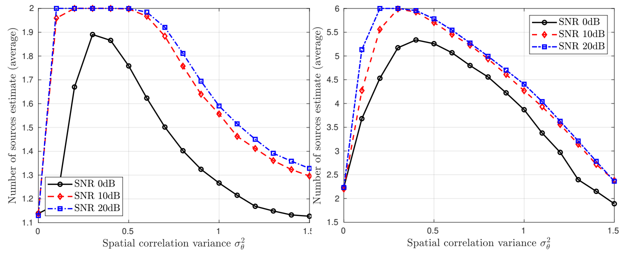
$$P_{\text{MUSIC}}(\phi) = \frac{1}{\mathbf{a}^H(\phi)\mathbf{Q}_2^H\mathbf{Q}_2\mathbf{a}(\phi)} \tag{9}$$

The peaks in $P_{\text{MUSIC}}(\phi)$ contain the AoAs.

5 AIC and MUSIC Limitations

Based on our discussions until now we can draw easily the first conclusion for the signal model. It is already clear that *the number of AoAs is not always equal to the rank of* $\mathbf{C_s}$ *since this matrix is now affected by the number of AoAs in* $(\mathbf{A} \odot \mathbf{B})$. More precisely, $\text{rank}((\mathbf{A} \odot \mathbf{B})\mathbf{H}_{\text{iid}}\mathbf{C_x}\mathbf{H}_{\text{iid}}(\mathbf{A} \odot \mathbf{B})^H)$ is affected mainly by three matrices (since $\mathbf{H}_{\text{iid}}$ is full rank). Clearly, its rank depends on the number of uncorrelated source signals in the vector $\mathbf{x}$ and the number of

[2]Note that as a covariance matrix $\mathbf{C_x}$ is Hermitian $\Rightarrow$ $\mathbf{C_s}$ is also a covariance matrix, hence Hermitian, since $\mathbf{s}$ is the result of linear processing of $\mathbf{x}$. For uncorrelated sources $\mathbf{C_x}$ is diagonal, full rank, and consequently $\mathbf{A}\mathbf{C_x}\mathbf{A}^H$ is already a valid diagonalization of $\mathbf{C_s}$.



(a) ϕ_1=30, N_{Tx}=2 (b) ϕ_1=30,ϕ_2=60, N_{Tx}=6 (3 antennas each)

Figure 2. Number of Antennas Estimation Results.

AoAs in $(\mathbf{A} \odot \mathbf{B})$. The beamsteering matrix is now $(\mathbf{A} \odot \mathbf{B})$. Hence, the original Vandermonde structure of $\mathbf{A}$ is compromised for this channel captured in (4). Spatial smoothing algorithms [7], that separate the ULA into L subarrays, are not helpful in this scenario. With spatial smoothing the objective is to restore the rank of $\mathbf{C_s}$. The smoothed estimate for L subarrays is:

$$\bar{\mathbf{C}}_{\mathbf{y}}^{(L)} = (\mathbf{A} \odot \mathbf{B})\mathbf{H}_{\text{iid}}\bar{\mathbf{C}}_{\mathbf{x}}^{(L)}\mathbf{H}_{\text{iid}}^H(\mathbf{A} \odot \mathbf{B})^H + C^2\mathbf{I} \tag{10}$$

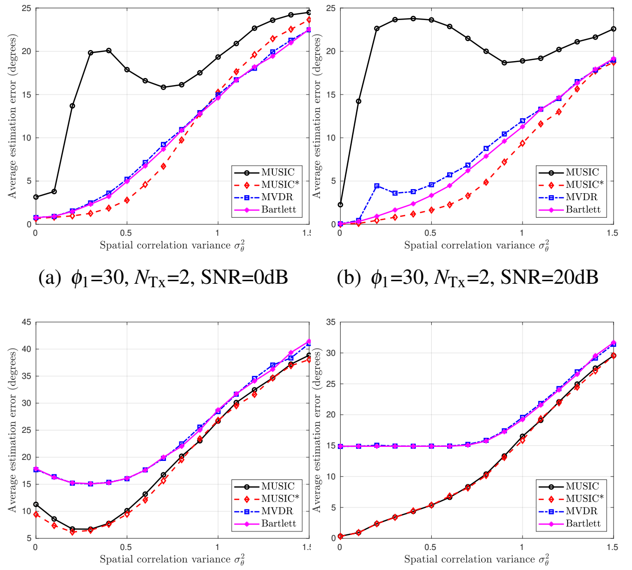
This means that even if smoothing restores the rank of the source covariance matrix, the rank of $\mathbf{A} \odot \mathbf{B}$ is not affected. Of course when the sources in $\mathbf{x}$ are uncorrelated spatial smoothing is unnecessary. Overall the use of AIC for estimating the number of AoAs is more challenging to analyze in this case and that is why we resort to simulations.

AIC Failure: There is situation where AIC fails to estimate both the number of antennas and AoAs (hence MUSIC also fails) even when there is no spatial correlation (i.e. $\mathbf{B} = \mathbf{I}$) This happens when $\text{rank}(\mathbf{C_x}) < \text{rank}(\mathbf{A})$ (indicating that I have more AoAs than actual uncorrelated wireless signals), allowing us thus to detect only a number of $\text{rank}(\mathbf{C_x})$ AoAs (the total number of actual AoAs is $\text{rank}(\mathbf{A})$). This can also be seen by noticing that now the number of $N_{\text{Rx}} - M$ solutions (dimension of the noise subspace) of the linear system in (6) will be higher than it should be, which means more zero-value eigenvectors, i.e. more solutions for the noise subspace. In real life this happens when $\text{rank}(\mathbf{C_x})$ is reduced due to the source signals being correlated (e.g. receiving a reflection of the original signal), requiring thus a smoothing algorithm [7] to make $\mathbf{C_x}$ full rank. Without smoothing, MUSIC will estimate the average AoA between correlated sources.

5.1 Channels With Spatial Correlation

6 Simulations & Verification

We configured a ULA with d=$\lambda/2$ (a necessary requirement for MUSIC), used a WiFi 5GHz carrier, and considered different number of antennas N_{Rx}. We also assumed we had access to 10 snapshots of $\mathbf{y}$ for estimating the covariance matrix.



(a) ϕ_1=30, N_{Tx}=2, SNR=0dB (b) ϕ_1=30, N_{Tx}=2, SNR=20dB

(c) ϕ_1=60, ϕ_2=30, N_{Tx}=2, SNR=0dB (d) ϕ_1=60, ϕ_2=30, N_{Tx}=2, SNR=20dB

Figure 3. AoA estimation error for one source/AoA with two antennas (a,b), that is M=1 and N_{Tx}=2, and two sources with two AoAs (c,d), that is M=2 and N_{Tx}=2).

Regarding the estimate of AoAs we explored two cases. First when we have one source/AoA with 2 antennas the results can be seen in Fig. 2(a). For 0 spatial correlation AIC can find the rank of $\mathbf{A} \odot \mathbf{B}$ which is equal to the number of AoAs. But when the spatial correlation is slightly increased AIC fails quickly and does not give the number of AoAs. Surprisingly, AIC estimates correctly now M since the structure of $\mathbf{A} \odot \mathbf{B}$ is compromised but becomes full rank (so does $(\mathbf{A} \odot \mathbf{B})\mathbf{C_x}(\mathbf{A} \odot \mathbf{B})^H$). For very high spatial correlation the rank approaches 1 again which means that it can be used now for AoA estimation. With one MIMO transmitter the AIC method may now either reveal the number of AoAs or the number of actual wireless sources which are two different things depending on level of antenna spatial correlation. For 2 users/AoAs with 3 antennas each the results in Fig. 2(b) we see that AIC only finds the AoAs for 0 spatial correlation but fails for higher values of it.

We next examine the estimation accuracy for a single AoA/source but with two antennas, i.e. in our model N_{Tx}=2. Recall that as σ_θ^2 is increased there is higher spatial correlation across the antenna elements leading to a deviation of the beamsteering matrix from the ideal form given in $\mathbf{A}$. Our results in Fig. 3(a),3(b) indicate that for every level of spatial correlation both the MVDR and Bartlett beamformers perform better. The reason behind this is that they estimate the angle for which the power of the incoming signal is maximized. On the other hand MUSIC is based on eigendecomposition that fails more easily when the channel matrix is not Vandermonde. To understand this even better we note that the performance of MUSIC with a genie-aided estimate of M, indicated as MUSIC*, performs the same as the two beamformers. But regardless of the used algorithm, spatial correlation reduces performance for all systems since it affects the rank of $\mathbf{C_y}$.

When two sources with AoAs $\pi/3, \pi/6$ are present, our results in Fig. 3(c) and Fig. 3(d) show that both beamformers perform the worst since they estimate the bearing which has the highest power and this is the average AoA between $\pi/3, \pi/6$. MUSIC performs substantially better. It is interesting that in low SNR conditions the best performance is not achieved for the lowest spatial correlation (with σ_θ^2=0). The reason is the poor estimate of M from AIC due to the low SNR, which coincides to give an accurate estimate for M for σ_θ^2=0.2. For 20dB in Fig. 3(d) M is estimated sufficiently well and the reason behind the poor performance at this point in spatial correlation as σ_θ^2 is increased.

7 Conclusions

Estimating the number of antennas and AoAs is not trivial with AIC since the produced result represents the correct information in certain cases while in some it does not. Overall we notice that for the model of an iid channel and a ULA, AIC provides the number of AoAs and antennas only when we have one antenna/user, but when we have more it provides only the AoAs. For correlated channels the problem is more complicated and the level of spatial correlation is a crucial metric that helps in choosing whether to use AIC for the two considered estimation problems. It is clear that there is a need for more robust schemes for estimating the number of antennas and AoAs with and without the presence of spatial correlation.

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