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High-order Hybridizable Discontinuous Galerkin Method for Multiphysics Modeling

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Multi-physical coupling process modeling and numerical simulation algorithm research are significant factors that promote the progress of information science and technology in the 21st century. In recent years, its applications in aerospace, advanced radar, wireless communications, and high-density integrated circuits have received unprecedented attention. There are electromagnetic, thermal, mechanical, wet, fluid, and other multi-physical fields causing strong nonlinear coupling processes in multi-scale heterogeneous functional structures, and dielectric constants, thermal conductivity, thermal expansion coefficient, and elastic modulus exhibit strong nonlinear cross-coupling effects under multi-physics. Unfortunately, traditional numerical methods such as the Finite Element Method (FEM) show low robustness, weak representation, and poor convergence facing such high-dimensional partial differential equations. Therefore, it is urgent to adopt multidisciplinary knowledge and hybrid methods to characterize the strongly nonlinear coupling process with high fidelity and clarify the physical mechanism.

In our work, an advanced numerical method is introduced to deal with this predicament, which is called Hybridizable Discontinuous Galerkin Method (HDG). HDG is proposed by mathematics in recent years and shows excellent characteristics in solving complex problems such as thermodynamics and fluid mechanics. Its core feature is to introduce a mixed variable to approximate the gradient value of the original unknown, and then convert a high-order equation into multiple low-order equations, reducing the complexity of single equation processing. The HDG method inherits the advantages of stability, convergence, self-adaptive discreteness, arbitrary high-order, and support for parallel computing architecture in the Discontinuous Galerkin Method (DG). Aiming at the shortcomings of the high computational cost of the DG method in the time-domain implicit solution, the HDG method introduces new degrees of freedom (DOF) on the mesh skeleton then divides the solution of the physical equation into the global problem and the local problem. The global problem only solves the DOFs on the mesh skeleton thus the number of DOFs and the coupling degree of the system matrix are much smaller than the DG method; Moreover, local problems are only solved inside the element thus the computational cost can be negligible. Therefore, the HDG method can greatly reduce the computational cost and memory requirement. In terms of convergence speed, the HDG method has the characteristics of optimal convergence, that is, by using p-order basis functions, the convergence speed can reach p+1; furtherly, after the elemental local post-processing, the convergence speed can even reach p+2, which is called superconvergence.

In all, we introduce the HDG method and perform the simulation of multi-physical systems. The numerical experiments show that our method is superior to the current mainstream algorithms in terms of stability, high-order convergence, and large-scale scalability.

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