Optimal Synthesis of Circular Ring Arrays for Targets Localization via Orbital Angular Momentum Vortex Beams

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Abstract

We propose a new way to use Orbital Angular Momentum (OAM) antennas for the synthesis of circular-ring arrays aimed at the localization of targets. Being able to generate a single null of the pattern in the spectral plane, the proposed solutions can achieve good localization performances as compared to the common techniques which lead to a whole line of zeroes in the spectral plane (or two perpendicular rows in case of 'delta square' patterns). The design procedure relies on two main steps: first, a Convex Programming (CP) optimization is performed in order to identify a continuous aperture source maximizing the localization performances through an optimal exploitation of the OAM modes and, then, a proper discretization of the continuous source into the sought circular-ring array is performed. Comparisons of radiation performances between the benchmark continuous aperture source and the corresponding designed array are given.

1. Introduction

The target localization in radar applications is usually exploited by resorting to difference (Δ) or 'delta-square' patterns. Accordingly, several design approaches have been developed by the Antennas Community [1]-[5]. They are usually aimed at generating a pattern having a null with the maximum slope at the boresight while subject to given upper bounds on the remaining part of the pattern [4]. In this respect, the radiation pattern of a OAM beam can be considered as a form of difference beam since it owns typical null depth [6].

Moreover, OAM vortex allows avoiding ambiguities along lines of the spectral plane. In fact, when considering planar sources, the source exhibits an antisymmetric behavior leading to a whole line of zeroes in the spectral plane (or two perpendicular rows in case of 'delta square' patterns) [1]. While convenient in terms of the beamforming network (as well as for easily commuting amongst the sum and difference modes), such a circumstance can imply ambiguities and, hence, loss of resolution in the azimuth direction when the delta pattern is along elevation (and vice versa).

A very critical point in the synthesis of OAM vortex beam is the control of the side lobes level (SLL) [6]. As a matter of fact, keeping as low as possible SLL ensure efficient work conditions in complex environments of ground object interference or electromagnetic interference. However, OAM generation by conventional methods leads to high SLL [7],[8]. Global optimization algorithms could represent a possibility towards such a criticism, but they need large time cost and computing resource. On the other side, a very powerful tool for the synthesis of low SLL difference pattern has been developed by Bayliss in [2], but it can be applied to linear sources.

In order to fill such a gap, we propose herein a novel synthesis procedure for circularly-symmetric power patterns whose azimuth cuts are Δ beams maximizing the localization performance. The developed procedure deals with the synthesis of continuous planar aperture sources through a CP algorithm relying on a field expansion exploiting at best the OAM beams properties. Then, starting from the achieved continuous sources, we show that a proper discretization of the latter into circular-ring arrays allows achieving equivalent (optimal) localization performances.

In the following, the mathematical background concerning the OAM beams properties herein required is given in Section II, while the proposed procedure is described in Section III and assessed in Section IV.

2. Mathematical background

By denoting with k' and ϕ the radial and azimuth coordinates in the spectral domain, and with ρ' and ϕ' the radial and azimuth coordinates spanning the aperture source, the radiated field $F(k', \phi)$ and the corresponding source $f(\rho', \phi')$ can be expanded in a multipole series as follows:

$$F(k', \phi) = \sum_{\ell=-\infty}^{+\infty} F_\ell(k')\, e^{j\ell\phi} \qquad (1.a)$$

$$f(\rho',\phi') = \sum_{\ell=-\infty}^{+\infty} f_\ell(\rho')e^{j\ell\phi'} \qquad (1.b)$$

where the contribution to the radiated field can be expressed as follow (see [6] for more details):

$$F_\ell(k') = \int_0^a f_\ell(\rho')J_\ell(k'\rho')\,\rho' d\rho' = H_\ell\{f_\ell(\rho')\} \qquad (2)$$

wherein $J_\ell(\cdot)$ indicates the ℓ-*th* order Bessel function of the first kind and a is the radius of the source.
By normalizing the variables ρ' and k' into the variables ρ and k belonging to the range [0,1], Eq.(2) can be rewritten, in an operator form, as follow:

$$F_\ell = A_\ell f_\ell \qquad (3)$$

Notably, the overall integral expression (2) is the Hankel transform [6],[9] of order ℓ of $f_\ell(\rho')$ and one can readily note in its kernel that $\forall\ell \neq 0$ $J_\ell(\cdot)$ has a $|\ell|th$ order zero in the origin. Therefore, $\forall\ell \neq 0$ and whatever the source, the corresponding far-field exhibits a null (with increasing size and depth for increasing values of $|\ell|$) in the boresight direction while still being different from zero elsewhere.

The above properties of the field suggest the opportunity of exploiting OAM beams for the synthesis of circularly symmetric difference pattern. Accordingly, a single OAM vortex is required for the scope since, if more than one vortex is used, then the circular symmetry of the generated power pattern would be lost. Moreover, the vortex cannot be neither the zero one since it is the only one leading to a non-zero field at boresight, nor the odd ones since they induce a field slope equal to zero at boresight [see Eq.(1.a)].

3. Synthesis procedure

By following a well-established strategy [10],[11], the proposed array-antenna design procedure is composed of two consecutive steps, i.e., the identification of the continuous aperture source achieving the best possible 'theoretical' radiation performances and, then, its discretization into the final array. These two steps are separately described in the two following subsections.

3.1 Step 1 - Synthesis of the optimal reference continuous source

The approach requires representing the visible spectrum in terms of the orthonormal singular functions of the radiation operator A_ℓ:

$$F_\ell(k') = \sum_{n=1}^{\infty} b_{\ell,n}u_{\ell,n}(k') \qquad (4)$$

$u_{\ell,n}$ being the left-handed singular functions (see [6] for more details) coming out from the singular values decomposition (SVD) of A_ℓ.Then, the continuous aperture source design can be solved as the following optimization problem:

$$\max_{b_{\hat{\ell},n}} \left\{ Re\left[\frac{dF_{\hat{\ell}}(k')}{dk}\right]_{k'=k_t} \right\} \qquad (5)$$

subject to:

$$Im\left[\frac{dF_{\hat{\ell}}(k')}{dk}\right]_{k'=k_t} = 0 \qquad (6)$$

$$Re\left[\frac{dF_{\hat{\ell}}(k')}{dk}\right]_{k'=k_t} \geq 0 \qquad (7)$$

$$|F_{\hat{\ell}}(k')|^2 \leq UB(k') \quad \forall k' \in \Omega \qquad (8)$$

where Re{·} and Im{·} denote the real and imaginary part, respectively, while $\hat{\ell}$ is the particular value of ℓ chosen by the user in order to perform the synthesis.

It is worth noting that the objective function (5) and constraints (6)-(7) allow the maximization of the amplitude of the derivative of $F_{\hat{\ell}}(k')$ in the target direction k'=k_t, while constraints (8) allow to control the SLL. Moreover, the overall constrained problem (5)-(8) is a convex programming one, and hence the globally-optimal solution can be effectively determined via local (fast) optimization procedures.

Interestingly, the problem (5)-(8) could also be repeatedly solved for different values of $\hat{\ell}$ in such a way to identify the vortex order guaranteeing the highest value of (5) for an equal SLL value. However, it is worth noting that $\hat{\ell} = \pm 1$ is expected to always guarantee the best performances as it leads to fields having at boresight a null width lower than any solutions achieved for $|\hat{\ell}| > 1$ (see results in Section IV).

Once the coefficients $b_{\hat{\ell},n}$ have been determined, the field component (4) and hence the overall spectrum (1.a) can be evaluated. Then, through an inverse Fourier transform, the corresponding source $f(\rho',\phi')$ can be also determined.

3.2 Step 2 - Sampling of the reference source

Once the optimal continuous source has been identified, it can be used both as a 'reference' and as a 'benchmark' for the design of the array. In fact, the source coming out from step 1 is exploitable not only in order to quickly identify the array elements' locations and excitations, but also to compare the array radiation performance with the maximum possible theoretical goals [10],[11].

In the case of circular-ring arrays, the discretization of the source is often performed in one of the two following different alternative ways, i.e.:

i. (for fixed-geometry arrays) by a-priori fixing a uniform array layout and exploiting only the elements excitations as degrees of freedom (see for instance [10]);

ii. (for isophoric sparse arrays) by using uniform-amplitude excitations and exploiting only the elements locations as degrees of freedom (see for instance [11]).

While noting that many further alternatives are available (such as using as degrees of freedom also the elements size and/or both their locations and excitations), in the following we exploited the alternative (i). Hence, after having fixed a proper array layout (taking also into account the desired number of elements), we applied the procedure in [10] to sample the continuous source into the elements' excitation values.

Obviously, as a 'classical' [11] circumstance, the higher the required the radiation performances, the higher the required source's variableness and consequent array's number of elements. In this respect, it is important to make the right trade-off between the final performances and the array number of elements, which comes into play anytime one aims at performing an 'optimal' synthesis of the array [12]-[14].

4. Numerical examples

In the following we report a numerical example validating the proposed technique. In particular, the aim is to compare the localization performance of the continuous source to the one of the corresponding array.

First of all, we solved problem (5)-(8) for different OAM mode number varies from $\hat{\ell} = 1$ to $\hat{\ell} = 3$ in order to identify the vortex order able to lead the best performance in terms of SLL and slope. From Fig. 1, accordingly to the statements given in [6], we can observe that $\hat{\ell} = 1$ is the most convenient one. In fact, the larger the OAM vortex, the lower the slope of the pattern at boresight.

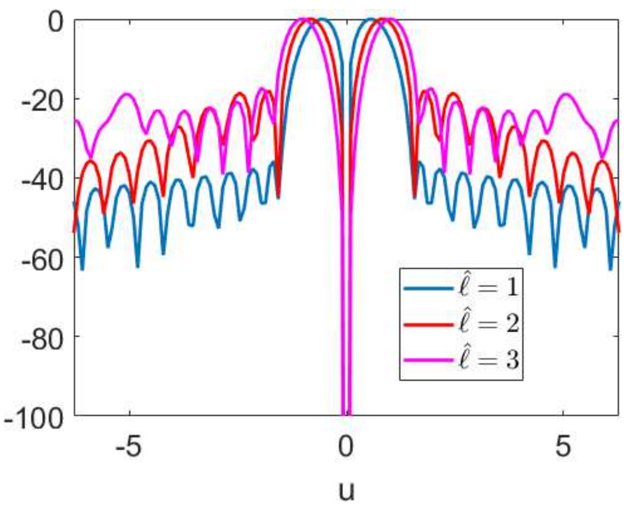

Figure 1. Radiation-pattern comparisons of OAM when mode number varies from $\hat{\ell} = 1$ to $\hat{\ell} = 3$.

After that, we discretized only the continuous source corresponding to $\hat{\ell} = 1$.

The considered continuous source has a radius $a = 5\lambda$, while the corresponding circular array layout is reported in Fig. 2. In particular, this latter is composed by 287 elements with a radial and azimuthal element spacing equal to 0.5λ. Moreover, we used isotropic elements, i.e., the power patterns shown in Fig. 3 are relative to the actual array factors.

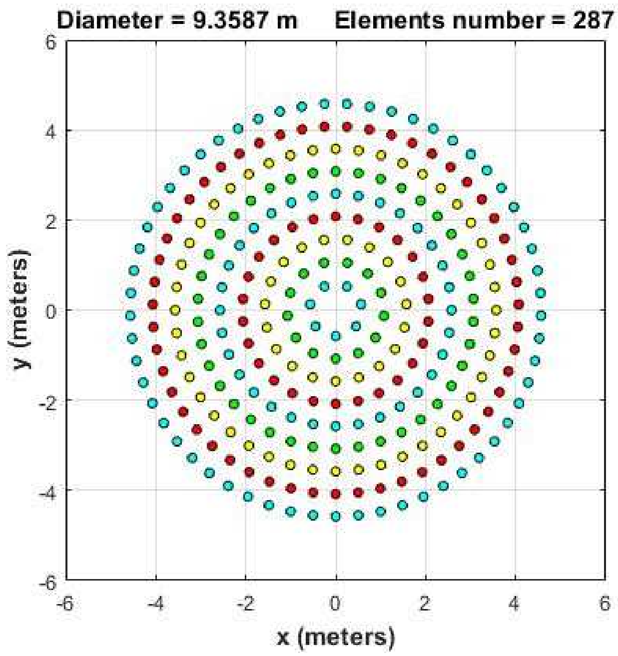

Figure 2. Circular array layout composed of isotropic elements and corresponding to the continuous source with radius $a = 5\lambda$.

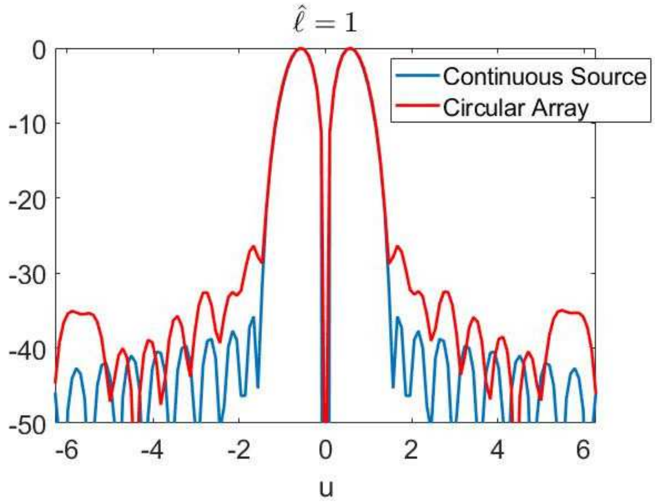

Figure 3. Radiation patterns of the reference continuous source and the ring array shown in Fig. 2 for $\hat{\ell} = 1$.

Fig. 3 shows radiation patterns comparison between the reference continuous source and the ring array for $\hat{\ell}$=1. As it can be seen, good localization performances are guaranteed, since a SLL equal to -26.4 dB is achieved for $|u|\geq 1.38$, which also testifies the effectiveness of step (i) of the devised synthesis procedure. This circumstance fully confirms the statements given in the previous Section.

Finally, it is clear that the synthesized array is able to guarantee localization performances very similar to the ones of the continuous sources, which testifies the effectiveness of the step (ii) of the proposed synthesis procedure.

Further details and examples will be given at the Conference.

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