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An effective ray-tracing by homing-in method and direct approach in anisotropic inhomogeneous ionosphere

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Abstract

The paper presents two approaches for the traditional two-point ray-tracing problem in anisotropic inhomogeneous ionosphere: the homing-in method based on global optimization and the direct approach based on Fermat's principle. While in most cases both methods are capable of finding all rays connecting the given transmitter and receiver, the direct approach tends to be more stable in locating highly divergent high rays, but the homing-in method usually offers a better tradeoff between the accuracy and computational cost of the low ray calculations. Joint application of the presented methods provides valuable means for solving applied problems of radio communication forecast and ionospheric models validation and adaptation.

1 Introduction

The ionosphere affects radio waves that are propagated within and through it. At the present time the geometrical optics approximation forms the basis for a numerical simulation of radio wave propagation in the ionosphere. The key point of modeling is a calculation of the "ray paths", which characterizes a wave front propagation. The ray information automatically provides important information about radio wave parameters such as amplitude, absorption, Doppler shift and etc. As a result, the problem is reduced to calculating the ray paths usually called as a ray tracing. Earlier, on the basis of the Hamiltonian formalism, an effective approach was created to solve the problem with initial conditions [1, 2]. A numerous models have been implemented for propagation problems in an inhomogeneous anisotropic ionosphere, which are successfully adopted in practice for the analysis of ionospheric events and radio communication forecast [3, 4, 5, 6].

On the other hand, the application formulates the problem of calculating radio waves between the known positions of receiver and transmitter. In fact, the problem is to calculate the ray path between the fixed two points. One of the main approaches to the boundary value problem is the widely used homing-in method [7, 8]. Based on this method different prediction system had been developed [9, 10]. However, the most natural approach is to solve a variation problem where boundary conditions are fixed by default. The approach to the solution of the variation equation have been proposed and have demonstrated their effectiveness, especially for highly divergent high rays [11]. Recently, a new variant of the direct optimization method for point-to-point ionospheric ray tracing was presented [12]. Various applications of this method to inhomogeneous isotropic ionosphere demonstrate its ability to resolve complex ray configurations including multi-path propagation where rays are close in the launch direction.

This paper is devoted to the implementation of the two approaches for isotropic and anisotropic ionosphere: an improved homing-in method based on global optimization and variational approach based on direct optimization of the phase path of the ray. The results obtained by the two methods for the analytical ionospheric model are compared. The prospects for their joint application to radio tomography are discussed.

2 Homing-in by the global optimization

For homing-in approach a ray tracing is performed by integrating Haselgrove's equations [1] in the way described in [3]. The cost function for optimization is defined as a following function of initial ray azimuth α and elevation ε:

$$C(\alpha,\varepsilon) = |\theta_{ray}(\alpha,\varepsilon) - \theta_{rec}|^2 + |\phi_{ray}(\alpha,\varepsilon) - \phi_{rec}|^2 + P_{space}$$

$$P_{space} = \begin{cases} \varepsilon^2, & \text{if} \quad h_{ray}(\alpha,\varepsilon) > h_{rec} \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

where $\theta_{ray,rec}$ and $\phi_{ray,rec}$ are the geographic coordinates of the final ray point and the intended receiver, respectively. Penalty P_{space} is introduced so that if a local optimization iteration ends up with a ray leaving the ionosphere, the penalty gradient becomes directed to lower elevation, leading the algorithm back into the viable search space. Correct solutions that satisfy the boundary conditions are therefore found at points where $C(\alpha,\varepsilon)=0$. The resulting cost function becomes rather complex to compute, rendering most optimization methods inefficient.

The cost function is then subjected to simplicial homology global optimization technique (SHGO) [13] as a preconditioning measure. SHGO constructs a simplicial complex (in our case, a triangulated mesh) out of a set of chosen vertices, giving a rough approximation of the surface of the cost function. When point-to-point ray tracing is considered, SHGO offers the following advantages when compared with other global optimization techniques: (a) it's derivative-free, requiring less cost function evaluations, and hence less traced rays; (b) it can determine multiple local minima with ease, especially useful when high and low rays are considered; (c) with increasing function sampling density, the number of candidate points for local optimization converges to a fixed value, avoiding numerous pointless local minima computations; and (d) most of its routines can be easily parallelized.

3 Direct variational approach

Here we present an extension of the generalized force approach [12] to point-to-point ray tracing in a magneto-active ionosphere. We start with the Fermat's principle:

$$\delta S = \delta \int_{t_A}^{t_B} n(\vec{r},\vec{u})(\dot{\vec{r}}\cdot\vec{u})dt = \delta \int_{A}^{B} \vec{n}(\vec{r},\vec{u})\cdot d\vec{l} = 0. \quad (2)$$

Here, A and B are end points, $\vec{r}$ and $\dot{\vec{r}}$ are the position vector and the unit tangent to the ray, respectively, $\vec{u}$ is the normal to the wavefront, dl is the length element along the ray, $n(\vec{r},\vec{u})$ is the refractive index defined by the Appleton–Hartree equation, $\vec{n}(\vec{r},\vec{u}) = n(\vec{r},\vec{u})\vec{u}$, and $d\vec{l} = dl\dot{\vec{r}}$. An approximate expression for the phase path functional is obtained by the midpoint rule:

$$S[\gamma] \approx S(\mathbf{r},\mathbf{u}) = \sum_{i=1}^{P+1} n(\vec{r}_{i-1/2},\vec{u}_i)\vec{u}_i\left[\vec{r}_i - \vec{r}_{i-1}\right], \quad (3)$$

where $\vec{r}_i = (x_i,y_i,z_i)$ is the position of the ith vertex, $\vec{r}_{i-1/2} = (\vec{r}_i+\vec{r}_{i-1})/2$, $\vec{u}_i$ is unit vector along impulse vector $\vec{p}_i$ given on the interval $[\vec{r}_i,\ \vec{r}_{i+1}]$, and $\vec{r}_0 = \vec{r}_A$, $\vec{r}_{P+1} = \vec{r}_B$. Here, the phase path functional is turned into a multidimensional function, $S = S(\mathbf{r},\mathbf{u})$, with $\mathbf{r} = (\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_P)$ and $\mathbf{u} = (\vec{u}_1,\vec{u}_2,\ldots,\vec{u}_{P+1})$. Considering that the impulse vector $\vec{p}_i$ of the ray and, accordingly, the vector $\vec{u}_i$ depend on $\vec{r}_i$, we can reduce the variational problem to optimization only with respect to the independent variables $\vec{r}_i$. In addition, the limitation of rotating and maintaining the unit norm of $\vec{u}_i$ should be taken into account. For this purpose, we use the method of Lagrange multipliers. The Lagrange function for the condition of $\vec{u}_i$ is given by the following formula:

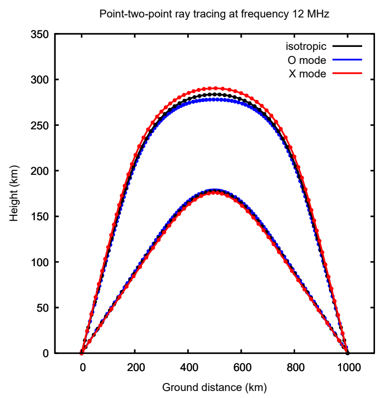

Figure 1. Results of ray calculations using direct variational approach (solid lines) and homing-in method (solid circles). The analytical ionospheric model is defined by Eq. (9). The magnetic field is equal to 0.5 T and directed upwards. The operating frequency is 12 MHz. Black, blue and red lines indicate the ray paths corresponding to the isotropic case, O-mode and X-mode, respectively.

$$S' = S + \sum_{k=1}^{P+1}\lambda_k\left(\sum_{\gamma=x,y,z}(u_k^{\gamma})^2 - 1\right), \quad (4)$$

where λ_k is the Lagrange multiplier. Using the local extremum condition $\partial S'/\partial\vec{u}_k = 0$, we write the expression:

$$\lambda_k = -\frac{1}{2}\left(\frac{\partial S'}{\partial\vec{u}_k}\cdot\vec{u}_k\right), \quad (5)$$

Formula (4) is the general expression for optimizing rays in an anisotropic ionosphere. According to Fermat's variational principle, ray paths correspond to extrema of general functional: minima for high rays and saddle points for low rays. Local minima search is based on the calculation of the first derivative of the objective function (4):

$$\mathbf{F}^{min} = -\frac{dS'}{d\mathbf{r}},\ \mathbf{R} = \frac{dS'}{d\mathbf{u}} \quad (6)$$

where $\mathbf{F}^{min} = \left(\vec{F}_1,...,\vec{F}_P\right)$ is the force vector that specifies the direction and displacement of vertices $\mathbf{r}$ to the local optimum, $\mathbf{R} = \left(\vec{R}_1,...,\vec{R}_{P+1}\right)$ is the force that specifies the rotation of vectors $\mathbf{u}$ on each interval of the piecewise linear curve. The "+" sign in expression (5) for R is justified by the choice of direction along the ray from transmitter A to receiver B. Saddle points search of functional (4) is carried out by the minimum mode following method [12]. The main idea of the method is based on the inversion of the only negative (minimum) mode of the objective function in the vicinity of the saddle point. The modified force formula for finding saddle points takes the following form:

$$\mathbf{F}^{saddle} = \begin{cases} -\mathbf{F}^{min} \quad \text{or} \quad \left(\mathbf{F}^{min}\cdot\mathbf{Q}_\lambda\right)\mathbf{Q}_\lambda, \\ \quad \text{if } \lambda \geqslant 0, \\ \mathbf{F}^{min} - 2\left(\mathbf{F}^{min}\cdot\mathbf{Q}_\lambda\right)\mathbf{Q}_\lambda, \\ \quad \text{if } \lambda < 0. \end{cases} \quad (7)$$

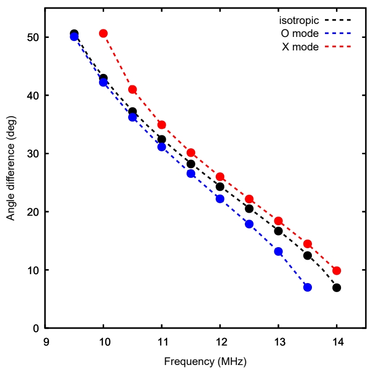

Figure 2. Result of initial angle differences between high and low rays with the same frequency obtained by direct variational approach (dashed lines) and homing-in method (solid circles). Black, blue and red colors indicate the isotropic case, O-mode and X-mode, respectively.

Here, λ is the minimal eigenvalue of the Hessian and $\mathbf{Q}_\lambda$ is the corresponding normalized eigenvector, the minimum mode. The numerical implementation is based on an iterative procedure of convergence in $\mathbf{r}$ along the generalized force $\mathbf{F}$ ($\mathbf{F}^{min}$ for minima or $\mathbf{F}^{saddle}$ for saddle points). Moreover, at each iteration, it is necessary to correct $\mathbf{u}$ along the force $\mathbf{R}$ in accordance with the condition $dS'/d\mathbf{u} = 0$. The convergence of the method can also be improved by projecting generalized force and applying the elastic force of the interaction of points between the vertices of a piecewise linear curve [12]. The implementation of the method is carried out by any methods of searching for local extrema: steepest descent, conjugate gradients, quasi-Newtonian methods, etc.

4 Results

Here we apply proposed direct variational approach and homing-in method to the point-to-point radio wave ray tracing in the anisotropic ionosphere where the electron density profile has a analytical form:

$$N_e(z) = N_0 \exp \frac{1}{2}\left[1 - \frac{z-z_0}{\Delta z_0} - \exp\left(-\frac{z-z_0}{\Delta z_0}\right)\right]. \quad (8)$$

Here, parameters $N_0 = 1.0 \cdot 10^6$ cm^{-3}, $z_0 = 300$ km and $\Delta z_0 = 75$ km are the peak electron density, peak height and thickness of F layer, respectively. The analytical approximation of the magnetic field is chosen equal to 0.5 T and directed upwards. Point-to-point ray tracing at the frequency 12 MHz using direct variational approach and homing-in method is presented in Fig.1. Here the two scenario of propagation are considered: isotropic and anisotropic (O-mode and X-mode) propagation. It is shown that calculation results given by both method for high and low rays are in complete agreement. The obtained results show the fundamental possibility of the proposed direct approach for calculating O- and X-modes in an anisotropic ionosphere. It is also should be pointed out that there is the possibility of

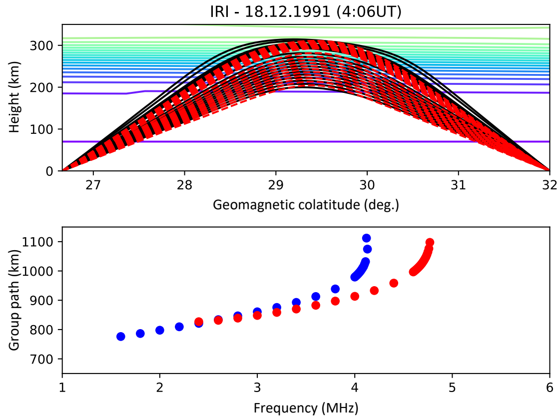

Figure 3. Ray paths and ionograms of near-vertical sounding synthesized from the IRI2012 model. Solid lines and blue dots correspond to the O-mode; dotted lines and red dots correspond to the X-mode. The electron density is presented within the range $(0-0.27)\times 10^{12}\ m^{-3}$ [15].

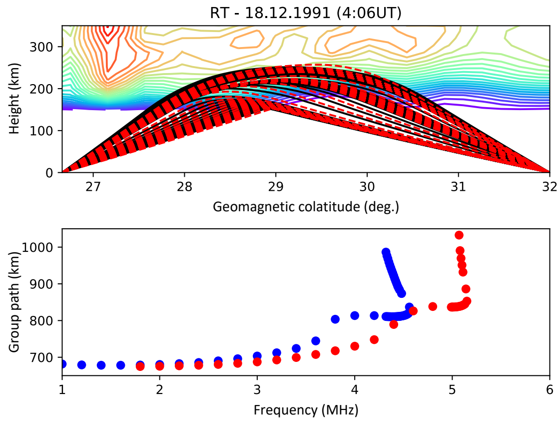

Figure 4. Ray paths and ionogram of near-vertical sounding synthesized from the RT-reconstruction data. The same designations as in the Fig. 3 are used [15].

an approximate calculation of anisotropic propagation using only isotropic ray tracing based on the frequency shift for a fixed group distance of a ray proposed in [14]. Another important aspect of method verification is the agreement at the initial radiation angles. Comparison of the obtained results is shown in Fig.2 as a frequency dependence of the difference between the angles of the radiation of the high and low rays. It can be seen that the angular difference obtained by the two methods is in a good agreement for the entire frequency range. The presented approaches have broad prospects for application in radio communication forecasting, validation and correction of ionospheric models based on radio tomography data and other experimental data. For example, Fig.3 and Fig.4 present the results of ray paths and calculations of synthesized near-vertical sounding ionograms for a hypothetical radio link using the IRI2012 model, as well as in the reconstructed ionosphere by the radio tomography data in the presence of artificial disturbances following Tsyklon rocket launch

[15]. Obviously, ionospheric climate models could not reproduce the real-time "weather", but they are a good basis for ionospheric correction.

5 Summary

The paper presents the homing-in method based on global optimization and further development of the direct variational approach for ionospheric ray tracing in an inhomogeneous anisotropic ionosphere. It is shown that for the direct approach, the optimization of the rays of an ordinary and extraordinary wave can be carried out based on the search for local minima and saddle points of an extended functional that takes into account not only the spatial position of the ray, but also the orientation of the wave front. The paper presents examples of two-point ray tracing in both isotropic and anisotropic ionosphere. Excellent agreement between the two methods has been obtained. The subject of further research is the joint application of the presented approaches for the problems of radio tomography and correction of ionospheric models.

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