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A Monte-Carlo Based Numerical Integration Method for Diffraction Computation in Ray Tracing

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Abstract

The computation of diffraction has been always a challenge in ray tracing simulations. Despite the popularity of the ray based methods such as geometrical theory of diffraction (GTD) and uniform theory of diffraction (UTD), their computational complexity grows rapidly as the number of cascaded diffraction phenomena increases. Though an efficient physical optics (PO) based bidirectional method has been introduced as an alternative, it still relies on GTD/UTD for multiple diffractions. In this work, a novel Huygens' principle based ray tracing method with an improved numerical integration technique is introduced. This method is expected to have lower computational cost and better numerical stability than GTD/UTD for multiple diffraction scenarios.

1 Introduction

A conventional ray tracing simulation launches rays in different directions from transmitters, with the expectation that some of these rays will be eventually captured by receivers after multiple reflections, refractions, or diffractions [1]. The rays captured by receivers identify valid wave propagation paths from transmitters to receivers, from which received electromagnetic (EM) fields can be calculated. Among the aforementioned ray propagation mechanisms (reflection, refraction, and diffraction), the diffraction is the most complicated [2] one, which usually requires secondary ray launches. Commonly utilized approaches of handling diffractions are based on the UTD [3], where secondary rays are launched for each ray that hits a diffraction edge. Since each diffracted ray can again be diffracted by another edge and invokes further ray launches, these approaches can lead to an exponential growth in number of rays. Besides, due to the law of diffraction [4], all diffracted rays from the same incident ray have the same diffraction angle equal to the incident angle, and thus are confined within a "Keller cone" rather than arbitrarily distributed [5], which makes them harder to hit the receivers. In order to overcome the shortcomings of ray based diffraction simulations, a reciprocity integral based bidirectional ray tracing method has recently been introduced in [6]. In this method, both transmitters and receivers launch rays towards an interaction surface adjacent to a diffraction edge. A reciprocity integral is then evaluated using the electric and magnetic fields associated with the rays hitting the surface [7]. The reciprocity integral is proportional to the received open-circuit voltage with the consideration of diffraction in a PO sense [7], therefore, the transfer function is obtained without secondary ray launches. Compared with the ray based diffraction handling, this method significantly reduces the number of ray launches without introducing errors due to large reception sphere [6] and greatly increases the chance of valid ray hits. However, diffraction caused by other edges that are not adjacent to the interaction surface still need to be calculated by UTD [7]. In order to extend this method to calculating multiple diffractions, while avoiding using UTD, a Huygens' principle based method is presented in this work.

The Huygens' principle based method can have multiple interaction surfaces, and each surface serves as a collection of equivalent sources for secondary ray launches according to the Huygens-Fresnel principle [8]. When assigning one surface to each diffraction edge, this Huygens' principle based approach is capable of evaluating multiple diffractions in a PO sense while avoiding the use of UTD. Although secondary ray launches are also required in this method, the computational complexity is usually lower and the results are more robust than that of UTD. This is because the core of the Huygens' principle based method is numerical integration, which implies that the diffracted fields are determined by the accumulation of a large number of rays, rather than relying on a single or very few important rays as in UTD. Therefore, no significant inaccuracy will be introduced when only a few rays are missed, unlike in UTD. Despite the advantages over UTD, there are mainly two challenges in the Huygens' principle based method when evaluating multiple diffractions. First, the cascading of interaction surfaces corresponds to a high dimensional integral, and second the integrand is generally highly oscillatory for high frequencies. The first issue is typically resolved by the Monte-Carlo method [9], but due to the second issue, a naive Monte-Carlo integration has poor convergence. In order to improve the convergence of the naive Monte-Carlo integration method, a new numerical integration method is introduced in this work, where the oscillation of the integrand is eliminated by local linear phase approximation.

2 Mathematical Formulation

The Huygens' principle interprets the wave propagation as a self-replicating process [10] where each wavefront can be thought of as a collection of new sources radiating secondary waves that add up to the original effect [8]. Therefore, the diffracted fields can be considered as generated by a collection of equivalent sources on the portion of the wavefront that is not blocked by the obstacle. In ray tracing, this idea is implemented by creating interacting surfaces adjacent to diffracting objects, where secondary sources are created according to the electromagnetic fields induced by the rays hitting these surfaces. In modern electromagnetics, this concept can be written as a surface integral

$$\mathbf{E}(\mathbf{r}) = -jk\mathbf{d} \times \left[\iint_{\Psi} \left(\mathbf{n} \times \mathbf{E}(\mathbf{r}')\right) \frac{e^{-jkr}}{4\pi r} ds' - Z_0 \mathbf{d} \times \iint_{\Psi} \left(\mathbf{n} \times \mathbf{H}(\mathbf{r}')\right) \frac{e^{-jkr}}{4\pi r} ds' \right], \tag{1}$$

where $\mathbf{E}(\mathbf{r}')$ and $\mathbf{H}(\mathbf{r}')$ are the electric and magnetic fields on the interaction surface Ψ, and

$$r = \left\|\mathbf{r}' - \mathbf{r}\right\|, \tag{2}$$

$$\mathbf{d} = \frac{\mathbf{r}' - \mathbf{r}}{r}. \tag{3}$$

The integral (1) can be interpreted as a summation of the fields generated by infinitesimal sources on Ψ, which naturally leads to the idea of making each ray-surface intersection a new source, so that the integral can be evaluated in a Monte-Carlo sense [10][11]. However, due to the highly oscillatory nature of the integrand in (1), the naive Monte-Carlo integration method has poor converge. In order to eliminate the oscillations, we subdivide the integral domain Ψ into small pieces $\Psi_1, \Psi_2, \cdots, \Psi_n$ ($\bigcup_i \Psi_i = \Psi$), so that the phase of the integrand is approximately linear. The criterion for the size of Ψ_i is exactly the well-known far-field condition [12]. In that case, for each small piece Ψ_i, the integral (1) can be approximated by

$$\mathbf{E}_i(\mathbf{r}) = \frac{-jke^{-jkr}}{4\pi r} \times \mathbf{d} \times \left[\iint_{\Psi_i} \left(\mathbf{n} \times \mathbf{E}(\mathbf{r}')\right) e^{jk\mathbf{d}\cdot(\mathbf{r}'-\mathbf{r}'_0)} ds' - Z_0 \mathbf{d} \times \iint_{\Psi_i} \left(\mathbf{n} \times \mathbf{H}(\mathbf{r}')\right) e^{jk\mathbf{d}\cdot(\mathbf{r}'-\mathbf{r}'_0)} ds' \right], \tag{4}$$

where $\mathbf{r}_0$ a representative point of the surface element Ψ_i. Due to the far-field condition, the phase of $\mathbf{E}(\mathbf{r}')$ and $\mathbf{H}(\mathbf{r}')$ within Ψ_i can also be approximated by a linear function of $\mathbf{r}'$. Therefore, the integrand can be written in a form of

$$\mathbf{E}(\mathbf{r}) = \iint_{\Psi_i} \mathbf{A}(\mathbf{r}') e^{j\mathbf{p}\cdot(\mathbf{r}'-\mathbf{r}_0)} ds', \tag{5}$$

where $\mathbf{A}(\mathbf{r}')$ is a smoothly varying function which can be approximated by a low order polynomial. When $\mathbf{A}(\mathbf{r}')$ is approximated by a polynomial, (5) can be evaluated analytically using integration by parts. As the area of the integral domain Ψ_i shrinks, the linear phase approximation is more accurate, but the computational complexity increases at the same time. For larger Ψ_i, the linear phase approximation (5) can be improved by averaging the integral from the linear phase approximation evaluated at multiple points. In that case, an improved linear approximation for Ψ with larger area can be written as

$$\mathbf{E}(\mathbf{r}) = \frac{1}{N} \sum_{j=0}^{N} \iint_{\Psi_i} \mathbf{A}_j(\mathbf{r}') e^{j\mathbf{p}_j\cdot(\mathbf{r}'-\mathbf{r}_j)} ds', \tag{6}$$

where $\mathbf{p}_j$ are the local linear phase factor at $\mathbf{r}_j$. In a practical ray tracing simulation, $\mathbf{r}_j$ are the ray-surface hit intersections, which are randomly distributed in Ψ_i if the rays are randomly launched, therefore, (6) can be considered as an approximation in a Monte-Carlo sense. The key advantage of this method is its simplicity of parallelization compared to a traditional quadrature method, because each ray-surface intersection contributes to the final fields independently, without the need for interpolation. This enables an efficient implementation for multiple surface scenarios.

3 Implementation in a Ray Tracer

A ray tracing simulation using the Huygens' principle based method contains at least two ray launching phases. In the first phase, a sufficient number of rays in all directions are launched from the transmitters, which propagate and bounce in a geometrical optics (GO) sense. Once a ray hits the nearest interaction surface, an equivalent source corresponding to a small piece of area containing the ray hit point is activated. Such pieces commonly consist of a single or multiple triangles of a mesh representing the interaction surface. In the second phase, the rays are launched by all those activated equivalent sources towards the other side of the interaction surface. These rays again propagate following the rules of GO, until they are captured by the receiver or encounter another interaction surface. This process is repeated if multiple interaction surface exists. Rays that are finally captured by receiver will then contribute to the received electromagnetic field.

The electric and magnetic field variation due to the GO propagation can be written as [12]

$$\mathbf{E}(s) = \mathbf{E}_0(\mathbf{d}) \sqrt{\frac{\rho_1 \rho_2}{(\rho_1 + s)(\rho_2 + s)}} e^{-jks}, \tag{7}$$

$$\mathbf{H}(s) = \frac{1}{Z_0} \mathbf{d} \times \mathbf{E}(s), \tag{8}$$

where s is the ray path length, and $\mathbf{E}_0(\mathbf{d})$ is the radiation intensity of the (equivalent) source launching this ray. If the ray is launched by an actual antenna, $\mathbf{E}_0(\mathbf{d})$ can be obtained by the radiation pattern of the antenna. If it is launched by a Huygens equivalent source, $\mathbf{E}_0(\mathbf{d})$ can be written as

$$\mathbf{E}_0(\mathbf{d}) = -jk\mathbf{d} \times \left[\iint_{\Psi_i} \left(\mathbf{n} \times \mathbf{E}(\mathbf{r}')\right) e^{jk\mathbf{d}\cdot(\mathbf{r}'-\mathbf{r}'_0)} ds' - Z_0 \mathbf{d} \times \iint_{\Psi_i} \left(\mathbf{n} \times \mathbf{H}(\mathbf{r}')\right) e^{jk\mathbf{d}\cdot(\mathbf{r}'-\mathbf{r}'_0)} ds' \right] \tag{9}$$

according to (4), which can be evaluated using (5) when the area of Ψ_i is sufficiently small, or using its Monte-Carlo variant (6) for larger Ψ_i.

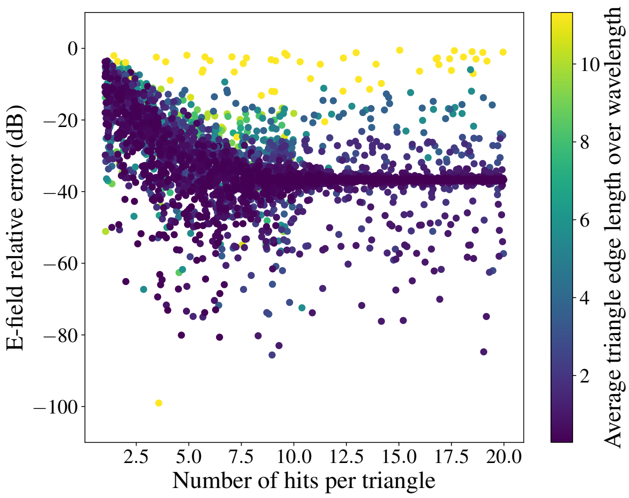

Figure 1. Relative error of linear phase interpolated Monte-Carlo integration method (the darkest color corresponds to average triangle edge length of 0.28 wavelength)

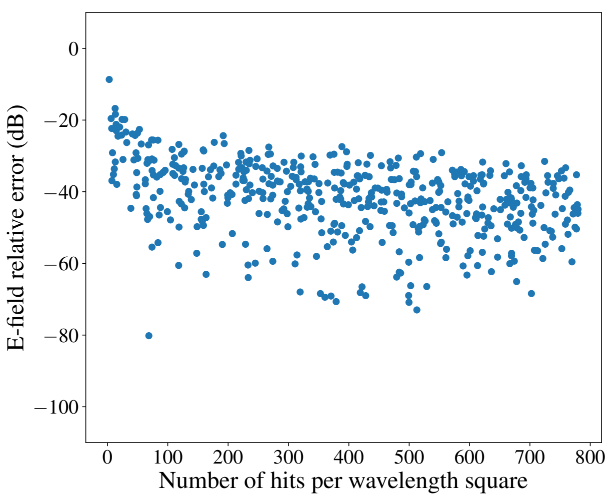

Figure 2. Relative error of naive Monte-Carlo integration method

4 Numerical Results

Numerical experiments for simple scenarios are carried out for proof of concept. For the considered scenarios, the linearly polarized isotropic antennas are used for both transmitter and receiver, and their polarizations are considered to be matched. All considered antennas have $50\,\Omega$ impedance and work with port currents of $0.025\,76\,\mathrm{A}$, which corresponds to an electric field magnitude of $1\,\mathrm{V\,m^{-1}}$ at $1\,\mathrm{m}$ distance.

The Huygens' principle based ray tracing is tested in a line-of-sight (LOS) scenario, where a transmitter and a receiver are placed at $(-3\,\mathrm{m}, 0\,\mathrm{m}, 0\,\mathrm{m})$ and $(3\,\mathrm{m}, 0\,\mathrm{m}, 0\,\mathrm{m})$, respectively. The electromagnetic fields are assumed to be directly transmitted from the transmitter to the receiver through space, therefore, the electric field at the receiver is calculated analytically using (7) as a reference. Meanwhile, by placing an interaction surface in between the transmitter and the receiver, the received electric field can be calculated using the Huygens' principle. The interaction surface is configured as a rectangular surface of the size $10\,\mathrm{m} \times 10\,\mathrm{m}$ and is partitioned into a triangular mesh. Each triangle of the mesh serves as an equivalent source whose radiation pattern is evaluated by (5) and (6). For different triangle sizes and numbers of ray hits, the accuracy of (5) and (6) is different. This is shown in Fig. 1, where the relative error in decibel (dB) versus the numbers of ray hits and triangle sizes divided by wavelength is plotted. Fig. 1 clearly shows a trend of accuracy increase with respect to the number of ray hits per triangle for almost all different sizes of the triangles, which shows the effectiveness of the Monte-Carlo sense average expressed by (6). For comparison, a naive Monte-Carlo integration method is also applied for the same configuration, where each ray hitting the Huygens plane becomes a secondary emitter. In that case, the Huygens surface does not have to be divided into small triangles, therefore, the ray hit density is measured by the average number of ray hits per wavelength square. The relative error versus the ray hit density is shown in Fig. 2. Note that in Fig. 1 the darkest color corresponds to a triangle mesh with average triangle edge length of 0.28 wavelength, which means that $-40\,\mathrm{dB}$ relative error can be achieved with high probability using roughly 150 hits per wavelength square. On the other hand, using the naive Monte-Carlo method, there is only around 50% chance of obtaining a relative error below $-40\,\mathrm{dB}$ even using 800 hits per wavelengh square as can be seen from Fig. 2. This shows the significant advantage of the newly proposed integration method over the naive Monte-Carlo integration method.

5 Conclusion

A Huygens' principle based ray tracing method with a novel numerical integration technique was presented. This method has potentially lower numerical complexity in diffraction calculation and is very flexible due to its ability of using multiple interaction surfaces. Numerical results have shown the feasibility of this method in simple scenarios, but without loss of generality. The same idea can be easily extended to more complicated situations.

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