

A multilevel \mathcal{H}^2 -based preconditioner for the Electric Field Integral Equation

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1 Extended Abstract

We present a low-complexity, data-sparse and kernel-independent preconditioner for solving dense linear systems $Ax = b$ arising from the discretization of the Electric Field Integral Equation (EFIE). We build fast solvers based on the recently developed \mathcal{H}^2 -matrix representation, that consists in replacing full dense matrix blocks by low-rank approximants computed from few entries of the original matrix without any knowledge of the underlying kernel [1]. Blocks that are admitted to have such low-rank approximation are selected on the basis of a simple *admissibility criterion* based on geometric information from the underlying mesh, such as

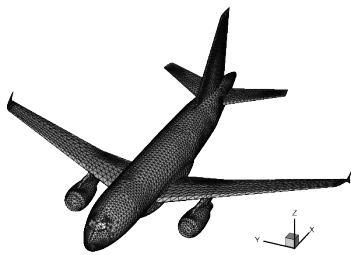
$$\max(\text{diam}(B_s), \text{diam}(B_t)) \leq \eta \cdot \text{dist}(B_s, B_t) \quad (1)$$

for some $0 < \eta$. In (1), B_s and B_t are rectangular boxes bounding the two clusters of mesh nodes denoted by s and t , respectively, and the distances (dist) between s and t and the diameters (diam) of these clusters are computed with respect to the Euclidean norm from the center of gravity of the box. The object is embedded in a large rectangular bounding box that is recursively partitioned into smaller boxes until the admissibility condition (1) is not satisfied, and a cluster tree data structure is produced. At this stage, a hierarchical \mathcal{H}^2 -matrix block-wise partitioning of the coefficient matrix A is constructed by associating the matrix block $(A_{ij})_{i \in s, j \in t}$ to the cartesian product $s \times t$ of the two clusters s, t . An admissible block A_{st} associated to the cluster of mesh nodes s and t is represented by the rank- k factorization

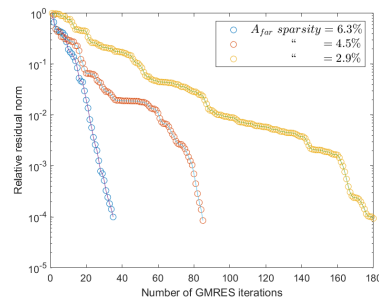
$$A_{st} = V_s S_k V_t^H,$$

where V_s of size $\#s \times k$ and V_t of size $k \times \#t$ (here symbol $\#$ denotes the set cardinality) are called *cluster bases* of clusters s and t , while S_k of size $k \times k$ is called *coupling matrix*. In the \mathcal{H}^2 -matrix representation, the cluster bases have a useful nested property: the bases V_s for a non-leaf cluster s in the tree can be expressed by its two children's bases V_{s_1} and V_{s_2} via two *transfer matrices* T_{s_1} and T_{s_2} . Because of this nested property, only the leaf bases and the transfer matrices need to be stored, with an obvious computational advantage.

In this talk, we present the fundamental steps of the approximate matrix factorization (used as a preconditioner for Krylov subspace methods [2]) yielding a complexity of $\mathcal{O}(n \log(n))$, n being the number of degrees of freedom (dofs), and we show a preliminary numerical convergence analysis on both academic and industrial problems.



Aircraft mesh. Courtesy of EADS-CCR
Toulouse.



Convergence histories on the Airbus
aircraft mesh.

References

- [1] Steffen Börm, *Efficient Numerical Methods for Non-local Operators*. European Mathematical Society Publishing House, December 2010.
- [2] Y. Saad, *Iterative Methods for Sparse Linear Systems*. SIAM, Philadelphia, 2nd edition, 2003.