



## Characteristic Eigenvalues and their Interpretation using Power Quantities

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For a fixed frequency and an arbitrarily shaped lossless object, characteristic mode (CM) analysis yields a set of characteristic eigenvalues and their corresponding characteristic eigencurrents. The fields radiated by the characteristic currents are orthogonal and the eigenvalues measure how well each mode radiates. In this presentation, we discuss how the characteristic eigenvalues can be interpreted in terms of power quantities when the object is either perfectly electrically conducting (PEC) or a homogeneous lossless dielectric.

For PEC objects, the theory based on the electric field integral equation [1] is elegant, well understood, and widely implemented. In this case, the characteristic eigenvalues give the ratio of the net reactive power to the radiated power of the fields radiated by the corresponding characteristic current. This follows from the complex Poynting theorem [1]. For PEC spheres, we can also show that the characteristic eigenvalues expressed using the Mie series solution [2] have the same physical interpretation.

For dielectric objects, there are several integral equation formulations that give the same characteristic eigenvalues [3] and for a sphere we also get the same eigenvalues using Mie series [2]. However, although several formulations agree on the eigenvalues, the published physical interpretations in terms of electromagnetic power are different [4–6]. To provide a better understanding of this confusing situation, we take a new look at the electromagnetic power quantities that follow from CM formulations based on Mie series [2], volume integral operators [7], and surface integral operators [6].

### References

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