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The Motion of Electrons under the Action of Inertial Forces in the Rarefied Medium

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Abstract

Present work is devoted to the electron-theoretical study in a novel problematic context with the help of numerical simulation. The motion of an electron in a rarefied dielectric medium, in a rotating reference frame, is described (the actions of classical inertial forces are taken into account). A special criterion for the occurrence of the electron drift motion in the rotating rarefied medium is derived. It was found that the radial current lifetime is complexly dependent on the ratio of the particle collision frequencies and angular velocity of rotation. An expression for the generalized eikonal was obtained. This expression takes into account the change in the dielectric medium conductivity over time.

1 Introduction

Present work is devoted to the novel electron-theoretical study. The general problem of the particles tracing in a medium under the action of an external force is solved. The main aim is to obtain the electrodynamics parameters of the system. The specific electrons dynamics have been considered for a long time by many researchers. In this context, we pay special attention to works [1] – [5]. Here, we are interested in a charged particles motion in a rarefied dielectric medium under the action of inertial force. That is why we place in the system a rotating reference frame in which centrifugal and Coriolis forces act. The main motivation for this study was the need to obtain a general eikonal equation for a rotating reference frame (for the time domain full-wave solution, similar to [6]). The state of the medium in the absence of the external electromagnetic fields should complement the eikonal from [7]. The practical significance of the obtained results is the most extensive, but for us, first of all, the possibility of more accurate moving medium remote sensing is important (see, *eg*, [8]).

A similar problem was considered by Shiozawa in [1]. Inspired by this work, we continue to study the electron theory with two fundamental differences. First, there is no external electromagnetic field in our case. And second, we postulate the presence of free charge carriers in a dielectric medium. The criterion indicated by Shiozawa for neglecting the action of centrifugal force on electrons bound to nuclei is not applicable for conduction electrons. Thus, this work expands and deepens the topic raised. The charged particles' presence effects in a moving medium are considered by Kong in [9]. Spectral characteristics of the radiation may be obtained from the kinematic characteristics of the particles [10].

A non-inertial reference frame may be associated with both material and matter-free medium. In these systems, the material and matter-free electromagnetic effects appear, respectively [11]. The classical eikonal equation can be used to describe these effects [12, 13]

$$[\nabla S(\mathbf{p},t)]^2 = N_\Sigma(\mathbf{p},t)^2, \qquad (1)$$

where $N_\Sigma(\mathbf{p},t)$ – general refraction index of medium at the coordinate $\mathbf{p}$ and time t. Matter-free effects are described in [7] and [11]. Material effects in the steady state (or with rest frame approximation) are described by bianisotropic medium equations [1, 9, 11]. And only transient processes (*eg*, electrons motion) in the medium remain unaccounted in (1).

The structure of this work is as follows: a detailed problem formulation is given in Sec.2. This section describes the drift motion of electrons. A special criterion for the occurrence of such a movement is derived. In the next section, the main dynamic characteristics of particles are given. Next, we very briefly describe the simulation performed and provide the main results in the conclusion.

2 Formulation

The center of a thin circular disc with the radius R is located in the (x,y,z) origin. The axis of rotation passes through the center of the disk, and the angular velocity vector $\vec{\Omega}$ is perpendicular to the $x0y$ plane (parallel to the z axis). The disc material is a non-ideal dielectric in which ohmic conduction may exist (see, *eg*, [14]). Moreover, the dielectric is not dense (rarefied medium). This condition provides the possibility of a nonzero displacement of an electron under the action of an inertial force. Hence, an essential *a priori* parameter is the conduction electrons concentration $n_e = \frac{N_e}{V} \neq 0$. In this work, we consider only the drift motion of electrons, i.e. movement with an average displacement along the force vector. The fact, confirmed by simulation, is that at achievable velocities, the drift of free electrons is possible only in a rarefied medium (most likely in a gas).

2.1 Electron drift

A centrifugal force acts on the electron as $\mathbf{F_{cf}} = \Omega^2 m_i \mathbf{r}$, where m_i – inertia electron mass. We describe the action of this force on an electron by an equivalent electric field: $\mathbf{E_{cf}} = \frac{\Omega^2 m_i \mathbf{r}}{e}$, where e – elementary charge. A current $\mathbf{J_{cf}} = \sigma \mathbf{E_{cf}}$, caused by the electrons drift under the action of inertial forces. By definition of the value we write an expression for the electron drift velocity: $\mathbf{v_d} = \mu_e \mathbf{E_{cf}}$. Electron mobility: $\mu_e = \frac{e\tau}{m^*}$, where m^* is the effective electron mass and relaxation time is $\tau = \frac{\varepsilon}{\sigma}$, where $\varepsilon = \varepsilon_r^* \varepsilon_0$ is absolute permittivity of the disc material [15] (p. 60). Hence,

$$\mathbf{v_d} = \tau k_m \Omega^2 \mathbf{r}, \tag{2}$$

where $k_m = \frac{m_i}{m^*}$.

According to the classical Drude model of conductivity, we may write that $\sigma = Q\mu_e$ (see, *eg* , [14]), where total charge is $Q = N_e e$ and N_e is a number of conduction electrons. From this expression we can get the electrons concentration:

$$n_e = \frac{\sigma}{QV\mu_e} = \frac{\sigma m^*}{QVe\tau} = \frac{\sigma^2 m^*}{QVe\varepsilon}. \tag{3}$$

It is necessary to mention a few words about the ε_r^* value. To characterize the electric induction and charge relaxation under the action of an external electric field, the relative permittivity in τ corresponds to the static field [15] (p. 60–61). All charged elements of matter (from the largest to the smallest) are polarized in an external electrostatic field. The equivalent field $\mathbf{E_{cf}}$ introduced by us coincides with the truly electric field only in action on conduction electrons, but not on dipoles, atoms, bound electrons, *etc*. Thus, we are forced to introduce the so-called effective dielectric constant ε_r^*. By definition of the value $\varepsilon_r^* = \frac{\mathbf{E_{cf}}}{\mathbf{E_{cf}}+\mathbf{E_d}}$, where $\mathbf{E_d}$ – macroscopic depolarizing electric field inside the medium. Let us take into account that $\mathbf{E_d} = \frac{\mathbf{P}}{\varepsilon_0}$. The dielectric polarizability, in the absence of an external field, only under the action of the centrifugal force was considered by Shiozawa in [1]. Assuming the centrifugal force negligence criterion to be valid, we have that $\mathbf{P} \to 0$, hence $\varepsilon_r^* = 1$.

Let us expand the consideration in (3) as follows [14]:

$$N_e = N_c exp\left[\frac{-E_g}{2k_B T}\right], \tag{4}$$

where N_c is the effective density of states of the conduction band, E_g is the energy band gap, k_B is the Boltzmann's constant and T is the temperature. Valuable expressions can be found in [16]. According to Stokes' law, we write down the friction force as

$$F_N = c_s \rho A v_d^2, \tag{5}$$

where c_s is a dimensionless coefficient, ρ is a dielectric density and A is a particle cross-section area. The following requirement must be met everywhere: $\mathbf{F_N} = -\mathbf{F_{cf}} = -e\mathbf{E_{cf}}$. Thus, a useful equality obtained [16]

$$eE_{cf} = c_s \rho A v_d^2. \tag{6}$$

The inert electron mass is an experimentally measured value. We will apply its gravitational value from the ratio $\frac{m_i g}{e} = 5.6 \times 10^{-11}$ V/m [17], where g – acceleration of gravity. Hence, $m_i \approx \frac{8.972 \times 10^{-30}}{g}$ kg. The quantity A in this case may be Thomson cross-section, for the electron equal to $A \approx 6.652 \times 10^{-29}$ m^2 .

2.2 Drift criterion

If the drift velocity is zero, then the value (5) vanishes, and equality (6) ceases to be true for non-zero Ω. Thus, at the beginning of the rotation, the electrons are not immediately involved in the drift motion. If their energy is greater than E_g, then particles begin to displace relative to the initial position. But the emerging interaction with the medium has the opposite effect. Thus, we introduce the striction force $\mathbf{F_s}$ into consideration. We will assume that

$$\mathbf{F_s} = -m_i f_c \mathbf{v_o}, \tag{7}$$

where f_c is the collision frequency and v_o is the linear velocity of a particle in oscillatory motion (co-directional with drift velocity). The value v_o may be interpreted in different ways, it is most convenient for us to take it equal to the minimum drift velocity (the velocity from which the electron drift begins). At the moment of the drift beginning, the Stokes force appears and the static friction force transforms into a dynamic one. Let us equate (5) and (7) at the specified moment and express the velocity

$$v_o = \frac{m_i f_c}{c_s \rho A}. \tag{8}$$

Expression (8) itself provides the value of the minimum drift velocity of particles in the system. However, we may get additional information from it, given that $f_c = A n_b v_o$, where n_b is the concentration of medium particles (background density number). Considering also that $\rho = n_b m_b$, where m_b is the mass of medium molecules we get

$$c_s = \frac{m_i}{m_b}. \tag{9}$$

3 Dynamics

To begin with, it is important to determine the current $\mathbf{J_{cf}}$ lifetime, *i.e.* the time it takes for the electron displacement equal to the disk radius. Using the standard kinematic relations, we can write that $t_l = \frac{2R}{v_d(r=R)} = \frac{2}{\tau k_m \Omega^2} = \frac{2R\sqrt{c_s \rho A}}{\Omega \sqrt{m_i R}}$. We may obtain useful notation by applying the criterion described in Section 2.2, and in particular the ratio (9) $t_l = \frac{2}{\Omega}\sqrt{R n_b A}$. If we take into account that v_0 is related to the linear velocity of rotation through the dimensionless parameter $\alpha(r)$, we get

$$t_l = \frac{2}{\sqrt{\alpha(r)}} \cdot \frac{\sqrt{f_c}}{\Omega^{3/2}} = B(r) \cdot \frac{\sqrt{f_c}}{\Omega^{3/2}}. \tag{10}$$

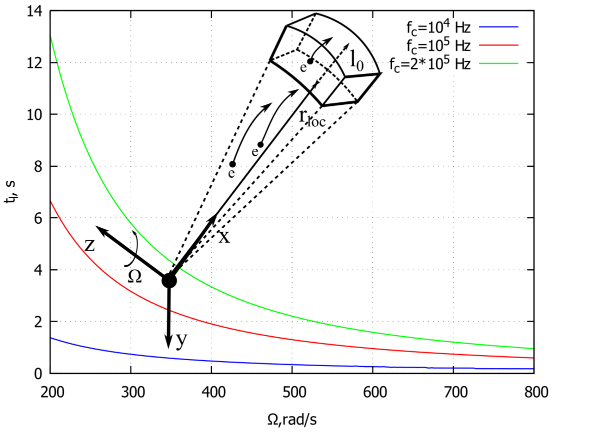

Figure 1. Unit of volume geometry definition and $t_l(\Omega)$ dependence for different f_c.

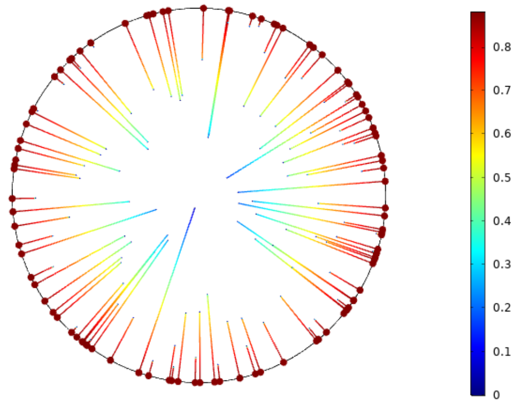

Figure 2. Particles trajectories tracing and its drift velocity ($R = 1$ m, $N_e = 100$, $\Omega = 1150$ rad/s, $f_c = 1.5 \times 10^6$ Hz).

It was found that B is linearly dependent on the $\frac{f_c}{\Omega[rad/s]}$ ratio as follows: $B = 0.046 \cdot \frac{f_c}{\Omega[rad/s]} + 36.713$. This expression is obtained for $r = R$. Fig. 1 shows the plots for $t_l(\Omega)$ function for different values of f_c according to (10).

As an explanation Fig. 1 also shows a randomly selected volume unit V_{loc} that is not located on the surface, thus $R > (r_{loc} + l_0)$, where r_{loc} is the shortest radius vector to the volume surface and l_0 is the shortest distance from one side to the other. Now we assume that (3) and (4) are constant throughout the entire volume of the disk. This statement is equivalent to the fact that the local electrons concentration (in V_{loc}) is constant over the time interval $[0; t_{loc}]$. In the time interval $T_{ex} = (t_{loc}; t_{loc2})$ electrons leave the volume. Let us denote the number of electrons in the V_{loc} during this time, as N_{eloc}. Finding the time interval T_{ex} is trivial. Exactly in this period the time dependence of the medium characteristics is observed, which must be taken into account in equation (1). Let us write down that $T_{ex} = (\frac{2r_{loc}}{v_d[r_{loc}]}; \frac{2(r_{loc}+l_0)}{v_d[r_{loc}+l_0]})$.

For the convenience, the electron drift velocity on the disk surface will be denoted as $v_s = \frac{2R}{t_l}$. Thus, the effective acceleration of an electron is $a^* = \frac{v_s}{t_l}$. The sequence of the next calculations may be as follows: $v_d(r) = \sqrt{2a^* r}$, $J_{cf}(r) = n_e e v_d(r)$, $N_{eloc}(r,t) = N_e - \frac{J_{cf}(r) S_{loc} t^2}{e}$, where S_{loc} is the volume cross-section area. Thus, in the time interval T_{ex}, the conductivity of an arbitrary volume of the medium is

$$\sigma(t) = N_{eloc}(t) e \mu_e. \qquad (11)$$

Hence, the value of $N_\Sigma(\mathbf{p}, t)$ from (1) becomes known, both in space and in time.

4 Discussion

Very useful information regarding the mobility of electrons in a dielectric material was obtained by the molecular dynamics approach in [18]. The electrodynamic interpretations of the electron kinematic characteristics are given in [10]. The main formulations of the present multiscale numerical experiment correspond to the standard discrete particle (electron) method [19]. Our first attempt to simulate the distribution of the radial currents in a rotating dielectric disk was made in [20] (in this work, only a rough estimate was made). In this case, we use the particle trajectories tracing technique with a help of a finite element solver. This approach may easily be expanded to a semiclassical theory if needed [3, 21].

The simulation was carried out in two ways (for the alternate modeling technique reference method): In the first model, electrons moved in a rarefied medium, taking into account their collision with the background particles without ionization. This model is based on an accurate description of particle-matter interaction effects (practically at all scales). In the second model, all the variety of these effects is replaced by one friction force (5) and (7). In all cases, the disk rotation was simulated directly, and the particle-particle interactions were taken into account (although for a small N_e this does not have a large effect). Particle tracing was carried out in two-dimensional space to save computational resources.

Fig. 2 shows the possible simulation visualization. The current lifetime in the simulation is $t_l = 3.010$ s, whereas the calculation (10) gives the values $t_l = 3.037$ s. The discrepancy in particle velocity is more significant (but not critical, taking into account the averaging of the value itself), so that $v_s = 0.88$ m/s in simulation and $v_s = 0.66$ m/s in calculation. This is due to the inaccuracy in taking into account the medium electrophysical parameters in the simulation (relaxation time, electron mobility, *etc.*). Thus, we assume that our approach gives more accurate results in this case.

5 Conclusion

In the present work, several important conclusions were made. It was found that the t_l value is complexly dependent on the $\frac{f_c}{\Omega}$ ratio (f_c and Ω are also not trivial related to each other). These two values are the only ones required initially. Thus, it is possible to design an experiment in which the medium electrophysical parameters can be obtained from f_c and Ω, for example, μ_e or m^*. Applying expression (11) to (1), we obtain the full eikonal. Together with expressions from [7], this allows us to carry out a full-wave simulation of the electromagnetic diffraction on a rotating dielectric disk.

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