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Matched waves for impedance boundaries

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Abstract

A matched wave in connection with a boundary is a plane wave that satisfies the boundary condition by itself. In other words, a single wave can exist, and its phase can propagate either towards the boundary or away from it. Using a four-parameter classification of impedance boundaries, the conditions for matched waves are discussed.

1 Introduction

The impedance boundary condition between tangential electric and magnetic fields reads

$$\mathbf{E}_t = \overline{\overline{\mathsf{Z}}}_s \cdot \mathbf{n} \times \eta_0 \mathbf{H} \tag{1}$$

on a surface with unit normal vector $\mathbf{n}$. Note the normalization: here the impedance dyadic is dimensionless. A well-known special case is the perfect electric conducting surface for which $\overline{\overline{\mathsf{Z}}}_s = 0$. However, due to the two-dimensionality, the impedance dyadic contains in general four independent parameters, which can be complex. Let us study how a plane wave is reflected from a boundary with condition (1).

2 Matched waves

In general the reflection from a boundary depends, in addition to the boundary parameters, also on the polarization and state of incidence of the incoming wave. Assuming a planar boundary with a given condition, there are constellations for a plane-wave excitation (incidence angle and polarization) when a single plane wave satisfies the boundary condition by itself. It is then called a *matched wave* [1]. These correspond to cases when the reflection coefficient or its inverse vanishes. In the following, let us look for the matched-wave conditions the impedance boundaries.

The condition (1) can be written as

$$\mathbf{a}_1 \cdot \mathbf{E} + \mathbf{b}_1 \cdot \eta_0 \mathbf{H} = 0 \tag{2}$$

$$\mathbf{a}_2 \cdot \mathbf{E} + \mathbf{b}_2 \cdot \eta_0 \mathbf{H} = 0 \tag{3}$$

with

$$\mathbf{a}_1 = \hat{\mathbf{x}}, \quad \mathbf{b}_1 = -Z_{xy}\hat{\mathbf{x}} + Z_{xx}\hat{\mathbf{y}} \tag{4}$$

$$\mathbf{a}_2 = \hat{\mathbf{y}}, \quad \mathbf{b}_1 = -Z_{yy}\hat{\mathbf{x}} + Z_{yx}\hat{\mathbf{y}} \tag{5}$$

Assume the coordinate system such that the planar impedance boundary is xy-plane and free space fills the half space for $z > 0$, the normal being $\mathbf{n} = \hat{\mathbf{z}}$. The incident plane wave impinges from the upper half space with wave vector

$$\mathbf{k}^i = \mathbf{k}_t - k_n \mathbf{n} \tag{6}$$

where $\mathbf{k}_t$ is the transversal part of the wave vector, being equal also in the reflected wave vector

$$\mathbf{k}^r = \mathbf{k}_t + k_n \mathbf{n} \tag{7}$$

Denote futher

$$\mathbf{c}_1^i = \mathbf{k}^i \times \mathbf{b}_1 - k_0 \mathbf{a}_1 \quad \mathbf{c}_1^r = \mathbf{k}^r \times \mathbf{b}_1 - k_0 \mathbf{a}_1 \tag{8}$$

$$\mathbf{c}_2^i = \mathbf{k}^i \times \mathbf{b}_2 - k_0 \mathbf{a}_2 \quad \mathbf{c}_2^r = \mathbf{k}^r \times \mathbf{b}_2 - k_0 \mathbf{a}_2 \tag{9}$$

$$\tag{10}$$

where $k_0^2 = \omega^2 \varepsilon_0 \mu_0$. As shown in [1], the matched-wave condition happens for

$$J^i = \mathbf{k}^i \cdot \mathbf{c}_1^i \times \mathbf{c}_2^i = 0 \tag{11}$$

for which the incident wave can exist by itself, and also

$$J^r = \mathbf{k}^r \cdot \mathbf{c}_1^r \times \mathbf{c}_2^r = 0 \tag{12}$$

in which case only a reflected wave exists.

3 Decomposition of the impedance dyadic

Following [2], let us decompose the surface impedance dyadic into four elements:

$$\overline{\overline{\mathsf{Z}}}_s = Z_I \overline{\overline{\mathsf{I}}} + Z_J \overline{\overline{\mathsf{J}}} + Z_K \overline{\overline{\mathsf{K}}} + Z_L \overline{\overline{\mathsf{L}}} \tag{13}$$

where the two-dimensional dyadics are (in the xy-base)

$$\overline{\overline{\mathsf{I}}} = \hat{\mathbf{x}}\hat{\mathbf{x}} + \hat{\mathbf{y}}\hat{\mathbf{y}} \quad \overline{\overline{\mathsf{J}}} = -\hat{\mathbf{x}}\hat{\mathbf{y}} + \hat{\mathbf{y}}\hat{\mathbf{x}} \tag{14}$$

$$\overline{\overline{\mathsf{K}}} = \hat{\mathbf{x}}\hat{\mathbf{x}} - \hat{\mathbf{y}}\hat{\mathbf{y}} \quad \overline{\overline{\mathsf{L}}} = \hat{\mathbf{x}}\hat{\mathbf{y}} + \hat{\mathbf{y}}\hat{\mathbf{x}} \tag{15}$$

Note that $\overline{\overline{\mathsf{I}}}$ and $\overline{\overline{\mathsf{J}}}$ are independent of the chosen base. The spherical coordinate angles are defined in the normal way:

$$\cos\theta = k_n/k_0, \quad \cos\varphi = \hat{\mathbf{x}} \cdot \mathbf{k}_t/k_t \tag{16}$$

4 Matched-wave conditions

4.1 Boundary of type $\overline{\overline{\mathrm{I}}}$

The simplest boundary condition is that only the dyadic $\overline{\overline{\mathrm{I}}}$ remains in (13): $Z_J = Z_K = Z_L = 0$. Then the impedance is isotropic and reciprocal, with amplitude Z_I. As is known [2], this boundary is lossless only if Z_I is imaginary. It is also known that the boundary is lossy if the real part of the impedance is positive, and active for negative values. Hence it is natural to start looking for matched wave from real Z_I values.

Due to the fact that the boundary is isotropic, the reflection coefficient is not dependent on the azimuth angle φ of the incident wave. The conditions for matched waves are the following connection between the incidence angle and the impedance parameter:

$$Z_I = \pm\cos\theta, \quad Z_I = \pm\frac{1}{\cos\theta} \tag{17}$$

where the upper signs correspond to zeros of the reflection, and lower signs for the case of existing reflection with zero incidence.

4.2 Boundary of type $\overline{\overline{\mathrm{J}}}$

For only $Z_J \neq 0$, we have the PEMC boundary [3] where the commonly-used admittance parameter M is connected to this normalized impedance as $Z_J = 1/(\eta_0 M)$. In this case the boundary is lossless for real values of Z_J. And as known, this is the only non-reciprocal impedance boundary condition.

If Z_J is imaginary, the boundary is always active, independently of the sign. This has also been noted in the study on lossy PEMC media in [4] where it was noted that PEMC cannot be lossy without an additional symmetric part to the impedance dyadic.

Again, the surface is isotropic (reflection is not dependent on φ). But what is interesting is that the matched wave condition is also independent of θ! The matching happens at $Z_J = \pm\mathrm{j}$. And both the zero and infinity of the reflection happen at the same time, independently of the incidence angle.

4.3 Boundaries of type $\overline{\overline{\mathrm{K}}}$ and $\overline{\overline{\mathrm{L}}}$

As is seen from expansions (14), impedance conditions with nonzero Z_K or Z_L make the surface anisotropic. Hence also the incidence and reflection directions for matched waves will depend on the azimuth angle φ. For these boundaries to be lossless it is required that Z_K and Z_L have to be purely imaginary. The boundaries are reciprocal for all parameter values.

Figures 1 and 2 show the conditions for matched waves in the $\theta - \varphi$ plane for $\overline{\overline{\mathrm{K}}}$ boundary with $Z_K = 2$ and $\overline{\overline{\mathrm{L}}}$ boundary for $Z_L = 1.2$. The two conditions (zero or pole of the reflection coefficient) can be seen to occur symmetrically in both cases: a 90° rotation in the azimuth plane interchanges the two matching conditions.

This also means that the sign of Z_K or Z_L does not tell whether the surface is active or passive. It depends on the incidence angle and polarization of the wave.

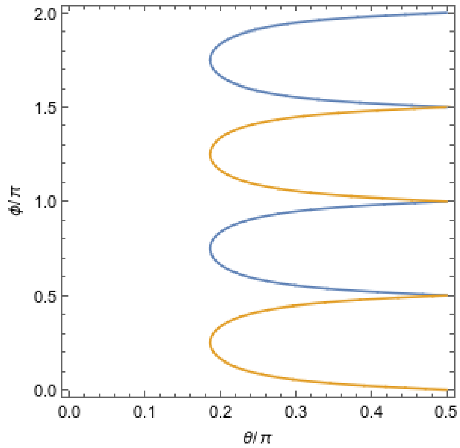

Figure 1. The conditions for a matched wave for a boundary condition with $\overline{\overline{\mathrm{Z}}}_s = 2\overline{\overline{\mathrm{K}}}$. Blue line: matched incident wave, orange line: matched reflected wave.

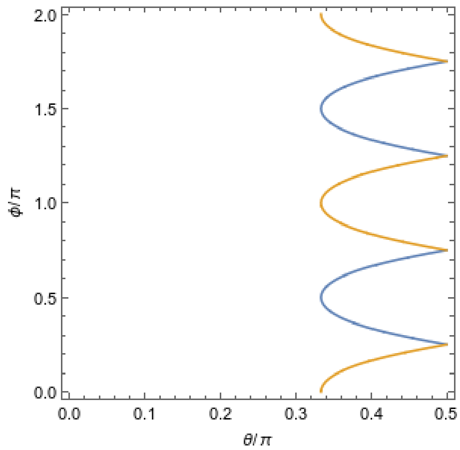

Figure 2. The conditions for a matched wave for a boundary condition with $\overline{\overline{\mathrm{Z}}}_s = 1.2\overline{\overline{\mathrm{L}}}$. Blue line: matched incident wave, orange line: matched reflected wave.

5 Conclusion

The conditions for existence of matched waves in connection with basic classes of impedance boundaries were discussed. In the presentation, we will extend the results to the polarization of matched waves and also discuss the perspectives of complex-values impedance parameters for which surface- and leaky-wave type matching may occur.

References

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