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Regularization – a mathematical tool to stabilize pulse recovery

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Due to the limited response time of electronics it is not possible to directly measure ultra-short laser pulses in the femtosecond range. Instead, one has to transform the unknown pulse such that an indirect measurement can be made. The difficulty now lies in the mathematical reversion of the transformation given the measured data. Examples for such transformations are the variants of FROG, SPIDER, and D-SCAN.

In the mathematical community, such a relation between the unknown, sought-after cause and its measurable effect, which are connected by a mathematical model describing the (forward) transformation, is called an inverse problem. Such inverse problems may be ill-posed. This means that no solution may exist to a given set of data, if a solution exists, it may be ambiguous; and even if the solution is unique small errors in the data may lead to arbitrarily large deviations in the solution. One might roughly say that this instability is due to the lack of information. Given a set of measured data, one can only hope to retrieve a certain amount of information about the unknown and/or the model. If this amount is exceeded, the solutions may become meaningless. A general tool to deal with these issues is regularization. Properly regularized algorithms will yield reasonable and stable approximations to the exact solution even when the data is insufficient or contaminated by noise.

In the first part of the talk we introduce the concept of ill-posedness and regularization in more detail and demonstrate the effects in simple numerical experiments. We show that it is often a rather simple extension of existing algorithms that are altered in a way that excludes solutions that are incompatible with user-specified a-priori knowledge of the unknown. The confidence in this a-priori knowledge is typically controlled by a real valued regularization parameter that has to be chosen appropriately. We discuss and give examples how regularization could be carried out in typical situations.

In the second part we move to a highly ill-posed variant of SD-SPIDER for which regularization is inevitable in order to obtain good approximations to the unknown laser pulse. We present our method of choice and describe how the specific measurement situation and the physical background knowledge helped to design a unique and successful retrieval algorithm [1,2].

We then move to D-SCAN. Our interest here lies in dealing with unstable pulse trains. Since it is no longer possible to recover the exact shape of the laser pulse, we present a model that allows to detect unstable pulse trains and to estimate the average length of the single pulses. Furthermore, our approach compensates for possible errors in the modeling of the transformation underlying D-SCAN. Due to the possible ambiguities it is again necessary to use regularization in order to obtain reasonable estimates for the unknown quantities.

1. S. Birkholz, G. Steinmeyer, S. Koke, D. Gerth, S. Bürger, B. Hofmann, “Phase retrieval via regularization in self-diffraction-based spectral interferometry,” *Journal of the Optical Society of America B* , **32**, 5, 2015, pp. 983-992, doi:10.1364/JOSAB.32.000983.

2. S.W. Anzengruber, S. Bürger, B. Hofmann, G.Steinmeyer, “Variational regularization of complex deautoconvolution and phase retrieval in ultrashort laser pulse characterization,” *Inverse Problems*, **32**, 3, 2016, 035002 (27pp), doi: 10.1088/0266-5611/32/3/035002