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Random processes metamodeling applied to dosimetry

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Abstract

Advanced computational tools take advantage of progress in high-performance calculation (HPC). Nowadays the wireless network design is simulated using software for the purpose of predicting coverage, capacity and throughput of planned networks. Despite these progress in HPC, uncertainty quantification requests enormous calculations which are often unaffordable due to the computational burden. Surrogate models have been investigated to overcome such a limitation and are used to date in most of the engineering domains. Previous study explored a 4G-induced exposure to Electromagnetic Fields in terms of propagation environments using stochastic city models. These stochastic models are governed by a few Constant Parameters (CP), such as, built surface, height of building or length of streets. The influence of these CP on the EMF exposure should be investigated by a sensitivity analysis. To do so, an efficient surrogate method dedicated to random processes is needed. Relative study is important but scant. To overcome such a limit, this paper proposes an innovative approach to built surrogate models for stochastic simulators based on Karhunen-Loève expansion combined with the polynomial chaos expansion to surrogate the covariance.

1 Introduction

To assess the day-to-day exposure generated by both uplink and downlink transmissions for an entire population in a geographical area, a new exposure metric called Exposure Index (EI) was developed in the framework of the European LEXNET project [1]. To characterize the EMF exposure using such an EI metric, many parameters, e.g., propagation environments, received and emitted powers, etc. should be taken into account. As a matter of fact, RF wave propagation in a geographical area plays an important role in assessing received and emitted powers as well as mobile throughputs. Meanwhile, building and terrain data vary from one city to another. One of the main challenges in RF waves propagation modeling is to integrate influences of these data into the propagation model. Theoretically, the received power P can be modeled in terms of the distance d between the transmitter and the receiver as well as the Path Loss Exponent (PLE, a.k.a., α), as follows:

$$P(d) = \beta - 10\,\alpha\, log_{10}(d) \tag{1}$$

Let us consider a received power vector $\mathbf{P}$ according to the corresponding distance vector $\mathbf{d}$. Through the approach of least square solution for linear systems, the path loss model can be characterized by parameter α and β. Indeed, α is linked to the energy attenuation between the transmitter and the receiver, which varies widely across the propagation environment (e.g., α equals to 2 in free space). Therefore, to cover propagation environment possibilities, stochastic geometry was used in modeling the building distribution and terrain topology [2]. City models were built stochastically and governed by the same rules of urbanization as shown in Table 1. One example selected from 500 3-D urban-based city samples is illustrated in Fig.1.

Table 1. *Mean values for morphological features of a typical urban city*

Parameters	Mean values
x_1: Street width	14 m
x_2: Building height	15 m
x_3: Building facade	50 m
x_4: Anisotropy	0.7

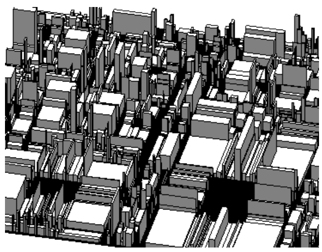

Figure 1. *3D stochastic city model*

Thus a statistical approach was proposed to estimate the probability law of the exponent α via ray-launching technique using stochastic city models. Such a ray-launching technique is often used to propagate EMF in urban areas [3]. Briefly, N rays were launched from an antenna installed at the center of a given sample city (500 samples in total), and they produce reflections. Hence, the EM attenuation map can be estimated by assessing the rays that hit the measure plane 1.5 m above the ground. Finally, the corresponding α can be estimated via Equation (1) by using this power map. Results derived from 500 simulations shows that a typical urban-based α follows a Gamma distribution. In the study of Huang et al.[4], this distribution was further used as input to evaluate the population RF exposure.

It should be noted that the population RF exposure is highly linked to the geographical region's characteristics (see Table 1). That is why, their influences on global EMF exposure should be investigated. However, in our case, at a given input value (i.e., building height, street width, building facade, anisotropy), the output is not one value but a probability density function (i.e., distribution of α) that needs to be characterized. As a consequence, surrogating a random one is an emerging question to be solved. The question of surrogating stochastic simulators has arisen only recently in the literature, mainly based on surrogating, only, the two first moment of the output distribution [5], or based on the assumption that the true model is a realization of a Gaussian process [6] which is a strong assumption that does not necessarily hold on practice. This study propose a general approach to surrogate the process with no assumption on the distribution.

2 Stochastic process modeling

Let $\mathscr{H}(t,\omega)$ be a random process of second order with zero mean, where t is a deterministic vector and ω belonging to the space of random events Ω, and $C(s,t)$ its covariance, this approach represents a metamodel based on the Karhunen-loève decomposition.

2.1 Karhunen-Loève Expansion

Karhunen-Loève (KL) expansion [7] is a spectral decomposition in a infinite linear combination of orthogonal functions as:

$$\mathscr{H}(t,\omega) = \lim_{p\to\infty} \sum_{i=1}^{p} \sqrt{\lambda_i}\xi_i(\omega)\phi_i(t) \tag{2}$$

where $\{\xi_i(\omega),\ i \in \mathbb{N}\}$ is a set of random variables to be determined, λ_n and $\phi_n(t)$ are, respectively, eigenvalues and eigenvectors of $C(s,t)$.

2.2 Surrogate modeling of a stochastic process

Traditionally KL decomposition is used to simulate and represent a stochastic process analogous to Fourier series representation of a function. In this work, KL is rather used to surrogate $\mathscr{H}$. As a first step we run simulations to build a numerical covariance of $\mathscr{H}$ denoted $\hat{C}$. Eigendecomposition of $\hat{C}$ was then carried out. The $\mathscr{H}$ surrogate, i.e. $\hat{\mathscr{H}}$, can be written as follows:

$$\hat{\mathscr{H}}(t,\omega) = \sum_{i=1}^{N} \sqrt{\lambda_i}\xi_i(\omega)\phi_i(t) \tag{3}$$

Where ξ_i are orthogonal Gaussian variables. The KL expansion is optimal in minimizing the mean square error [8].

2.3 Surrogate model of the covariance

Stochastic simulators are often time consuming, in our context one run takes 3 hours. Building the covariance matrix requires $N.M$ runs, N is the number of points whereas M denotes the realizations on each point. That is why we need to surrogate the covariance. Deterministic approaches like Kriging, Polynomial Chaos (PC) interpolation [9] or support vector machines (SVM) can be carried to surrogate $\hat{C}$. Let $\tilde{C}$ be the PC surrogate of $\hat{C}$, thus KL decomposition will be carried on $\tilde{C}$ instead of $\hat{C}$.

2.4 Case study

We consider a simulated Gaussian process defined on $[0,1]\times\Omega$ and the corresponding covariance $\hat{C}$ on N points. A PC surrogate model of $\hat{C}$ is constructed, the KL decomposition is then applied to surrogate the stochastic process. On a given new point t^* we can predict the probability distribution as in Fig.2.

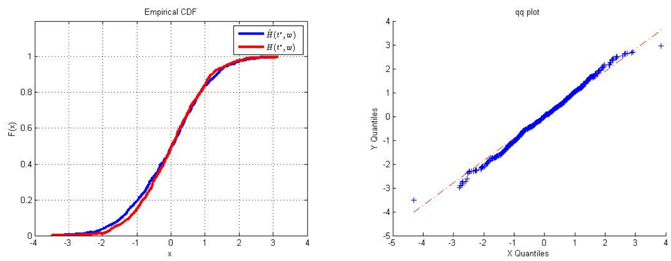

Figure 2. *left, Comparison of empirical Cumulative Density Function (blue curve) with the surrogated one (red dashed curve), right, quantile-quantile plot comparing the two probability distributions*

3 Conclusion

This work proposed an innovative approach of stochastic metamodeling using deterministic metamodeling approaches with KL spectral decomposition. The accuracy of the metamodel depends on N and M as well as the precision of the expansions. Flowing works will present the application of this approach to the exposure estimation and to the α assessment.

References

[1] N. Varsier *et al.*, "A novel method to assess human population exposure induced by a wireless cellular network," *Bioelectromagnetics*, vol. 36, no 6, pp. 451-463, 2015.

[2] T. Courtat, L. Decreusefond, and P. Martins, “Stochastic simulation of urban environments. Application to Path-loss in wireless systems,” arXiv preprint arXiv:1604.00688, 2016.

[3] J. Wiart, “Propagation des ondes radioélectriques en milieu urbain dans un contexte microcellulaire. Analyse par la GTD et validation expérimentale,” Ph.D. dissertation, University of Paris 6, Paris, France, 1996.

[4] Y. Huang and J. Wiart, “Simplified Assessment Method for Population RF Exposure Induced by a 4G Network,” *IEEE Journal of Electromagnetics, RF and Microwaves in Medicine and Biology*, vol. PP, no 99, 2017.

[5] B.Iooss and M. Ribatet, “Global sensitivity analysis of computer models with functional inputs,” *Reliability Engineering and System Safety*, vol. 25, no 94(7), pp. 1194-1204, 2009.

[6] B. Ankenman, B. Nelson, and J. Staum (2009). Stochastic Kriging for simulation metamodeling. *Operations Research*, no 58, 371-382.

[7] Roger G. Ghanem and Pol D. Spanos,*“Stochastic Finite Elements, A Spectral Approach”*, DOVER PUBLICATIONS, New York, 2003.

[8] O. P. Le Maître and O. M. Kino, *“Spectral Methods for Uncertainty Quantification, With Applications to Fluid Dynamics”*, Springer, Berlin, 2010.

[9] G. Blatman and B. Sudret, “An adaptive algorithm to build up sparse polynomial chaos expansions for stochastic finite element analysis,” *Probabilistic Engineering Mechanics*, vol. 25, no 2, pp. 183-197, 2010.