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Generalized Telegrapher's Equations for Buried Curved Wires

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Abstract

The paper deals with the calculation of transient currents and voltages along curved wire structures buried in a lossy ground using the generalized Telegrapher's equations. The influence of a lossy half space is taken into account by the simplified reflection coefficient arising from the modified image theory (MIT). The concept of the scattered voltage is computed via boundary element formalism provided the current distribution is already known. Computational examples are given for the transient voltage induced along the grounding grid and grounding system for wind turbines (WTs).

1. Introduction

Electromagnetic modeling of buried wire configurations has numerous applications, e.g. in the analysis of underground cables, or grounding systems [1]–[4]. For most of the applications dealing with straight buried wires, the transmission line (TL) approach is used (e.g. [3], [5-8]), primarily in the frequency domain, due to the simplicity of the formulation and computational efficiency. The TL approach, on the other hand, is mostly limited to straight wire structures and fails to account for the radiation effects at higher frequencies [3].

An alternative is to use the antenna theory (AT or full-wave) approach, whose main disadvantage is the complexity of formulation and high computational cost [3] [5]. A possible way to reduce the computational cost is to combine TL and AT approaches, a task that has been carried out in [2] for the case of straight buried wires. Moreover, it is rather important to find a clear theoretical relationship between the standard telegrapher's equations and integral equations arising from the wire antenna theory including the effect of a lossy ground.

The present work extends the work carried out in [2] to the analysis of curved wires buried in a lossy ground [2]. The influence of a lossy ground is taken into account via the simplified reflection coefficient arising from the Modified Image Theory (MIT) appearing within the related Green's function. The application of the generalized telegrapher's equations for buried wires is of particular importance in the transient analysis of grounding systems as the concept of scattered voltage is included within the formulation which is not the case in standard antenna theory. This concept allows a straightforward determination of the transient voltage which is necessary for the evaluation of input impedance and step-voltage at the ground surface.

Some illustrative computational examples pertaining to the transient voltage induced along realistic grounding system configurations are presented in the paper.

2. Formulation

The generalized telegrapher's equations for curved wire configurations can be derived by extending the simple case of a straight buried wire. A straight thin wire of length L and radius a buried in a lossy ground at depth d, see Figure 1, is considered.

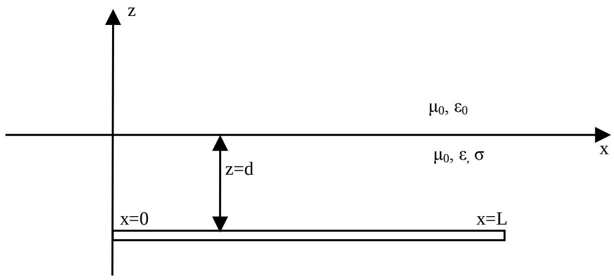

Figure 1. Straight thin wire buried in a lossy ground

The buried wire is excited either by a plane wave (underground cable), or by an equivalent current source (grounding system). The current and voltage induced along the wire are obtained by solving the generalized telegrapher's equations derived in [2]:

$$\int_0^L \frac{\partial I(x')}{\partial x'} g(x,x')dx' + j4\pi\omega\varepsilon_{eff} V^{sct}(x) = 0 \qquad (1)$$

where $I(x')$ is the current distribution along the wire, and $g(x, x')$ stands for the corresponding Green's function. The rigorous form of the Green's function is expressed in terms of the Sommerfeld integrals whose evaluation is rather time consuming [5]. An approximate approach is to use the Fresnel reflection coefficient [2] leading to the following total Green's function

$$g(x,x') = g_0(x,x') - \Gamma_{ref}\, g_i(x,x') \qquad (2)$$

where $g_0(x, x')$ is the free-space Green's function

$$g_o(x,x') = \frac{e^{-j\gamma R_o}}{R_o} \tag{3}$$

and $g_i(x, x')$ arises from the image theory and is given by

$$g_i(x,x') = \frac{e^{-j\gamma R_i}}{R_i} \tag{4}$$

In (3) and (4), R_o and R_i denote the corresponding distance from the source to the observation point, respectively, and the propagation constant of the lossy ground is given by

$$\gamma = \sqrt{j\omega\mu\sigma - \omega^2\mu\varepsilon} \tag{5}$$

The corresponding Fresnel reflection is of the form [5]

$$\Gamma_{ref} = \frac{\frac{\varepsilon_0}{\varepsilon_{eff}}\cos\theta - \sqrt{\frac{\varepsilon_0}{\varepsilon_{eff}} - \sin^2\theta}}{\frac{\varepsilon_0}{\varepsilon_{eff}}\cos\theta + \sqrt{\frac{\varepsilon_0}{\varepsilon_{eff}} - \sin^2\theta}} \tag{6}$$

which accounts for the presence of a lossy half-space.
The complex permittivity of the ground is given by

$$\varepsilon_{eff} = \varepsilon_r\varepsilon_0 - j\frac{\sigma}{\omega}$$

(7)
and argument θ is defined as:

$$\theta = \text{arctg}\frac{|x-x'|}{2d} \tag{8}$$

An alternative is to use the simplified reflection coefficient arising from modified image theory:

$$\Gamma_{MIT} = \frac{\varepsilon_{eff} - \varepsilon_0}{\varepsilon_{eff} + \varepsilon_0} \tag{9}$$

The next step is to extend the formulation (1) to the case of an arbitrarily-shaped wire. The curved wire with the corresponding image is shown in Fig 2.

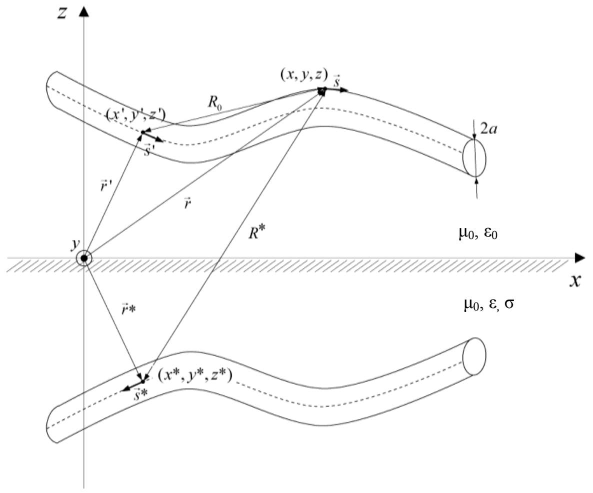

Figure 2. Buried wire of arbitrary shape and its image

Assuming the potential in the remote soil to be zero and by using straightforward mathematical manipulations, the following expression for the voltage between any point on the wire ($\vec{r}$) and remote soil is obtained:

$$\int_{C'}\frac{\partial I(s')}{\partial s'}\cdot g_0(\vec{r},s')ds' - \int_{C'}\frac{\partial I(s')}{\partial s^*}\cdot\Gamma_{MIT}\, g_i(\vec{r},s^*)ds' + j4\pi\omega\varepsilon_{eff}V^{sct}(\vec{r}) = 0 \tag{10}$$

in which the geometrical parameters are defined in Fig. 2. Integral expression (10) represents the 2nd telegrapher's equation for curved wires. Note that r stands for the observation point position, while s' and s^* are related to source variables over the buried wire and image in the air, respectively. Now the extension to the multiconductor case is straightforward

$$\sum_{k=1}^{N_w}\left\{\int_{C'}\frac{\partial I_k(s')}{\partial s'}\cdot g_{0k}(\vec{r},s')ds' + \Gamma_{MIT}\int_{C'}\frac{\partial I_k(s')}{\partial s^*}\cdot g_{ik}(\vec{r},s^*)ds'\right\} + j4\pi\omega\varepsilon_{eff}V^{sct}(\vec{r}) = 0 \tag{11}$$

where N_w denotes the total number of wires.
The 1st generalized telegrapher's equation for curved wires could be derived in a similar manner. Once the current distribution along the given wire configuration is determined by solving the corresponding integro-differential equation, it is possible to evaluate the scattered voltage (10) or (11), respectively. The current $I_n^e(\zeta)$ over the n-th segment is expressed via basis functions f_{ni}, and complex coefficients I_{ni}. Featuring the use of isoparametric elements, it follows [4]

$$I_n^e(\zeta) = \sum_{i=1}^{n} I_{ni} f_{ni}(\zeta) = \{f\}_n^T\{I\}_n \tag{12}$$

where n is the number of local nodes per element.
The boundary element formalism applied to (11) yields

$$V(\vec{r}) = -\frac{1}{j4\pi\omega\varepsilon_{eff}}\sum_{i=1}^{N_w}\sum_{n=1}^{N_g}\sum_{k=1}^{nl} I_k^{ei}\int_{-1}^{1}\frac{\partial f_k^e(\zeta')}{\partial\zeta'}\cdot\left[g_0^i(\vec{r},s') - \Gamma_{MIT}\, g_i^i(\vec{r},s^*)\right]d\zeta' \tag{13}$$

The transient voltage is obtained by applying the Inverse Fourier Transform (IFT) to (13).

3. Computational examples

The first set of numerical results deals with a grounding system composed of 60m x 60m grid (10m x 10m square meshes), with conductor radius a=0.007m, Fig 3. The grid is buried at depth d=0.5m in a lossy ground with a conductivity σ=1mS/m and a relative permittivity ε_r=10.

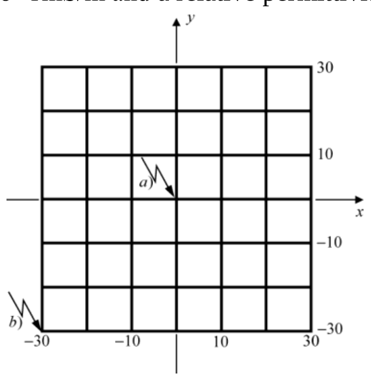

Figure 3. Geometry of a grounding grid

The grounding grid is excited at its center by a double exponential current source

$$i(t) = I\left(e^{-\alpha t} - e^{-\beta t}\right) \qquad (13)$$

where: $I = 1.0167kA$, $\alpha = 0.0142\mu s^{-1}$ and $\beta = 5.073\mu s^{-1}$.
Figure 4 shows the spatial distribution of the voltage induced along the grounding grid. for the case of center injection at various time instants.

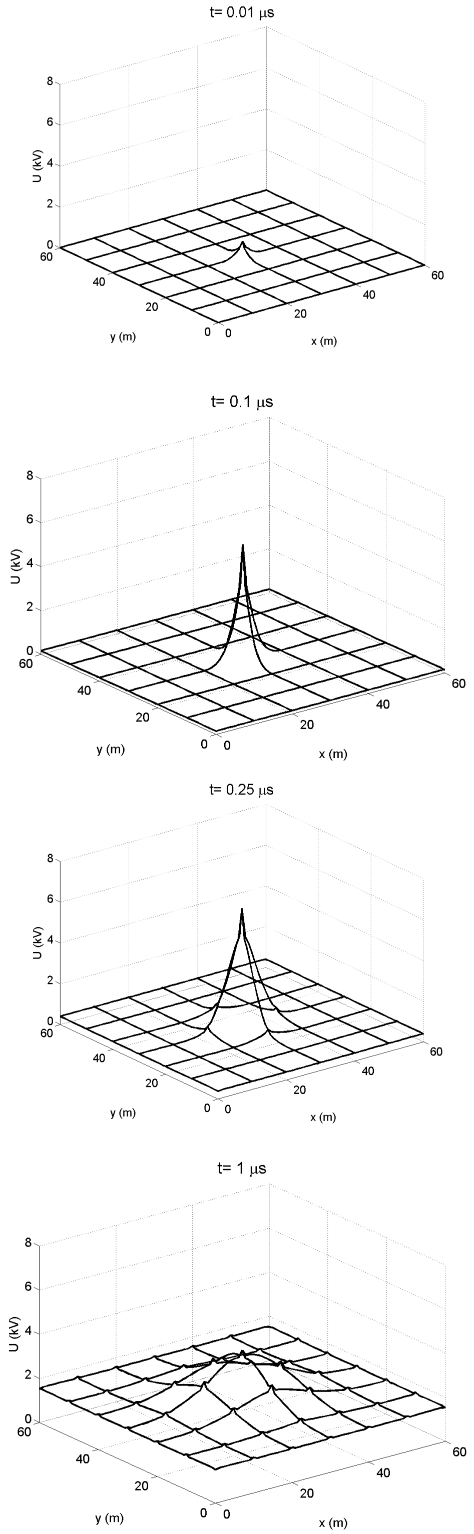

Figure 4. The voltage distribution along the grounding grid (center injection) at various instants of time

The obtained numerical results are in a good agreement with the results published in [1].
The next example deals with a typical grounding system of a wind turbine (WT), shown in Fig. 5. The WT grounding system, composed of 2 copper rings with radii 3.25m and 6.8m (buried at depths 5cm and 55cm) is placed in a homogenous soil of conductivity σ=0.8 mS/m and a relative dielectric constant ε_r=9.

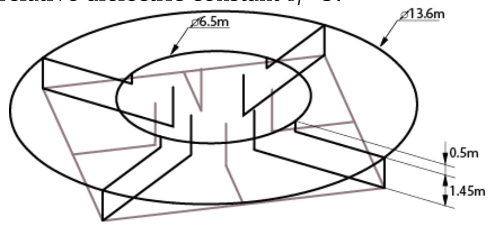

Figure 5. Typical configuration of WT grounding system

Note that a rectangular grounding grid of wire length 9.6 m buried at depth 2 m is located inside bigger ring. Horizontal wires (Fe/Zn 30x4mm) of length 28m, 29m and 9m are placed radially, as depicted in Fig. 5. The WT is excited by the lightning current given by the double exponential function (13) with: I_0=1.1043A, $\alpha=0.07924\cdot10^{6}s^{-1}$, $\beta=4.0011\cdot10^{6}\ s^{-1}$, representing a 1/10 μs pulse, as shown in Fig 6.

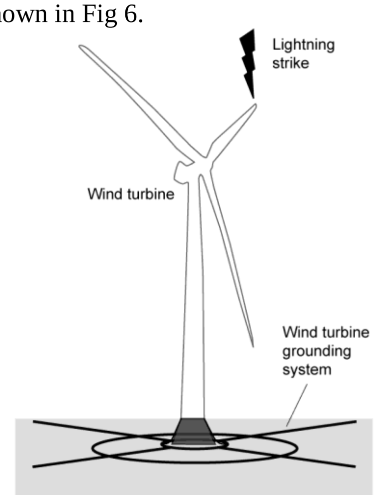

Figure 6. WT excited by a lightning return-stroke pulse

The time-domain waveform of the lightning return-stroke current pulse is shown in Figure 7.

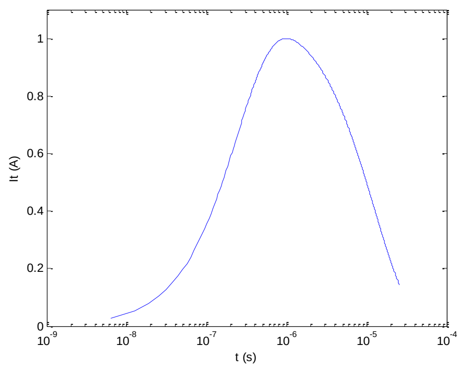

Figure 7. The lightning return-stroke current pulse

Figure 8 shows the spatial distribution of the voltage at the ground surface above the WT grounding system.

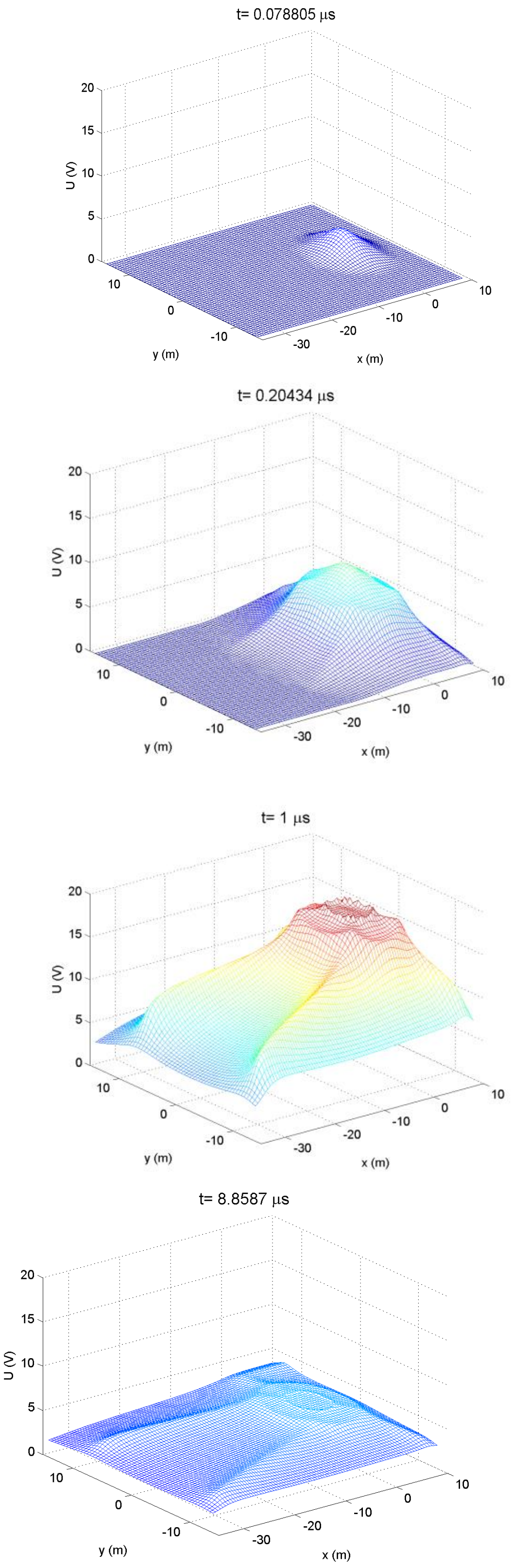

Figure 8. The voltage distribution induced along the WT grounding system (center injection) at various time instants.

4. Concluding remarks

A generalized form of the frequency domain telegrapher's equations for electromagnetic field coupling to curved wires buried in a lossy ground is presented in this paper. Once the current distribution along the grounding system is determined, the transient voltage along the grounding grid is calculated by integrating the current distribution over the grounding structure. The transient voltage along the grounding structures is computed by applying the Inverse Fourier Transform (IFT).

Future work will deal with the generalized telegrapher's equations for buried wires of arbitrary shape using the rigorous Sommerfeld integral approach to account for the presence of a lossy half-space.

5. References

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