



Quantitative Statistical Analysis for Wave Propagation in Chaotic Environments: A Stochastic Green's Function - Integral Equation Method

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1 Extended Abstract

In recent years, wave chaos has received increasing attentions due to its distinguished characteristics and unique behaviors, including chaotic sensitivity, ergodicity, and broadband spectra of dynamics in phase space. One representative class of wave-chaotic problems is confined electromagnetic (EM) systems, e.g. the antennas and electronics within large and complicated enclosures. Applications include computer cases, reverberation chambers, indoor wireless channels, etc. In the high-frequency regime, the complex boundary of the enclosure can lead to high modal density and high modal overlap. Wave solutions inside these enclosures show strong fluctuations that are extremely sensitive to the exact geometry of the enclosure, the location of internal sensors and electronics, and the operating frequency. Minor differences in the detailed configuration of the system, or small numerical errors in the computation, can result in significantly different EM field distributions within the enclosure.

Another general class of problems is wave propagation in complicated scattering environments. Related applications include wireless communication in metropolitan areas, diffusive random media, speckle phenomena in radar imaging, etc. As the wavelength is much shorter than the typical size of the structures (scatterers) in the environment, the wave scattering process may exhibit chaotic ray dynamics. Consider two propagating rays with slightly different incident directions. Their trajectories start out very close to each other and separate significantly over time. In a finite number of bounces the results become completely different, even though the scattering environment is deterministic.

Evidently, the difficulty of determining precise environmental information and the high variability of wave properties lead to a conceptual limitation for deterministic approaches. The emphasis of this work is placed on the fundamental mathematical models and algorithms for the statistical wave analysis of wave-chaotic environments, leveraging wave chaos physics and random statistical analysis. A novel stochastic Green's function (SGF) method is proposed, which quantitatively describing the universal statistical property of chaotic media through random matrix theory (RMT). Based on SGF, we investigate a stochastic integral equation formulation for the statistical characterization of wave interactions within wave-chaotic environments. The theoretical investigation is supported by experimental verification and validation.