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Improvements to the Principal Components Generalized Projections algorithm: Operator Formulation

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I present an improvement to the principal components generalized projections (PCGP) algorithm used for Frequency Resolved Optical Gating (FROG) inversion that still maintains the speed and simplicity of the PCGP algorithm while moving toward an operator-based formulation.

Frequency Resolved Optical Gating (FROG) uses the ultrafast laser pulse to interrogate itself for full characterization [1-3]. This is accomplished by spectrally resolving the autocorrelation of the pulse to be measured formed by the nonlinear interaction between two pulse replicas, which we call the probe and the gate in this work. The resulting data is a spectrogram of the pulse usually referred to as the FROG trace.

Because only the intensity of the FROG trace (spectrogram), can be measured, a two-dimensional phase retrieval algorithm is used to obtain the phase of the FROG data set [1-3]. Recovery of the phase allows extraction of the pulse characteristics from the FROG data set. FROG phase retrieval algorithms iterate between two constraints. The first constraint is the FROG trace intensity and the second is a mathematical form constraint constructing the FROG trace from the pulse(s). The FROG trace constraint is applied in the frequency domain while keeping the phase. The mathematical form constraint is applied in the time domain. Obtaining the next guess for the pulse to apply the mathematical form constraint differentiates the algorithms. The most stable and reliable algorithms obtain the next guess by minimizing the distance between the set formed by the constrained data and the set formed by the mathematical form constraint on each iteration [1-3]. These generalized projections algorithms can either be designed to minimize a function or can use eigenvectors to find the next guess.

The eigenvector approach is called the Principal Components Generalized Projections Algorithm (PCGP), which depends on the idea that an outer product of two vectors contains all of the points required to construct a time domain FROG trace [2,3]. By rearranging the points of the outer product matrix using a one-to-one, invertible transform (usually by row rotations), the time domain FROG trace is constructed. The magnitude squared of the Fourier transform of the columns of the time domain FROG trace constructs the FROG trace.

Reversing the procedure creates the outer product form matrix, which is the basis of the operator formulation. Because the FROG trace can be formed from the outer product of a single pair of vectors, the resulting outer product form matrix should be rank 1. In practice, it never is rank 1. Therefore, the best rank 1 estimate for the outer product form matrix, O, where v and w are unit vector solutions of the pair of Hermitian eigenvalue-eigenvector problems:

$$OO^*v = \sigma_1^2 v, \quad O^*Ow = \sigma_1^2 w \tag{1}$$

This result is the foundation for the power method of the PCGP FROG retrieval algorithm when v is replaced by the probe, and w is replaced by the gate [2,3].

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