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Inverse Laplace and the Quasi-Conformal Transformation.

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1 Extended Abstract

Transformations optics (TO) offers an approach that can allow us to re-imagine and manipulate the electromagnetic world around us with relative ease [1]. This breakthrough was made possible by the form invariance of Maxwell's equations under Coordinate Transformation(CO) that constructs an equivalence between the geometric and material spaces. For a given desired EM field behaviour, the CO can determine the permissible permittivities and permeabilities that can successfully implement this spatial mapping. The first generation of TO methodologies was severely restricted by the resultant anisotropic solutions and characterised by their narrow bandwidths and lossy behaviour. By restricting the available transformations to those that satisfy the conditions laid down by conformal mapping, all-dielectric isotropic material distributions are possible [2]. Such conformality or angle-preservation requires the coordinate transformation to meet the Cauchy-Riemann conditions [3]. Under these terms, for a TE-polarised wave with electric field normal to the plane of propagation the constitutive parameters for the transformation medium may be simplified to $n' = 1/\sqrt{(}\det(\mathbf{A}))$. Therein $\mathbf{A}$ is the Jacobian matrix for the CO from the virtual to the physical space. This simplification is critical to making the material amenable to fabrication using commercially available additive manufacturing approaches. While versatile, the conformal mapping can't guarantee boundary conditions that may be critical to device performance. The Quasi-Conformal Transformation Optics approach (QCTO) resolves this issue allowing for the stipulation of Dirichlet boundary conditions that can ensure impedance matching to free-space [4]. Typically these are imposed in combination with Neumann boundary conditions to compensate and guarantee that the mapping will be orthogonal to these boundaries within a Laplace solution. These maps approximate the Cauchy-Riemann relations and introduce an in-plane material tensor element that is no longer equal. However, in general, if these perturbations are small they can be ignored, albeit with a reciprocating reduction in device performance [5].

In this work, we interrogate the benefits and limitations of the Inverse Laplace QCTO approach, a subset of the QCTO where spatial transformation is defined not from the physical space (regular shape) to the virtual (arbitrary-shaped) but vice-versa [6]. Of particular interest is how flexible this approach is to generating designs that can readily be manufactured using commercially available 3d print technologies.

References

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