2nd AT-RASC, Gran Canaria, Spain, 28 May–1 June 2018

Theoretical analysis of 2D spectra of radar returns

M. Saillard*[(1)], F. Platzer[(1)(2)], and V. Fabbro[(2)]
(1) Mediterranean Institute of Oceanography, Aix-Marseille University, University of Toulon, CNRS, IRD, UM 110, CS 60584, 83041 Toulon cedex 9, France
(2) ONERA, The French Aerospace Lab, Electromagnetism and Radar Dept., 2 Avenue Edouard Belin, BP 74025, 31055 Toulouse Cedex 4, France

Abstract

The work presented here aims at interpreting the pattern observed at low frequencies in wavenumber-frequency representations (dispersion diagrams) of radar returns. This pattern is sometimes referred to as the "group line" in the literature and is known to result from non-linearities. To this end, we have analytically calculated the patterns generated by various non-linear terms of second order in sea surface height and compared them with numerical simulations of the backscattered field or with an empirical law.

1 Introduction

Time series of microwave radar cross section (RCS) from sea surface are almost routinely used to image sea surface and extract informations about sea state, current or wind fields [1]. Such radars operate in the microwave range and illuminate the surface under garzing incidence from a vessel or a platform. Data processing is often based on a Fourier transform in both space and time. The resolution in range of the radars are less than a few meters while time series duration allows a good resolution in frequency. The resulting 2D spectrum exhibits some characteristic patterns of gravity waves, including a bright line in accordance with their dispersion relation in deep seas, some energy along the harmonics and a so-called "group line" at low time and space frequencies. The group line results from the non-linear dependence of the cross-section with respect to surface parameters.

In general, the group line is supposed not to bring useful information about sea state. However, if coherent radars are used, one can also map the Doppler velocity (first-order moment of the Doppler spectrum). In this case, whether the group line can be interpreted as the signature of breaking waves has been investigated [2], since the slope of the group line seems to fit the velocity of breakers predicted by Phillips Λ function [3].

In this paper, we present a detailed theoretical analysis of the group line generated by various second-order non-linear functions of sea surface height and its derivatives. The theoretical predictions are then compared to synthetic data from numerical solution of the scattering from one-dimensional sea-like surfaces and are used to interprete experimental results previously analyzed in the literature [4].

2 Analytical results

This section focuses on the group line generated by various nonlinear functions of a random variable with Gaussian probability density and given power spectrum $S(K)$, typically the surface height of a linear superposition of sinusoidal waves, or the associated distributions of slopes or velocities. Since the radar resolution in range is about a meter, or more, smaller scales have been filtered out from S. In these conditions, it is shown that the group line fills a part of the (k,ω) plane of which shape is controlled by the (low) frequency of the most energetic wave on one side, by the radar resolution in range on the other side. The distribution of energy inside this area depends on the kind of non linear term under consideration. An example is given in figure 1, which exhibits the group line derived from the analytical calculation of the 2D Fourier transform of η^2, where $\eta(x,t)$ denotes the time varying boundary between air and sea water. The plot has been restricted to the upper right quadrant, since the spectrum is centro-symmetric and all the waves go in the same direction here. To translate this 2D map in

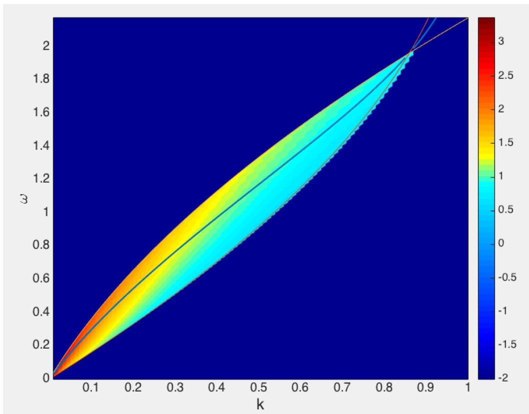

Figure 1. Group line associated with η^2 (log scale) for a band-limited K^{-3} wave spectrum. The low-frequency cut-off is $K_p = .14$ and the radar range resolution is $K_u = 1$ rad/m. The blue line represents the weighted average value, $<\omega>(k)$

terms of "line" or "slope", a weighted average $<\omega>$ is calculated for each k, and the mean slope $<\omega>/k$ between 0 and k is considered. In figure 1, $<\omega>$ has been plotted in blue, according to the following formula

$$<\omega>=\sqrt{gk}\frac{\left[\frac{\frac{\omega}{\sqrt{gk}}}{1-\left(\frac{\omega^2}{gk}\right)^2}-\frac{1}{4}\ln\frac{1+\frac{\omega}{\sqrt{gk}}}{1-\frac{\omega}{\sqrt{gk}}}-\frac{1}{2}\arctan\frac{\omega}{\sqrt{gk}}\right]_{\omega_p}^{\omega_u}}{\left[\frac{1}{1-\left(\frac{\omega^2}{gk}\right)^2}\right]_{\omega_p}^{\omega_u}} \tag{1}$$

where $\omega_p = \sqrt{g(K_p+k)} - \sqrt{gK_p}$ and $\omega_u = \sqrt{gK_u} - \sqrt{g(K_u-k)}$ represent the upper and lower boundary of the domain covered by the group line (also plotted in figure 1). The same kind of calculations can be performed for the square of the slope, η_x^2. In this case, since the spectrum decays very slowly, as K^{-1}, the result strongly depends on the radar resolution and the energy spreads out in a more homogeneous way, leading to smaller average values $<\omega>$.

$$<\omega>=\sqrt{gk}\frac{[\omega]_{\omega_p}^{\omega_u}}{[\ln\omega]_{\omega_p}^{\omega_u}} \tag{2}$$

This clearly appears in figure 2, where the surface parameters are the same as in figure 1. The difference between the group lines associated with η^2 and η_x^2 is also illustrated in figure 3, which compares both slopes $<\omega>/k$.

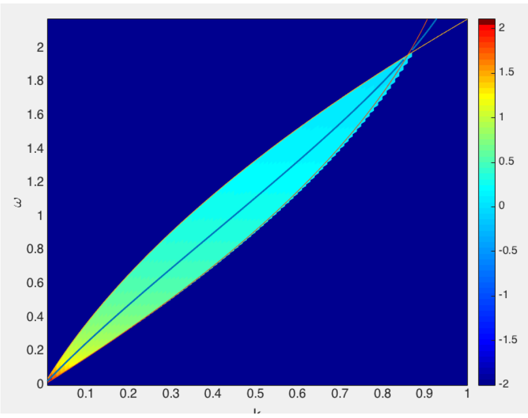

Figure 2. Same as figure 1, but for η_x^2.

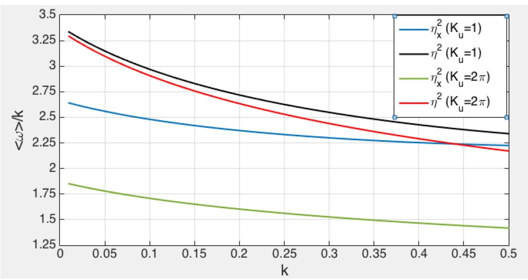

Figure 3. Slope of the group line $<\omega>/k$ versus k, for η^2 (black: $K_u = 1$; red: $K_u = 2\pi$) and η_x^2 (blue: $K_u = 1$; green: $K_u = 2\pi$).

The high sensitivity of the group line of η_x^2 to the high frequency cut-off is also exhibited in figure 3, where the two cases $K_u = 1$ and $K_u = 2\pi$ have been considered. The higher K_u , the smaller slope. This results from the low decay of the spectrum of η_x, of which rms depends on K_u.
On the contrary, the slope of the group line of η^2 is hardly changed, because most of the energy is concentrated along the line $\omega = \omega_p(k)$. It is also worth to notice that the limit when k tends to 0 of $<\omega>$ in equation (1) is $0.4\sqrt{gk}\sqrt{k/K_p}$, leading to slope at origin $0.4c_\phi(K_p)$ for η^2 in figure 3, where $c_\phi(K_p) = \sqrt{g/K_p}$ denotes the phase velocity of the dominant wave. This is a little bit less than their group velocity $c_g = c_\phi/2$, because of the weight of slower waves of sea spectrum. The 0.4 coefficient is thus the signature of the K^{-3} decay of the spectrum. Let us also notice that the slope decreases with k.
Similarly, if one considers the spectrum of horizontal (or vertical) velocities, which decays as K^{-2}, one gets $c_\phi(K_p)/3$ as slope at origin for the group line associated with the square of the velocity.
When mixing two quantities through their product, one gets slopes ranging between the slopes of the square of each. For instance, the slope of the group line at origin associated with $\eta\eta_x$, which combines K^{-3} and K^{-1} spectra, is $c_\phi(K_p)/3$, the same as for horizontal velocity. Calculations for higher order non linearities have not been performed because, of their complexity on the one hand, they do not strongly modify the structure of the group line on the other hand. An interpretation may be that, when more than two terms (gravity waves, here) are introduced, the main contribution, since no resonance effect between three or four waves is considered here, comes from the two-term interference.

3 Simulated data of backscattered field

Time series of the backscattered field have been generated with the help of a numerical solver based on a rigorous boundary integral formalism, devoted to scattering from one-dimensional rough surfaces illuminated under grazing incidence [5]. It has already been used to investigate Doppler spectra [6], from short time series of the backscattered field. Here, we are recording the contribution from each small range cell during about a minute, and a two-dimensional Fourier transform is then performed.
Sea surface is described by Pierson Moskowitz spectrum and computations are performed for horizontal polarization at L band, to reduce computation time, without loss of generality since experimental results do not exhibit a frequency or polarization dependent behavior. Creamer's approach is used to represent hydrodynamic non-linearities.

The dispersion diagram for a 7 m/s wind speed and a radar range resolution of 1m is plotted In figure 4. Though noisy, the main expected features are there, with energy along the dispersion curve and its harmonics, and along something like a piece of line going through the origin. The slope of this group line can be roughly estimated to 2.3 m/s. However, the low dynamics of the numerical data makes accurate comparison with theory difficult to achieve. If, in the

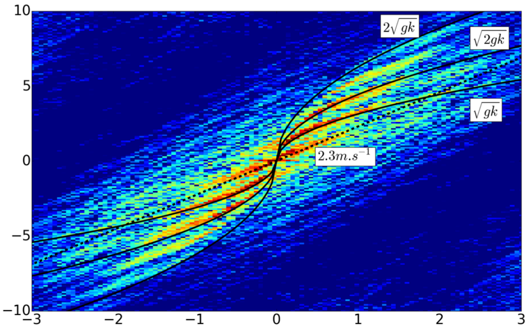

Figure 4. Dispersion diagram of the backscattered amplitude for sea surface with wind speed 7 m/s, illuminated under grazing incidence and H polarization at L band. The resolution in range is 1 m. horizontal axis: wavenumber; vertical axis: frequency

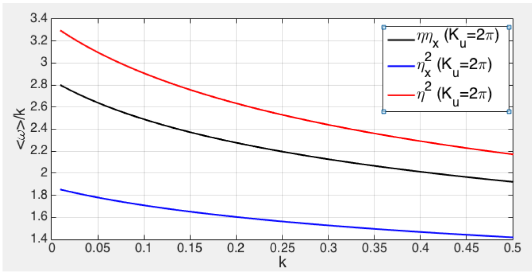

Figure 5. Slope of the group line $<\omega>/k$ versus k, for η^2 (red), η_x^2 (blue) and $\eta\eta_x$ (black), for $K_u = 2\pi$.

previous section, a threshold is introduced to reduce the dynamics of the spectrum, the shape and the slope of the group line will be modified. This would first alter the lower boundary, which is less energetic, and induce an overestimation of the slope. The same occurs with experimental data.

Nonetheless, if we compare with the predictions of the previous section (K_p and K_u are almost the same), it appears from figure 3 that the numerical result lies between the slopes attached to η^2 and η_x^2. In fact, it is closer to the slope associated with $\eta\eta_x$ which ranges from 2.8 m/s at origin down to 1.9 m/s for $k = 0.5$ (see figure 5), $[-0.5, 0.5]$ being the k interval of bright points of the group line in figure 4.

Let us try to link this result with scattering theory. First, the scattering amplitude under grazing incidence behaves as the square of the grazing angle [5], which is directly related to the average slope of the radar cell. This is in favor of a term of the type of η_x^2. But it is also known that the main contribution to backscattering comes from cells located on the top of the surface profile, which is referred to as shadowing. Combining both makes the radar return behave as a non linear function of η and η_x. Numerical simulation of the dispersion diagram of $\eta_x^2 \times P_\eta$, where P_η is a ray tracing shadowing function, have permitted us to retrieve the same result as with the scattered field.

Beyond the physical interpretation, the main result of this section is that the dispersion diagram of the scattering amplitude presents significant energy along the group line, even for a weakly non-linear sea surface at low wind. In addition, the slope lies in the range predicted by analytical results for non-linearities involving η and η_x. Therefore, any quantity built upon the scattering amplitude will generate a bright group line, whether waves are breaking or not.

4 Comparison with experimental data

Frequency-wavenumber spectra of velocity, defined as the first moment of the Doppler spectrum, have been studied in [3], where the slope of the group line is assimilated to the most probable velocity of breakers c_0. In [4], the authors have fitted this set of velocity data as a second order polynomial in $c_\phi(K_p)$

$$c_0 = 0.22 + 0.39 c_\phi - 0.008 c_\phi^2 \tag{3}$$

It is interesting to notice that the first order coefficient is very close to the slope at $k = 0$ of $<\omega>$ in eq. (1), which is equal to 0.4. This suggests to compare c_0 with the slope associated with η^2. Since the second order derivative of $<\omega>$ at $k = 0$ is negative, we have found a small value of k which allows us to fit the polynomial, as shown in figure 6. Although choosing $k = 0.11$ has no physical meaning,

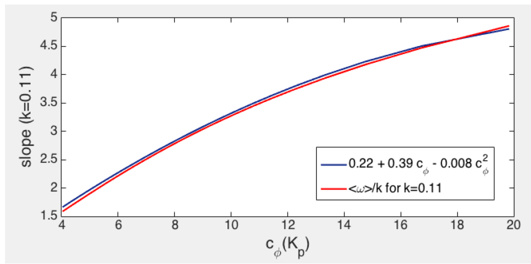

Figure 6. Slope of the group line $<\omega>/k$ (eq. (1)) versus phase velocity of the dominant wave $c_\phi(K_p)$ (red), for $k = 0.11$, versus the empirical formula proposed in [4] for the most probable velocity of breakers (blue).

noticing that the group line in figure 1 of [3] concentrates energy between $k = -0.2$ and $k = 0.2$, it makes sense to estimate the slope around $k = 0.1$. But the most important here is that we get the same behaviour and the same order of magnitude by considering that the group line of the dispersion diagram of the average Doppler velocity results from a η^2 term.

5 Conclusion

The characteristics of the group line have been explicitely linked to the spectrum of waves and to the radar resolution in range, at least for simple non-linear functions of η. Though accurate comparison with numerical simulations or experimental data would also require to take the low dynamics of their 2D spectra into account, the theoretical predictions are consistent with them. In particular, the behavior of the slope of the group line associated with

η^2, with respect to phase speed of the dominant wave, fits well the velocity of breakers as estimated in [4]. However, group lines are not always the signature of breaking: as the backscattered amplitude is basically a non-linear function of η, its dispersion diagram exhibit a group line, of which slope appears to be lower if breaking does not occur.

6 Acknowledgements

The authors acknowledge the Délégation Générale de l'Armement (DGA) for financial support.

References

[1] H. Dankert, J. Horstmann and W. Rosenthal, "Wind and Wave Retrieval using Marine X-band Radar-Image Sequences" , *IEEE Journal of Oceanic Engineering*, 2005, doi 10.1016/10.1109/JOE.2005.857524.

[2] W. J. Plant and G. Farquharson, "Origins of features in wave number-frequency spectra of space-time images of the ocean," *Journal of Geophysical Research: Oceans*, **17**, 2012, pp. 2156-2202.

[3] W. J. Plant, "Whitecaps in deep water" , *Geophysical Research Letters*, **39**, 2012, pp.1944-8007.

[4] V. Irisov, W. Plant, "Philip's Lambda function: Data summary and physical model," *Geophys. Res. Lett.*, **43**, 2016, pp. 2053-2058, doi=10.1002/2015GL067352.

[5] D. Miret, G. Soriano and M. Saillard, "Rigorous Simulations of Microwave Scattering From Finite Conductivity Two-Dimensional Sea Surfaces at Low Grazing Angles," *IEEE Transactions on Geoscience and Remote Sensing*, **52**, June 2014, pp. 3150-3158, doi=10.1109/TGRS.2013.2271384.

[6] D. Miret, G. Soriano, F. Nouguier, P. Forget, M. Saillard and C. A. Guérin, "Sea Surface Microwave Scattering at Extreme Grazing Angle: Numerical Investigation of the Doppler Shift," *IEEE Transactions on Geoscience and Remote Sensing*, **52**, November 2014, pp. 7120-7129, doi=10.1109/TGRS.2014.2307893.