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Polarimetric Requirements for Epoch of Reionization Observations

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Abstract

Radio observations of the 21 cm (1420 MHz) fine-structure splitting of the ground state of neutral hydrogen have been an essential component of astronomy for over 60 years. Because the expansion of the universe encodes information about the distance to an emitter in the redshift of its wavelength, it is possible to observe the redshifted 21 cm line and obtain fully three-dimensional information about the location of neutral hydrogen at early times. Recent scientific interest in observing the highly redshifted (50 - 250 MHz) emission of neutral hydrogen has been spurred by the promise of a unique view into the first billion years of the universe's history, by revealing – in three dimensions, and over time – the formation of the first stars, galaxies, and black holes through their effect on the thermal and ionization state of the intergalactic medium. Measurements of this signal will require an unprecedented understanding of the properties of low-frequency radio interferometers and careful data analysis to separate the strong intervening "foreground" emission from other, nearer, astrophysical sources from the desired signal. A powerful tool for this separation is by contrasting the spectral "smoothness" of the foreground signal, primarily synchrotron emission, from the highly spectrally structured cosmological line signal. This signal is intrinsically unpolarized, but because Faraday-rotated polarized emission is spectrally structured in a way difficult to distinguish statistically from the cosmological signal, a high degree of isolation of polarized leakage from Stokes Q and U to I is necessary. A thorough understanding of the instrument's polarization properties are necessary to separate Faraday-rotated astrophysical emission from the expected signal and to achieve high dynamic range imaging. We review the crucial elements of polarized interferometry and the current state of the art.

1 Introduction

The current generation of epoch of reionization experiments seeks to make a statistical detection of the power spectrum of brightness-temperature fluctuations in neutral hydrogen (HI) during the Epoch of Reionization (EoR; for a scientific review see [1]). A number of groups around the world are currently pursuing a detection of this signal (e.g. LOFAR[1], MWA[2], PAPER[3], HERA[4]) This detection must be made in the presence of foreground emission from our Galaxy and intervening galaxies which is orders of magnitude brighter. There are various approaches to allow the measurement in the presence of these foregrounds, which focus more heavily on modeling and subtraction in the image domain [2], avoidance via spectral smoothness in the Fourier domain [3], or hybrid approaches [4].

A feature of observed polarized emission that is not present for the unpolarized case is Faraday rotation. This is due to the index of refraction of an ionized, magnetized plasma being birefringent, and so the left-and right-circular polarizations of an electromagnetic wave passing through such a plasma undergo different phase shifts, such that the phase difference $\Delta\varphi$ of the light becomes

$$\Delta\varphi = \frac{e^2}{(m_e c^2)^2}\lambda^2 \int n_e(s) B_{\parallel}(s) \mathrm{d}s \equiv \lambda^2 \Phi, \qquad (1)$$

where n_e is the electron density of the plasma, $B_{\parallel}$ is the component of the magnetic field along the line of sight, e and m_e are the electron charge and mass, λ^2 is the wavelength of the incident light, and the integral extends along the line of sight. Equation 1 defines the rotation measure Φ. Faraday rotation affects the linear components of the Stokes parameters such that a polarized source with intrinsic Stokes Q and U, when viewed through a magnetized plasma, will have measured Stokes parameters Q' and U'[5]

$$Q'(\hat{s},\nu) + iU'(\hat{s},\nu) = e^{-2i\lambda^2\Phi(\hat{s})}(Q(\hat{s},\nu) + iU(\hat{s},\nu)) \qquad (2)$$

Note that the rotation measure in general depends on the direction of observation, but the frequency dependence has been factored out. It is this frequency structure which can mimic exactly the cosmological modes of the HI signal.

For image-based approaches to measuring the EoR, the problem of polarization is primarily the need to build a polarized instrument and sky model, including Faraday rotation, in order to isolate and remove the polarized foreground signal. The requisite complexity of the model and its accuracy remains a subject of active investigation. For avoidance techniques, there is little concern about polarized

[1]`www.lofar.org`
[2]`www.mwatelescope.org`
[3]`eor.berkeley.edu`
[4]`www.reionization.org`
[5]We use the physics convention $i = \sqrt{-1}$.

emission with $\Phi \approx 0$, since this maps to modes which are already corrupted by unpolarized foregrounds, but sources with larger Φ will contaminate the putative EoR power spectrum. Thus Faraday rotation is uniquely problematic, as there is no additional discriminant against spectral structure. Regardless of the approach taken, any effort in instrument design which cleanly separates the Stokes parameters minimizes the effect of polarization on the final EoR spectrum, as we show below.

2 Interferometric Response to Polarization

To capture all of the instrument effects, it is necessary to use a fully polarized description of an interferometer; we follow the basic conventions laid out in [5]. This will also naturally include wide-field effects. We begin by defining the Stokes parameters on the sphere. For astronomical purposes we express the incident electric field $\vec{E}(\hat{s},\nu)$ from direction $\hat{s}$ at frequency ν in the Right Ascension and Declination basis (unit vectors $\hat{e}_\alpha$, $\hat{e}_\delta$), allowing us to define the coherency tensor field

$$\mathscr{C} = \langle E_\delta^* E_\delta \rangle \hat{e}_\delta \otimes \hat{e}_\delta + \langle E_\alpha^* E_\delta \rangle \hat{e}_\alpha \otimes \hat{e}_\delta + \langle E_\delta^* E_\alpha \rangle \hat{e}_\delta \otimes \hat{e}_\alpha + \langle E_\alpha^* E_\alpha \rangle \hat{e}_\alpha \otimes \hat{e}_\alpha \tag{3}$$

or as a matrix

$$\mathscr{C} = \begin{bmatrix} \langle E_\delta^* E_\delta \rangle & \langle E_\alpha^* E_\delta \rangle \\ \langle E_\delta^* E_\alpha \rangle & \langle E_\alpha^* E_\alpha \rangle \end{bmatrix} \tag{4}$$

where we the direction $\hat{s}$ and frequency ν dependence of the fields is implicit. The coherency field in terms of the Stokes parameters on the sphere is then

$$\mathscr{C} = \begin{bmatrix} I(\hat{s},\nu)+Q(\hat{s},\nu) & U(\hat{s},\nu)-iV(\hat{s},\nu) \\ U(\hat{s},\nu)+iV(\hat{s},\nu) & I(\hat{s},\nu)-Q(\hat{s},\nu) \end{bmatrix} \tag{5}$$

Each polarized feed p of an antenna j responds to incident radiation from infinity in the direction (α,δ) with an complex vector antenna pattern

$$\vec{A}_j^p(\hat{s},\nu) = A_{j,\delta}^p(\hat{s},\nu)\hat{e}_\delta + A_{j,\alpha}^p(\hat{s},\nu)\hat{e}_\alpha \tag{6}$$

where $(\hat{\delta},\hat{\alpha})$ define an orthogonal coordinate system on the sphere. This antenna pattern is proportional to the far-field beam patterns, by the reciprocity theorem. The antenna patterns can be assembled into a Jones matrix per antenna as

$$\mathscr{J}_j = \begin{bmatrix} A_{j,\delta}^p(\hat{s},\nu) & A_{j,\alpha}^p(\hat{s},\nu) \\ A_{j,\delta}^q(\hat{s},\nu) & A_{j,\alpha}^q(\hat{s},\nu) \end{bmatrix} \tag{7}$$

We can then express the fully-polarized visibility equation as

$$\mathscr{V}_{jk} = \begin{bmatrix} V_{jk}^{pp} & V_{jk}^{pq} \\ V_{jk}^{qp} & V_{jk}^{qq} \end{bmatrix} = \int \mathscr{J}_j \mathscr{C} \mathscr{J}_k^\dagger \exp(-2\pi i\nu\vec{b}\cdot\hat{s}/c)d\hat{s} \tag{8}$$

where the integration is taken over the full sphere. From the visibility matrix we can then define the Vokes[6] visibilities, in analogy with the Stokes parameters,

$$V_{jk}^I = \mathrm{Tr}(\sigma_I \mathscr{V}_{jk}) \quad V_{jk}^Q = \mathrm{Tr}(\sigma_Q \mathscr{V}_{jk}) \tag{9}$$

$$V_{jk}^U = \mathrm{Tr}(\sigma_U \mathscr{V}_{jk}) \quad V_{jk}^V = \mathrm{Tr}(\sigma_V \mathscr{V}_{jk}). \tag{10}$$

where the σ_α-matrices are the Pauli matrices labeled by the appropriate Stokes parameter:

$$\sigma_I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \sigma_Q = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \tag{11}$$

$$\sigma_U = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_V = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}. \tag{12}$$

By expanding Equations 9 and 10 we find the that the Vokes visibilities are naturally described by the Mueller matrix elements. For each of $a \in \{I,Q,U,V\}$, we have

$$\begin{aligned} V_{jk}^a(\nu,t) &= \int \mathscr{M}_{ab}(\hat{s},\nu)\exp(-2\pi i\nu\vec{b}\cdot\hat{s}/c)d\hat{s} \\ &= \int (\mathscr{M}_{aI}I + \mathscr{M}_{aQ}Q + \mathscr{M}_{aU}U + \mathscr{M}_{aV}V)\exp(-2\pi i\nu\vec{b}\cdot\hat{s}/c)d\hat{s} \end{aligned} \tag{13}$$

where

$$\mathscr{M}_{ab}(\hat{s},\nu) = \mathrm{Tr}(\sigma_a \mathscr{J}_j \sigma_b \mathscr{J}_k^\dagger) \tag{14}$$

are the Mueller matrix elements.

3 Requirements for the Primary Beam

The EoR signal is expected to be highly unpolarized, and thus we seek an estimate of Stokes I to measure it. Image-based approaches attempt to avoid polarized contamination by inverting Equations 8 or 13 via Fourier methods or otherwise (e.g,[7, 8, 9]). Foreground avoidance techniques use the Vokes-I visibility directly [10]. In either case, the requirement for reducing the leakage between Stokes parameters due to the wide-field beam can be seen fairly straightforwardly from Equation 13:

$$\mathscr{M}_{IQ} = \mathscr{M}_{IU} = \mathscr{M}_{IV} = 0 \tag{15}$$

which also implies that there is some basis $(\hat{\theta},\hat{\phi})$ wherein

$$A_{j,\theta}^p(\hat{s},\nu) = A_{j,\phi}^q(\hat{s},\nu); \; A_{j,\theta}^q(\hat{s},\nu) = A_{j,\theta}^q(\hat{s},\nu) = 0 \tag{16}$$

This is an ideal limit that is probably not achievable in practice, but defines the goal. While Equation 15 turns out to be basis independent, Equation 16 is not. To define a figure of merit independent of basis which can be used to evaluate situations near but not at the ideal, we can use the condition number of the Jones matrix $\kappa(\mathscr{J})$ or the invariant quantity $\mathscr{M}_{IQ}^2 + \mathscr{M}_{IU}^2 + \mathscr{M}_{IV}^2$. These form the bases for the intrinsic cross-polarization figures of merit

$$\mathrm{IXR_J} = \left(\frac{\kappa(\mathscr{J})+1}{\kappa(\mathscr{J})-1}\right)^2 \tag{17}$$

and

$$\mathrm{IXR_M} = \frac{|\mathscr{M}_{\mathrm{II}}|}{\sqrt{\mathscr{M}_{\mathrm{IQ}}^2 + \mathscr{M}_{\mathrm{IU}}^2 + \mathscr{M}_{\mathrm{IV}}^2}} \tag{18}$$

as defined by [11].

[6] We use "Vokes" throughout this work to refer to the linear combinations of visibilities that are formed as proxies for the real "Stokes parameters" as defined by the IEEE [6] in order to clearly differentiate between quantities that have distinctly different properties.

4 Discussion and Conclusions

It is clear that the fidelity with which Equations 8 or 13 can be inverted at every pixel on the sphere, and thus the accuracy of the image-based approached to minimizing contamination, is directly related to $\kappa(\mathscr{J})$. It is not clear from the current literature how good the IXR must be over what range of the field of view to suppress polarized emission to the level required. Studies with LOFAR appear to show acceptable performance for polarized leakage, with simulated values below the expected EoR signal [12, 13, 14]. For the case of foreground avoidance, simulations have been performed of the expected behavior of polarized leakage directly into the power spectrum for PAPER [15, 16], with the conclusion that the instrument likely has leakage at levels above the expected EoR signal.

However, part of the difficulty in setting a requirement for polarization is that there is considerable uncertainty as to the strength of the polarized emission and its properties. The catalog of polarized point sources observed at frequencies below 300 MHz has been relatively small [17, 18]. There is some evidence for systematic depolarization of steep-spectrum point sources towards low frequencies [19], causing low polarization fractions ($\ll 1\%$) below 300 MHz. This causes a problem in both estimating contamination, and also in that there are relatively few polarized calibrators available, particularly at the low angular resolution and sensitivity of EoR arrays [20] as opposed to the high resolution possible with LOFAR, which has recently greatly enlarged the catalog of point sources [21]. The diffuse emission has been measured [22, 23, 24, 25, 12, 18] and appears to show high polarization fraction with low rotation measure. How much this varies from field is uncertain [14].

It is clear nevertheless that optimizing polarization performance in the instrument is highly desirable. This must be done without compromising spectral smoothness, which is key to preventing contamination of the EoR signal by the bright Stokes I foreground. Optimizations to maximize the IXR as a function of the antenna parameters have been performed for some cases of interest to the SKA [26, 27]. Separately, detailed studies of instrument spectral smoothness have been undertaken by HERA [28, 29] and the SKA [30, 31, 32]. A goal for future studies would be the simultaneous optimization of polarization performance and spectra smoothness to obtain an antenna design which is optimal for EoR studies. Designing better instruments stands to make the considerable challenges of EoR detection and measurement easier and maximizes the impact on EoR science.

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