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Physical Interpretation of the Orthogonality Sampling Method

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Abstract

The orthogonality sampling method has been recently introduced in inverse scattering literature as a way to retrieve morphological properties of unknown targets. Both the simplicity and the possibility of being exploited also for a single incidence make this method more appealing than other *qualitative* methods. However, notwithstanding these interesting features, no physical interpretation is attributed to the method. For this reason, in this contribution we firstly attempt to formulate a physical interpretation of the method, in order to take full advantage of its flexibility and to fully understand its limitations.

1. Introduction

Qualitative methods aim at recovering presence, position and shape of the unknown targets by just processing the measurements of the fields which they scatter if illuminated by means of some given incident fields. Usually, they achieve the aforementioned information in a simple and effective way as they partially avoid the difficulties related to the solution of the underlying inverse scattering problem. However, while avoiding the difficulties related to non-linearity, they give up to the possibility to retrieve the electromagnetic properties of the scatterers.

Many different strategies have been developed in literature, among them it is worth to mention the multiple signal classification, the decomposition of time reversal operator, the linear sampling method (LSM), the factorization method (FM) and, finally, the orthogonality sampling method (OSM) [1-4]. Another promising and recently introduced approach based on equivalence principles and Compressive Sensing is described in [5].

The OSM shows some similarity with respect to both the LSM and the FM. In fact, in the OSM the scenario under investigation is sampled into an arbitrary grid of points and an indicator function, whose behavior provides the support information, is computed over this grid. However, OSM exhibits a significant robustness to noise as the indicator function is simply computed as the modulus of the scalar product between the measurements of the far field pattern and a test function, without no need of regularization technique [4].

Moreover, the OSM is more flexible than the LSM and the FM, which require data to be acquired under a multi-view multi-static configuration, as its application is possible also for single-view, multifrequency, multi-view, and a combination of them [4].

Due to its relevance and straightforwardness, the approach has been recently extended to the case of near-field measurements performed on a circle enclosing the scatterers [6]. Nevertheless, the physical understanding of the basis of the method, its limitations (in term of electromagnetic properties and sizes of treatable targets), as well as measurement requirements is not yet completely understood. A possible attempt can be represented by the analysis introduced in [7], where an innovative interpretation of the indicator function as the zeroth order Fourier coefficient of the far-field pattern of the scattered field (in a suitably translated coordinate system) is suggested.

Starting from these considerations, some preliminary thoughts on the physics underlying OSM are briefly given in this contribution. Throughout the paper we consider the canonical 2D scalar problem (TM polarized fields) and we assume and drop the time harmonic factor $exp\{j\omega t\}$.

2. The rationale of Orthogonality Sampling Method

In the OSM, the key role is played by the *reduced scattered field* $E_s{}^{red}$, which is defined as scalar product between the measurements of the far field pattern and a properly defined test function. By following [4], its expression can be given as:

$$E_s{}^{red}(\boldsymbol{r}, \hat{\boldsymbol{r}}_t, k_b) = \int_\Gamma E_s{}^{\infty}(\hat{\boldsymbol{r}}_m, \hat{\boldsymbol{r}}_t, k_b)\, e^{jk_b \boldsymbol{r}\cdot\hat{\boldsymbol{r}}_m}\, d\Gamma \qquad (1)$$

where

- $E_s{}^{\infty}(\hat{\boldsymbol{r}}_m, \boldsymbol{r}_t, k_b)$ is the far-field pattern;
- $\boldsymbol{r}$ is the generic position on an arbitrary grid sampling the region under test Ω, where the unknown targets are hosted;
- $\hat{\boldsymbol{r}}_t$ is the unit vector which identifies the angular position of the antennas, probing Ω and located on a closed curve Γ in the scatterers far-zone;
- $\hat{\boldsymbol{r}}_m$ is the unit vector which identifies the measurements angular position;

- $k_b = \omega\sqrt{\mu_b \varepsilon_b}$ is the wavenumber in the host medium at the frequency ω.

The scalar product in (1) assays the *orthogonality* relation between the measured far field pattern E_s^{∞} with a test function which corresponds to the Green's function (apart from a constant γ at a fixed frequency):

$$G_b^{\infty}(\hat{\boldsymbol{r}}_m, \boldsymbol{r}, k_b) = \gamma(k_b)\, e^{-jk_b \boldsymbol{r}\cdot\hat{\boldsymbol{r}}_m} \tag{2}$$

as computed in far-field zone, which is given by plane waves back-propagated into the region under test [4].

Once the reduced scattered field is computed by means of (1), the OSM indicator function $I(\boldsymbol{r})$ is defined as [4]:

$$I(\boldsymbol{r}, k_b) = \int_{\Gamma} \left| E_s^{red}(\boldsymbol{r}, \hat{\boldsymbol{r}}_t, k_b) \right|^2 d\,\hat{\boldsymbol{r}}_t \tag{3}$$

which is expected to exhibit large values for sampling points belonging to the targets support and limited value outside it.

Note that in equations (1) and (3) single frequency and multi-view data are considered. In case of multifrequency measurements data, the indicator (3) is computed by simply integrating over the relevant frequency band B with $k_b \in B$, i.e.:

$$I_{MF}(\boldsymbol{r}) = \int_B \int_{\Gamma} \left| E_s^{red}(\boldsymbol{r}, \hat{\boldsymbol{r}}_t, k_b) \right|^2 d\,\hat{\boldsymbol{r}}_t\, dk_b \tag{4}$$

3. Physical interpretation of the Reduced Scattered Field

As recalled in the introduction, the ultimate relation between the reduced scattered field and the unknown support of the contrast function χ is still an open question. A very first interpretation is to look at (1) as a superposition of plane waves back-propagated into the region under test [4].

An original and more insightful understanding can be deduced from an alternative definition of the reduced scattered field. This latter can be obtained by substituting into (1) the explicit expression of the far-field pattern given by [8]:

$$E_s^{\infty}(\hat{\boldsymbol{r}}_m, \hat{\boldsymbol{r}}_t, k_b) = \int_{\Omega} G_b^{\infty}(\boldsymbol{r}_m, \boldsymbol{r}', k_b)\, W(\boldsymbol{r}', \hat{\boldsymbol{r}}_t, k_b) d\boldsymbol{r}' \tag{5}$$

where $W = \chi E$ is the contrast source induced in the region under test by the probing antennas, which relates the relevant total field E to the contrast function $\chi(r) = \epsilon_s(r)/\epsilon_b - 1$, wherein ϵ_s and ϵ_b are the complex permittivities of the targets and the background medium, respectively.

By taking advantage from the properties of the Bessel functions and from the Funck-Hecke formula, the reduced scattered field, at a fixed frequency and for a fixed illumination condition, can be interestingly rewritten as the convolution product between the contrast source and the Bessel function of zero order J_0 [4], i.e.:

$$E_s^{red}(\boldsymbol{r}) = \int_{\Omega} \gamma\lambda_0\; J_0(k_b|\boldsymbol{r} - \boldsymbol{r}'|)\, W(\boldsymbol{r}')\, d\boldsymbol{r}' \tag{6}$$

where λ_0 is a constant related to the Funck-Hecke formula.

By assuming, without loss of generality, that the region under test is a disc and considering that the Fourier transform of the Bessel function is given by a distribution of Dirac impulses centered on k_b [9], the reduced scattered field can be finally interpreted (but for a constant) as the restriction to the circle of radius k_b of the Fourier transform of the contrast sources. Such a circumstance has two interesting consequences.

Firstly, as the contrast sources have the same support as the targets, it explains why the OSM indicator works as observed. In fact, it is like plotting (a superposition of the amplitudes of) the contrast sources.

Second, it is known that a further interpretation can be inferred in the spectral domain as the radiating components are those located on a circle of radius k_b [10]. As a consequence, the reduced scattered field E_s^{red} can be related to the radiative part of the sources induced by the different illuminations. Such a circumstance also allows to foresee the expected limitations of OSM, which will be related to the presence of significant non-radiating component of the contrast sources (which will leave no trace into the reduced field) [11].

A further Fourier-based interpretation can be deduced in the case of Born regime [12], wherein the total field can be approximated by just the incident field. Under such hypothesis, the convolution product in (6) can be rewritten as it follows:

$$E_s^{red}(\boldsymbol{r}) = \int_{\Omega} \gamma\lambda_0\; J_0(k_b|\boldsymbol{r} - \boldsymbol{r}'|)\, \chi(\boldsymbol{r}') e^{-jk_b\hat{\boldsymbol{r}}_t\cdot\boldsymbol{r}'}\, d\boldsymbol{r}' \tag{7}$$

Again, by exploiting the properties of Fourier transform and the Bessel functions, the reduced scattered field can be related in the spectral domain to the Fourier transform of the contrast function in a suitably translated coordinate system and limited to the circle of radius k_b.

More insightful understanding and details as well as some interesting performance characteristics will be given at the conference.

7. References

1. F. Cakoni and D. Colton. "Qualitative methods in inverse scattering theory", Springer-Verlag, Berlin, Germany, 2006.
2. H. Ammari, E. Iakovleva, D. Lesselier, G Perrusson, "MUSIC-type electromagnetic imaging of a collection of small three-dimensional inclusions" SIAM Journal on Scientific Computing, vol 29 n. 2, pp. 674-709, 2007.

3. H. Tortel, G. Micolau, and M. Saillard, “Decomposition of the time reversal operator for electromagnetic scattering”, J. Electromagn. Waves Appl., vol. 13, n. 5, pp. 687-719, 1999.
4. R. Potthast, “A study on orthogonality sampling”, Inverse Prob., vol. 26, n. 7, 2010.
5. M. Bevacqua and T. Isernia, "Shape reconstruction via equivalence principles, constrained inverse source problems and sparsity promotion," Progress In Electromagnetics Research, Vol. 158, 37-48, 2017.
6. M. N. Akıncı, M. Çayören ve I. Akduman, “Near-Field Orthogonality Sampling Method for Microwave Imaging: Theory and Experimental Verification”, IEEE Trans. Microw. Theory Techn., vol. 64, n.8, pp. 2489-2501, Aug. 2016.
7. R.Griesmaier, “Multi-frequency orthogonality sampling for inverse obstacle scattering problems”, Inverse Problems, vol. 27, 2011.
8. D. Colton and R. Kress. “Inverse Acoustic and Electromagnetic Scattering Theory”, Springer-Verlag, Berlin, Germany, 1998.
9. R. Piessens, “The Hankel Transform,” The Transforms and Applications Handbook, §9, Alexander D. Poularikas, Boca Raton: CRC Press LLC, 2000.
10. M. Bertero and P. Boccacci. Introduction to Inverse Problems in Imaging, Institute of Physics, Bristol, UK, 1998.
11. A. J. Devaney, and E. Wolf, “Radiating and Nonradiating Classical Current Distributions and the Fields They Generate”, Phys. Rev. D, vol. 8, n.4, 1973.
12. A. J. Devaney, “Geophysical diffraction tomography,” IEEE Trans. Geosci. Remote Sensing, vol. GE-22, no. 1, pp. 3–13, 1984.