2nd URSI AT-RASC, Gran Canaria, 28 May – 1 June 2018

Evaluation of Time Domain Physical Optics Integral on Curved Surfaces

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Abstract

A scheme to evaluate the physical optics (PO) integral in closed form over the curved triangular patches is presented. The PO integral is interpreted as a Radon transform, as a result the PO integral over the curved triangle surface is reduced to a line integral. This line is determined as the intersection of the plane, which is formed by the propagation direction of the incident wave and the observation direction, and curved triangular surface (integration domain). The line integral in time domain can be evaluated in closed-form. A numerical example that shows the validity and accuracy of the scheme is presented.

1 Introduction

The physical optics (PO) approximation involves the evaluation of an integral with a highly oscillatory kernel, called PO integral. The PO integral can be evaluated in closed-form when the scatterer is modeled with flat triangular patches [1, 2]. For the curved surfaces, the PO integral is evaluated approximately [3, 4] or using numerical procedures which might require dense sampling for electrically large surfaces.

The Radon transform interpretation allows to reduce the dimension of the integration domain and to find closed-form expressions to radiation integrals in time domain, e.g. in time domain surface [5] and volume integral equations [6] or PO approximation for different source/observation configurations [2, 7, 8, 9]. In [2], closed-form expression to the PO integral over a flat triangular surface is evaluated. In particular, the surface integral is reduced to a line integral over the intersecting curve of the flat triangular surface and the plane formed by the incident plane wave's propagation and observation directions. This intersection results in a straight line, hence the PO integral can be easily evaluated analytically. This idea is then extended to NURBS surfaces but the intersection and the line integral are determined numerically [7].

In this work, a scheme to evaluate the PO integral, which is interpreted as a Radon transform, is presented for curved (quadratic) triangular surfaces in time domain. The intersection of the above mentioned plane and curved triangular surface, that yields the line integral, is determined as a quadratic curve in terms of the barycentric (area) coordinates [10]. Depending on the triangle, plane and barycentric coordinates used for coordinate transformation, this curve can be classified as an ellipse, a hyperbola, etc. The limits of the line integral can be determined using the intersection points of the intersecting curve and the curved triangle's edges. Using a proper coordinate transformation, e.g. elliptic coordinates for ellipses, the line integral, hence the PO integral, can be evaluated analytically. Note that the resulting expressions are obtained in exact-form without any approximation or assumption.

2 Formulation

Let S denote illuminated surface of a perfect electrically conducting (PEC) scatterer, and S is discretized with curved triangles $S = \bigcup S_n$. The scattered electric field [7] due to an impulsively excited plane wave is

$$\mathbf{E}^{\text{sca}}(\mathbf{r},t) = -\frac{1}{2\pi c}\frac{\partial_t \delta(t-r/c)}{r} * \mathbf{E}^{\text{sca}}_{\text{rc}}(\hat{\mathbf{k}}_s,t), \qquad (1)$$

where ∂_t and "$*$" denote time derivative and convolution, respectively, c is the speed of light, $\delta(\cdot)$ is the Dirac delta function, and $\mathbf{E}^{\text{sca}}_{\text{rc}}(\hat{\mathbf{k}}_s,t)$ is the range corrected electric field:

$$\mathbf{E}^{\text{sca}}_{\text{rc}}(\hat{\mathbf{k}}_s,t) = \hat{\mathbf{k}}_s \times \hat{\mathbf{k}}_s \times (\hat{\mathbf{k}}_i \times \hat{\mathbf{p}}) \times \sum_n \mathbf{h}_n(t). \qquad (2)$$

Here $\hat{\mathbf{k}}_s$ is the observation direction, $\hat{\mathbf{k}}_i$ and $\hat{\mathbf{p}}$ are the propagation direction and polarization of the incident plane wave, respectively, and $\mathbf{h}_n(t)$ function is defined as

$$\mathbf{h}_n(t) = \int_{S_n} \hat{\mathbf{n}}(\mathbf{r}')\delta\left(t - \frac{2}{c}\mathbf{k}_r \cdot \mathbf{r}'\right) d\mathbf{r}', \qquad (3)$$

where $\hat{\mathbf{n}}(\mathbf{r})$ is the outward unit normal vector to S_n, and $\hat{\mathbf{k}}_r = (\hat{\mathbf{k}}_i - \hat{\mathbf{k}}_s)/2$. In (3), shape of S_n determines the shape of the intersecting curve. If S_n is a flat surface, then $\hat{\mathbf{n}}(\mathbf{r})$ is constant, therefore $\mathbf{h}_n(t)$ in (3) is reduced to integration of a scalar function, also intersection of the plane and the flat triangle yields a straight line as shown in [2]. For curved surfaces, $\hat{\mathbf{n}}(\mathbf{r})$ is not constant inside the integration domain S_n. Hence $\hat{\mathbf{n}}(\mathbf{r})$ should be taken into account while evaluating $\mathbf{h}_n(t)$.

Assuming that $t = t_0$, $\hat{\mathbf{k}}_r$, and $\mathbf{r}_i$, $i = 1,\ldots,6$, nodes of the curved triangle are known, the steps of the scheme to evaluate $\mathbf{h}_n(t)$ is given as follows:

Step 1: The Dirac delta function in (3) defines a plane at a given time $t = t_0$, Inserting the barycentric coordinates defined for the curved triangle, into the plane equation results in a quadratic equation. Type of the quadratic equation determines the type of intersecting curve in the barycentric coordinate system [10], e.g. hyperbola, ellipse, etc. If there is no intersection, then $\mathbf{h}_n(t_0) = 0$. In addition, the intersection points on the curved triangle can be determined using the quadratic equation easily. With this step, the PO integral over curved triangle surface is reduced to a line integral.

Step 2: To parametrize the intersecting curve, a second coordinate transformation is applied, considering the type of the curve: e.g. elliptic coordinate transformation for ellipse.

Step 3: The parametrized line integral, hence the PO integral, can be evaluated in closed-form. The intersection points determined in Step 1 can be used as the limits of the integration.

Note that for every type of intersection curve there are different closed-form expressions. Since these expressions occupy large space they are not presented here for brevity.

3 Numerical Example

In this section a numerical example is presented to validate the proposed scheme. A curved triangle on a discretized unit sphere is chosen, as shown in Fig. 1(a). The nodes of the curved triangle are $\mathbf{r}_1 = (1,0,0)$, $\mathbf{r}_2 = (0.877, -0.291, 0.381)$, $\mathbf{r}_3 = (0.886, -0.464, 0)$, $\mathbf{r}_4 = (0.968, -0.157, 0.194)$, $\mathbf{r}_5 = (0.903, -0.353, 0.194)$, $\mathbf{r}_6 = (0.971, -0.239, 0)$. For comparison, the curved triangle is divided repeatedly into 6 flat and 6 curved triangles, e.g. the curved triangle shown in Fig. 1(a), divided into 6 flat triangles shown in Fig. 1(b) and 6 curved triangles (not shown). Then these 6 curved triangles are subdivided into 6 flat triangles as shown in Fig. 1(c) and so on. Closed-form expressions given in [2] are applied to these flat triangles.

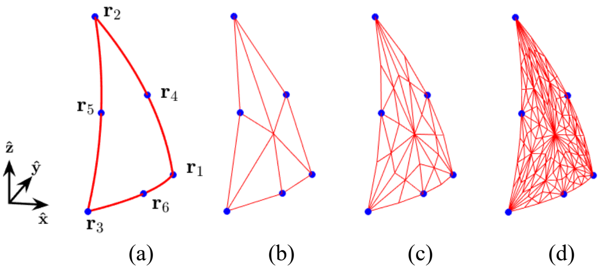

Figure 1. Test triangles: (a) the curved triangle, and (b)-(d) 6, 36, 216 subdivided flat triangles.

For the example, the propagation direction of the incident wave $\hat{\mathbf{k}}_i = -\hat{\mathbf{z}}$, observation direction $\hat{\mathbf{k}}_s = \hat{\mathbf{z}}$, time step size is chosen as $\Delta t = 2.855$ ps. In this setup, the plane is directed to $\hat{\mathbf{k}}_r = \hat{\mathbf{z}}$, starts to intersect with the curved triangle at $t = -2.5409$ ns, and leaves the triangle at $t = 0$ s.

Fig 2. plots the norm of the range corrected electric field $\left|\mathbf{E}_{\mathrm{rc}}^{\mathrm{sca}}(\hat{\mathbf{k}}_s, t)\right|$, obtained by applying the proposed scheme to the curved triangle in Fig. 1(a) and the scheme presented in [2] to 6, 36, 216 flat triangles in Fig. 1(b)-(d). Fig. 3 plots the root mean square error between the result for the curved triangle and the results obtained for 6 – 7776 flat triangles. As seen in Fig. 2 and 3, the results obtained using flat triangles are converging to the results obtained for the curved triangle while the number of the flat triangles increases. Since the both methods use closed-form expressions only source of error is the geometric modeling. Using more flat triangles represents the curved surface better, as a result the error reduces while the number of flat triangles increases.

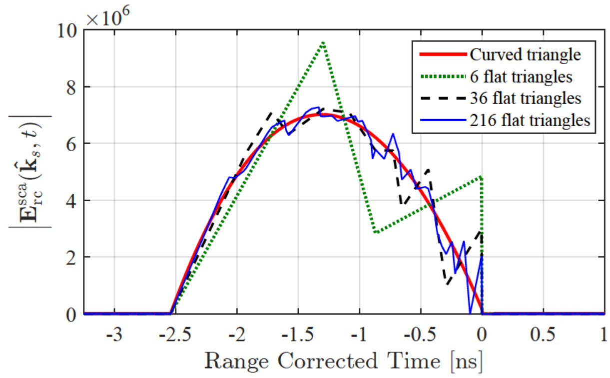

Figure 2. Norm of the range corrected scattered electric field.

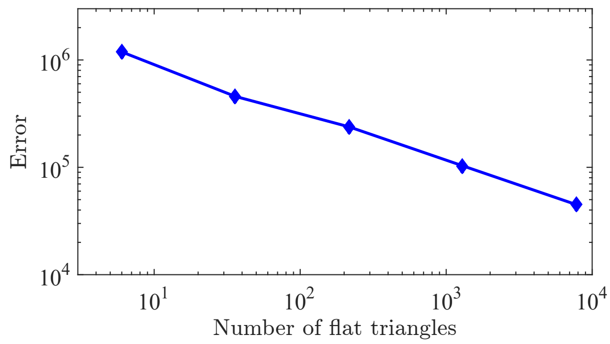

Figure 3. Error in the range corrected scattered electric field.

4 Conclusions

In this work, a scheme for the evaluation of the time domain PO integral on curved triangles is proposed. The scheme allows reducing the PO integral over the curved surface to a line integral, then this line integral can be evaluated in closed-form using proper coordinate transformations. A preliminary numerical example is presented to show the accuracy of the proposed scheme.

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