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A Physical Insight into Complex-Source Beam Diffraction by a Wedge

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Abstract

An extension of the Uniform Geometrical Theory of Diffraction (UTD) has been recently proposed to analyze Complex-Source Beam (CSB) diffraction by a perfectly electrically conducting (PEC) wedge. In the context of high-frequency techniques, the proposed simple and compact solution has been written in terms of incident, reflected and diffracted contributions, so that it can be directly applied to calculate the scattering from more complex geometries with edges. The purpose of this paper is to show the accuracy of the above approach through comparisons with a rigorous solution obtained by a multipole expansion of the scattered field. A physical insight into the involved phenomena is also gained.

1. Introduction

As well known, when the dimensions of a scattering object become large with respect to the operating wavelength, high-frequency asymptotic techniques can be efficiently applied to analyze many practical scattering problems. Among the above high-frequency techniques, one which has been extensively used for its simplicity and numerical stability is the Uniform Geometrical Theory of Diffraction (UTD) [1]. In this context, the high-frequency analysis of diffraction from simple canonical objects opens the way to calculate the scattering from more complex metallic or material objects [2, 3]. More recently, attention has been focused on a specific UTD extension, which allows to consider evanescent wave scattering from wedges [4, 5], although the problem of analyzing propagation and scattering of evanescent waves has been always of strong interest [6, 7]. The latter situation may occur for instance when the wedge is illuminated by a surface wave, a lateral wave, or a wave propagating in a lossy medium [8]. Basically, it has been shown in [4, 5] that a simple UTD type solution can be obtained by a direct analytic continuation of the corresponding UTD formulation for sources in real space, by allowing angles and distances to have complex values. It is observed that the proper discontinuities in the diffracted field are provided by the standard UTD transition function, when it is extended to complex arguments. These discontinuities exactly compensate for those exhibited by the incident and reflected field, when crossing the incident and reflection shadow boundary, respectively. The main problems in this procedure consist in the interpretation of physical phenomena in real space. This latter aspect represents the main object of this paper. It will be analyzed in the following sections, after demonstrating that the derived diffraction coefficients provide very accurate results, when compared with a rigorous multipole expansion of the field.

2. UTD Diffraction Coefficients

A time factor $\exp(j\omega t)$ will be assumed and supressed throughout the analysis. Let us refer to the geometry reported in Fig. 1, where a PEC wedge is illuminated by a complex-source beam (CSB) with complex-valued source coordinates $(\tilde{\rho}', \tilde{\phi}')$.

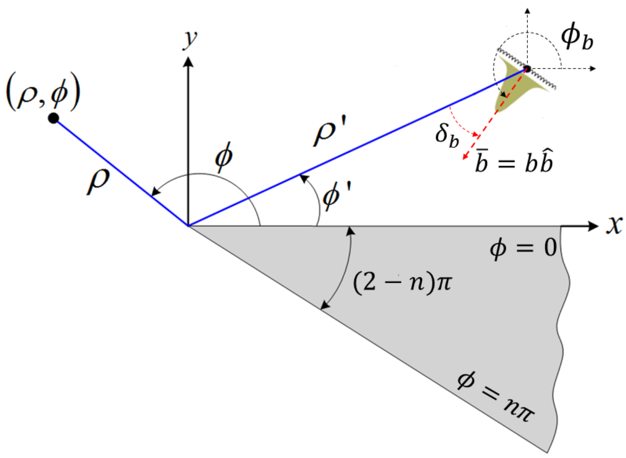

Figure 1. Geometry for CSB diffraction by a PEC wedge. The complex source-coordinates are given by $(\tilde{\rho}', \tilde{\phi}')$. The axis of the beam is pointing along $\hat{b}$, forming an angle ϕ_b with respect to the x-axis. The angle δ_b (which can be positive or negative) denotes the tilting of the beam axis with respect to the direction of the edge. The interior wedge angle is WA=(2-n)π .

As proposed in [4, 5], the uniform high-frequency solution can be obtained by following a standard UTD procedure. In particular, the total field in the exterior region is represented as the summation of the incident, the reflected, and the diffracted contributions. Then, suitable expressions for the diffracted contribution can be obtained by applying a uniform asymptotic evaluation of the diffraction integral following the Pauli-Clemmow method [9, 10], and retaining all terms of order $K^{-1/2}$ in the asymptotic evaluation.

Eventually, the diffracted contribution can be cast in the following form:

$$u_{s,h}^{d}(\rho,\tilde{\rho}';\phi,\tilde{\phi}') \approx u^{i} D_{s,h} \frac{e^{-jk\rho}}{\sqrt{\rho}}, \tag{1}$$

where:

$$u^{i} = \frac{-j}{4}\sqrt{\frac{2j}{\pi k}}\frac{e^{-jk\tilde{\rho}'}}{\sqrt{\tilde{\rho}'}} \quad . \tag{2}$$

In (1), the sub-scripts s,h are corresponding to the soft and hard case, respectively. Eventually, the diffraction coefficients in (1) assume the following form:

$$\begin{aligned} D_{s,h} = &\left[d^{+}\left(\tilde{\beta}^{-}, z_p^{+}(\tilde{\beta}^{-})\right) + d^{-}\left(\tilde{\beta}^{-}, z_p^{-}(\tilde{\beta}^{-})\right) \right] \\ &\mp \left[d^{+}\left(\tilde{\beta}^{+}, z_p^{+}(\tilde{\beta}^{+})\right) + d^{-}\left(\tilde{\beta}^{+}, z_p^{-}(\tilde{\beta}^{+})\right) \right] \end{aligned} \tag{3}$$

where:

$$\begin{aligned} d^{\pm}\left(\tilde{\beta}, z_p^{\pm}(\tilde{\beta})\right) = &\frac{-e^{j\pi/4}}{2n\sqrt{2\pi k}} \left\{ \cot\left(\frac{\pi \pm \tilde{\beta}}{2n}\right) F\left(kL\tilde{a}^{\pm}(\tilde{\beta})\right) + \right. \\ &+ \left(-\sqrt{\frac{\rho+\tilde{\rho}'}{\gamma\left(z_p^{\pm}(\tilde{\beta})\right)}} \frac{n}{\cos\left(z_p^{\pm}(\tilde{\beta})/2\right)} + \cot\left(\frac{\pi+\tilde{\beta}}{2n}\right) \right) \cdot \\ &\left. \cdot \left(1 - F\left(kL\tilde{a}^{\pm}(\tilde{\beta})\right)\right) \right\} \end{aligned} \tag{4}$$

with:

$$\gamma(z) = \sqrt{\rho^2 + \tilde{\rho}'^2 - 2\rho\tilde{\rho}'\cos z} \quad , \tag{5}$$

$$L = \frac{\rho\tilde{\rho}'}{\rho+\tilde{\rho}'} \quad , \tag{6}$$

$$\tilde{a}^{\pm}(\beta) = 2\cos^2\left(\frac{z_p^{\pm}(\tilde{\beta})}{2}\right). \tag{7}$$

In particular, in eqs. (3), (4) and (7):

$$z = z_p^{\pm} = -\tilde{\beta} + 2nN^{\pm}\pi \,, \tag{8}$$

where $N^{\pm}$ denotes the closest pole to one of the saddle points, namely the integer that most nearly satisfies the following equation:

$$N^{\pm}(\tilde{\beta}) = \frac{\pm\pi + \mathrm{Re}(\tilde{\beta})}{2\pi n}. \tag{9}$$

In eq. (2), for a complex-valued $\tilde{\rho}'$, u^{i} approximately represents the CSB impinging on the edge. We note that the function $F[x]$ appearing in eq. (4) is the conventional UTD transition function, extended to complex arguments x. The properties of this function are exhaustively described in the Appendix of [4].

Also, it is worth pointing out that the structure of eq. (4) highlights two contributions: *i)* the first one, corresponding to the first line of the same eq. (4) can be interpreted as a standard UTD term; *ii)* the second contribution, appearing in the second and third line of equation (4) has been defined as a "slope-like" term, since it is similar to the conventional slope diffraction coefficient. Finally, it is important to note that in the case of a PEC wedge illuminated by a CSB, no situation has been observed where the "slope-like" term provides a significant contribution to the total field.

3. Numerical Results

To demonstrate the accuracy of the proposed high-frequency solution, a set of numerical results is presented in this section. In all the cases shown, data obtained by the proposed high-frequency technique are compared with reference data derived by a rigorous multipole expansion of the field. In particular, the amplitude of the total field in the presence of a PEC wedge is plotted in Fig. 2. There, data obtained by the UTD solution (red asterisks) are compared with those given by the multipole expansion (dotted blue line). The interior wedge angle is 70 degrees. Geometrical and electrical parameters are reported in the figure. As shown in the figure, the UTD data are in practice overlapped to the rigorous solution. Data are relevant to the TM polarization case.

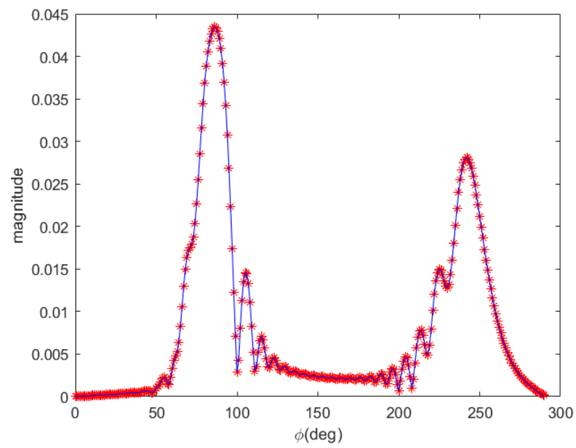

Figure 2. Amplitude of the total field as a function of the observation ϕ: UTD solution (red asterisks); multipole expansion solution (dotted blue curve). Geometrical and electrical parameters: $\rho' = 20\lambda$, $b = 20\lambda, \rho = 10\lambda, \phi' = 80°, WA = 70°, \delta_b = -4°, TM\ case.$

Another example is reported in the following Fig. 3. This time, reference is made to the other polarization case, namely the TE case. The main objective of the example under consideration is to show that the accuracy of the asymptotic solution is surprisingly still very good even when the distance of the observation point from the edge of the wedge is reduced to $0.25\,\lambda$. This time, the interior wedge angle is 45 degrees. The other geometrical and electrical parameters are reported in the figure. Again, the UTD and the reference solution match very well.
A final example is reported in Fig. 4. There, the amplitude of the total field is plotted as a function of the observation angle ϕ at a constant distance ($\rho = 10\lambda$) from the edge of the wedge. Different curves are represented for several values of the beam scan angle δ_b. Geometrical and electrical parameters are reported in the description of the figure. Again, the TM polarization case is analyzed.
In this last case, only the curves obtained by the UTD solution are plotted in the figure for the safe of clarity. However, it is worth observing that the corresponding reference multipole expansion solutions are exactly overlapping to the high-frequency curves shown, confirming the excellent accuracy of the proposed technique.

5. A Physical Insight into the Solution

The advantage of introducing a uniform ray technique to the scattering analysis under consideration is the possibility of gaining a deep physical insight into the diffraction phenomenon. By following a similar approach to that defined in [4, 11, 12, 13], peculiar effects can be observed in the scattering mechanism. In particular, we are especially interested in identifying the location of the actual incident and reflection shadow boundaries and their shape. In the case of inhomogeneous plane wave illumination of the wedge, the shadow and reflection boundaries are displaced from their classical locations, when the incident plane wave is homogeneous [4]. The extent of this displacement depends on the inhomogeneity shown by the incident field. Moreover, the transition regions adjacent to the incident and reflection shadow boundaries are bounded by an ellipse [4].
Similar investigations are in progress for the case analyzed here, when the wedge is illuminated by a CSB.

6. Conclusions

A uniform asymptotic solution for the diffraction of a CSB by a PEC wedge has been obtained in the format of the UTD. In particular, compact expressions have been given for the UTD diffraction coefficients. Numerical comparisons with a rigorous multipole expansion solution have shown that the asymptotic solution is accurate, even when the observation point is only $\lambda/4$ away from the edge of the wedge.

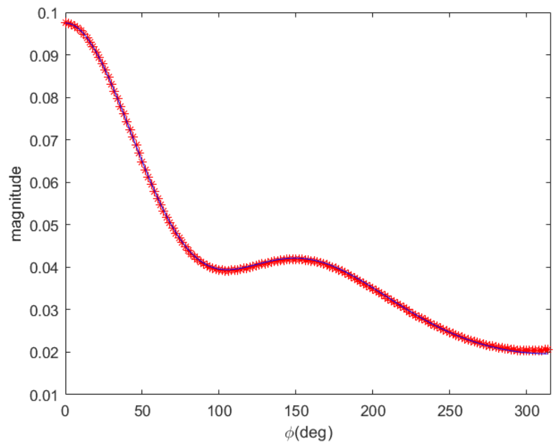

Figure 3. Amplitude of the total field as a function of the observation angle ϕ: UTD solution (red asterisks); multipole expansion solution (blue curve). Geometrical and electrical parameters: $\rho' = 6\lambda,\ b = 10\lambda, \rho = 0.25\lambda, \phi' = 45°, WA = 45°, \delta_b = 5°, TE\ case.$

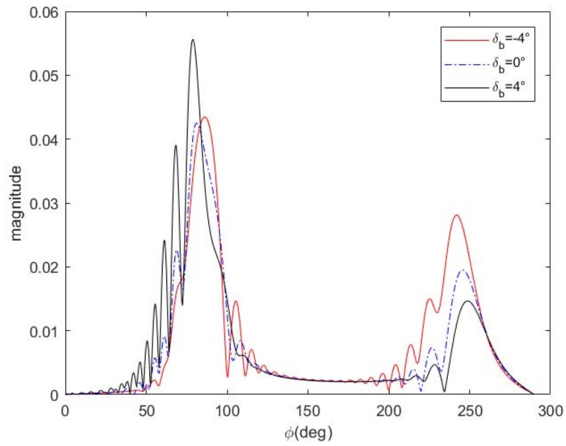

Figure 4. Amplitude of the total field as a function of the observation angle ϕ and for several values of the scan angle δ_b. Geometrical and electrical parameters: $\rho' = 20\lambda,\ b = 20\lambda, \rho = 10\lambda, \phi' = 80°, WA = 70°, TM\ case.$

7. References

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