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Identification of Low and High Ionospheric Rays by a Direct Variational Method

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Abstract

Direct optimization of the optical path functional is a promising approach to the point-to-point ionospheric ray tracing problem. The approach involves a systematic transformation of the ray trajectory to an optimal configuration satisfying the Fermat's principle, while the endpoints are kept fixed according to the boundary conditions. Here, a strategy is proposed for the identification of both high and low rays using a direct variational approach. High rays are obtained by minimizing the optical path of ionospheric radio rays. Low rays which correspond to saddle points of the optical path are found using the minimum mode following method, where the saddle points are essentially converted to local minima. The method is applied to a point-to-point ionospheric ray tracing, where the propagation medium is obtained with the International Reference Ionosphere model.

1. Introduction

Point-to-point ionospheric ray tracing problem involves finding all relevant radio rays between transmitter and receiver. The most traditional approach to this boundary problem is the numerical solution of the eikonal equation combined with the shooting method also known as homing-in approach [1]. However, homing-in approach may suffer from convergence problems when applied to a realistic 3D ionosphere [2]. Coleman [3] proposed an alternative strategy making use of variational methods based on direct optimization of the radio ray optical path, where some initially defined trajectory is iteratively transformed to an optimal one while its end points are kept fixed, satisfying the boundary conditions automatically. High rays are identified simply by minimizing the optical path. Low rays, however, need a special treatment since they do not satisfy the Jacobi test for a minimum and, therefore, can not be found by direct minimization technique [3]. Although both high and low rays are stationary radio wave trajectories, analysis presented in Ref. [4] demonstrated that the former correspond to the minima of the optical path functional, while the latter correspond to the first order saddle points, which are difficult to locate. The low rays can still be found if the trajectory is divided at the apex and separate minimization calculations are performed for each segment of the radio ray [5]. This scheme is, however, only possible if position of the apex is known. Newton-Raphson (NR) method can be used to find the low rays, as advocated by Coleman [3]. However, the NR method converges to any stationary point of an object function and does not discriminate between minima, maxima and saddle points of all orders. The definite identification of the low rays is equivalent to the location of the first order saddle point, stationary points of an object functions which are minima to all but one degree of freedom, with respect to which it is a maximum [4, 5].

Here, the direct variational method of Coleman is revisited in a sense that both high and low rays are found as a result of the same optimization procedure guided by a generalized force. For high rays, the force is simply a negative gradient of the optical path functional. For low rays, the negative gradient needs to be modified so as to effectively convert a saddle point to a local minimum, as explained below. This conversion technique is known in the literature as the minimum mode following (MMF) method [6, 7]. The direct variational method is used to identify all high and low rays between Kaliningrad and Tromsö for the daytime summer solstice on 22.06.2014, where the propagation medium is obtained with the International Reference Ionosphere model.

2. Direct variational method

The optical path of the radio ray in isotropic medium is given by the following equation:

$$S = \int_A^B n(\vec{r})dl \qquad (1).$$

Here, the integration is performed along the curve γ, which connects transmitter A and receiver B; $n(\vec{r})$ — refractive index at point $\vec{r}$; dl — the length element along γ. Continuous curve γ is then represented as a polygonal line connecting N vertices and the integral in Eq. (1) is discretized using trapezoidal rule so that the optical path functional becomes a multidimensional function of positions of the vertices:

$$S \approx \frac{1}{2}\sum_{i=1}^{N-1}(n_{i+1}+n_i)\,|\vec{r}_{i+1}-\vec{r}_i| \qquad (2).$$

Here, $\vec{r}_i$ is a position of the ith vertex and $n_i \equiv n(\vec{r}_i)$ is a refractive index at $\vec{r}_i$. According to Fermat's principle, ionospheric point-to-point ray tracing then reduces to an identification of stationary points of function S. For practical applications, stationary points of only two types are relevant, i.e. minima and first order saddle points. The former correspond to high ionospheric rays while the latter correspond to low rays [5].

This can be visualized as a two-dimensional map, for which a reduced description of the model can be used. This is accomplished by choosing a three-point representation of the radio wave trajectory, where two points are fixed according to the boundary conditions and the third one defines the apex position (hypothetical reflection point). Each segment of the trajectory is now a minimum of the optical path functional. With this representation, the radio ray is completely defined by two essential variables – horizontal and vertical coordinates of the apex point – and a contour map of the optical path can be constructed.

Resulting contour map of the optical path is presented in Fig. 1, which demonstrates that high rays correspond to minima of the optical path, while the low rays correspond to saddle points. This explains why high rays can be reliably identified by direct minimization of the optical path. Saddle points are, however, difficult to locate. The difficulty arises from the need to minimize the optical path with respect to all but one degree of freedom for which a maximization should be carried out and it is not known a priori which degree of freedom should be treated differently.

2.1 Identification of High Rays

Discrete representation of high ionospheric rays can be obtained by simply minimizing the optical path given by Eq. (2). The minimization is guided by a negative gradient or generalized force, $\boldsymbol{F}$, components of which are given by the following equation:

$$F_i^{\alpha} = -\frac{\partial S}{\partial r_i^{\alpha}} \qquad (3).$$

Where $i = 2, \ldots, N-1$ and $\alpha = x, y, z$. Observe that the first and the last points do not move upon the minimization since they correspond to the transmitter and receiver. While various numerical, gradient-based methods are available for finding minima of an object function. The velocity projection optimization (VPO) method is used here (see, for example, Ref. [8]).

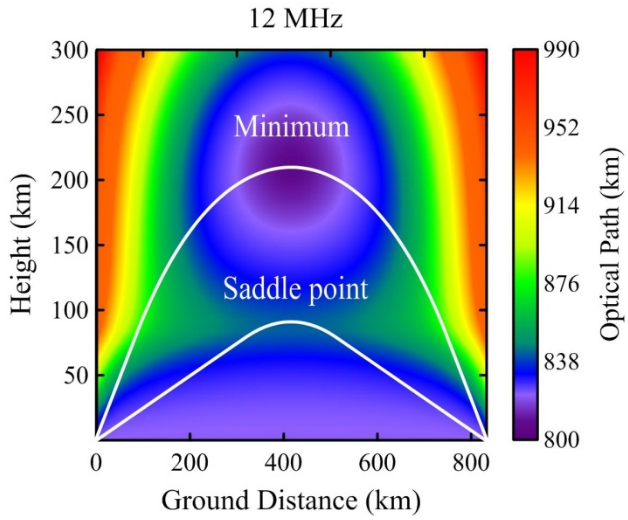

Figure 1. Contour maps of the optical path for frequency 12 MHz in the parabolic-layer ionosphere. White solid lines show radio wave trajectories (high and low rays).

The method essentially represents simulation of damped dynamics, where the effect of damping is modeled by projection of the generalized velocity on the direction of the force. The final, relaxed configuration of vertices gives a discrete representation of the high ionospheric ray. Some initial trajectory is needed to start the calculation. This can be done in various ways, but the simplest method is to generate a linear interpolation between the endpoints. When two or more radio ray trajectories exist between the same receiver and transmitter, the optimization procedure will most likely lead to convergence to the trajectory closest to the initial path. In order to find all high radio wave rays in such a situation, some sampling of the various trajectories needs to be carried out.

2.2 Identification of Low Rays

Low ionospheric rays are the first order saddle points of the optical path functional. Therefore, they can not be identified by following the negative gradient force $\boldsymbol{F}$ in the optimization procedure. However, the force can be modified according to the MMF technique so that a saddle point is effectively converted into a local minimum. This modified force can then be used in any available gradient-based optimization method to locate low rays.

A more detailed description of the MMF method is as follows. In the neighborhood of the sought-for low ray, the minimum eigenvalue, λ, of the Hessian of the optical path (see Eq. (2)) is negative. This statement follows from the definition of the first order saddle point as well as from the assumption that the optical path and its derivatives are continuous functions. Inversion of the minimum mode component of the negative gradient force at points near a first order saddle point leads to a modified force $\widetilde{\boldsymbol{F}}$ which corresponds to the neighborhood of a local minimum:

$$\widetilde{\boldsymbol{F}} = \boldsymbol{F} - 2(\boldsymbol{F} \cdot \boldsymbol{Q}_\lambda)\boldsymbol{Q}_\lambda \qquad (4).$$

Here, $\boldsymbol{Q}_\lambda$ is a normalized eigenvector corresponding to the eigenvalue λ, so called minimum mode. The modified force defined in Eq. (4) can be used in combination with a gradient-based optimization method such as VPO to locate low ionospheric rays.

Optimization procedure for the identification of the low rays can be initiated in the vicinity of the high ray minimum, where all eigenvalues of the Hessian are positive. The ray needs to be moved into a region in the configuration space where the minimum eigenvalue of the Hessian becomes negative. In practice, this can be achieved by e.g. following the gradient force. Then the force guiding the optimization needs to be switched to $\widetilde{\boldsymbol{F}}$, which relaxes the vertices of the discretized trajectory γ on the low ray. Observe that the difference in the identification of high and low rays is only in the definition of the generalized force.

3. Results

The direct variational method is applied here to the point-to-point ray tracing between Kaliningrad (54.57º N, 20º E) and Tromsö (65.65º N, 18.57º E) at 10 MHz. The electron density in the propagation medium is given by IRI-2007 model for 12:00 UT on June 22, 2014. Results of the ray tracing are presented in Fig. 2. Four radio wave rays have been found, two high and two low rays, which is consistent with a well-defined two-layer structure in electron density vertical profile.

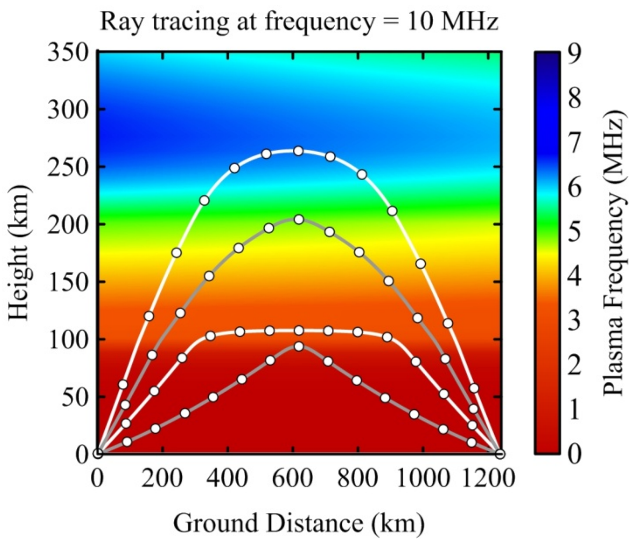

Figure 2. Point-to-point ray tracing between Kaliningrad (54.57º N, 20º E) and Tromsö (65.65º N, 18.57º E) at 10 MHz for the daytime summer solstice on 22.06.2014. Low rays are shown with gray curves while high rays are shown with white curves. Circles along the curves show position of vertices in the discrete, polygonal representation of radio rays.

High rays are calculated by setting initial guesses for the radio wave trajectory at the altitudes of F2 and E layer peaks and then relaxing the trajectory to a local minimum under the influence of the negative gradient force (see Eq. (3)). After all high rays have been found, the optimization procedure for the low ray search is initiated. It starts in the vicinity of the minima of the optical path functional and exploits the gradient force to shift the trajectory towards the basin of attraction of a saddle point, i.e. a region in the configuration space where the minimum eigenvalue of the Hessian becomes negative. The force guiding the optimization procedure is then transformed according to the MMF method (see Eq. (4)).

The final, relaxed ray trajectories are shown in Fig. 2 with white (high rays) and gray (low rays) curves. The results obtained with the direct variational method have been verified by a conventional homing-in approach. Excellent agreement between the two methods has been obtained.

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5. References

1. R. M. Jones, J. J. Stephenson, “A versatile three dimensional ray tracing computer program for radio waves in the ionosphere,” *NASA STI/Recon Technical Report*, **76**, 1975, 25476.

2. N.N. Kalitkin, *Numerical Methods,* Nauka, Moscow, Russia, 266 – 276, 1978, (in Russian).

3. C. J. Coleman, “Point-to-point Ionospheric Ray Tracing by a Direct Variational Method,” *Radio Science*, **46**, 5, October 2011, RS5016, doi: 10.1029/2011RS004748.

4. I. A. Nosikov, M. V. Klimenko, P. F. Bessarab, and G. A. Zhbankov, “Investigation of optical path functional for high and low ionospheric radio rays,” in URSI Asia-Pacific Radio Science Conference (URSI AP-RASC), IEEE, August 2016, pp. 1317-1320.

5. I.A. Nosikov, M.V. Klimenko, P. F., Bessarab, and, G. A. Zhbankov, “Application of the nudged elastic band method to the point-to-point radio wave ray tracing in IRI modeled ionosphere”, *Advances in Space Research*, available online 19 December 2016, doi: 10.1016/j.asr.2016.12.003.

6. G. Henkelman, H. Jónsson, “A Dimer Method for Finding Saddle Points on High Dimensional Potential Surfaces Using Only First Derivatives,” *Journal of Chemical Physics*. **111**, July 1999, 7010, doi: 10.1063/1.480097.

7. M.P. Gutiérrez, C. Argáez, H. Jónsson, “Improved Minimum Mode Following Method for Finding First Order Saddle Points,” *Journal of Chemical Theory and Computation*, **13**, 1, December 2016, pp 125–134, doi: 10.1021/acs.jctc.5b01216.

8. P. F. Bessarab, V. M. Uzdin, H. Jónsson, “Method for finding mechanism and activation energy of magnetic transitions, applied to skyrmion and antivortex annihilation,” *Computer Physics Communications,* **196**, November 2015, pp. 335–347, doi: 10.1016/j.cpc.2015.07.001.