2nd URSI AT-RASC, Gran Canaria, 28 May – 1 June 2018

Influence of Channel Estimation Error on Noise Enhancement Suppression with Multilevel Spreading Codes

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Abstract

Zero-forcing frequency-domain equalization (ZF-FDE) has been used to combat frequency-selective fading caused by multipath propagation. The ZF-FDE is effective when the matrix representing the propagation channel is well-conditioned; when ill-conditioned, however, it causes noise enhancement and, as a result, deteriorates the transmission performance. We have recently proposed a new system and a new multilevel spreading code to suppress the noise enhancement. The proposed system regards the spreading code matrix as multiple-input multiple-output (MIMO) propagation channel and performs the ZF-FDE, multiplying the channel matrix by the spreading code matrix. The proposed system was once evaluated on an assumption of no error in estimating the channel matrix. If the channel estimation error is present, the proposed system quality deteriorates. In this paper, we investigate the influence of channel estimation error on the bit error rate of the proposed system by computer simulation.

1. Introduction

Wireless communication is affected by multipath constituted by reflected and diffracted waves with different time delays, and transmission quality deteriorates. In such multipath channel, the received signal is a superposition of several delayed and scaled copies of the transmitted signal, giving rise to frequency-selective fading, which may degrade the transmission performance. Zero-forcing frequency-domain equalization (ZF-FDE) has been proposed to combat frequency selective fading [1]. However, The ZF-FDE causes noise enhancement, when the matrix representing the propagation channel is ill-conditioned.

Application of the ZF-FDE to direct sequence code division multiple access (DS-CDMA) transmission has been studied in [2]-[4]. We have recently proposed a new system and a new multilevel spreading code to suppress the noise enhancement [5]. First, an arbitrary binary spreading code is represented by a matrix. Next, the matrix is singular-value decomposed. Then, all the singular values are replaced by a positive constant, and a reverse operation to the singular-value decomposition generates a new matrix, which is amply well-conditioned. The new matrix represents a multilevel spreading code, which can suppress the noise enhancement. However, the perfect channel estimation was considered. In this paper, we apply a Gaussian approximation to the channel estimation error and evaluate how the channel estimation error affects the BER performance of the proposed system.

2. Conventional System

Transmission system model for DS-CDMA using the ZF-FDE (conventional system) is illustrated in Fig. 1. Throughout this paper, discrete time representation of transmitting and receiving signals is used. The transmitting DS-CDMA signal s is written by

$$s = dC, \qquad (1)$$

where d is the diagonal matrix $(U \times U)$ whose diagonal elements are the data symbol sequences:

$$d = \begin{bmatrix} d_0 & & & 0 \\ & d_1 & & \\ & & \ddots & \\ 0 & & & d_{U-1} \end{bmatrix} \qquad (2)$$

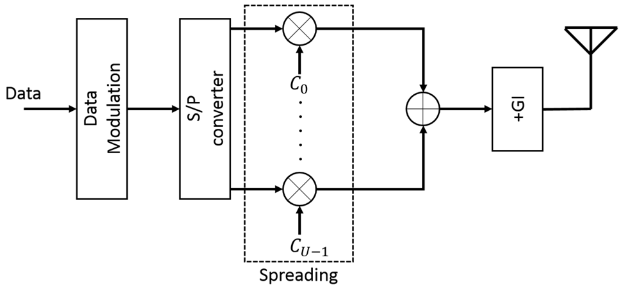
and C is the spreading code matrix $(U \times SF)$:

$$C = \begin{bmatrix} C_{0,0} & C_{0,1} & & C_{0,SF-1} \\ C_{1,0} & C_{1,1} & & \\ \vdots & & \ddots & \\ C_{U-1,0} & & \cdots & C_{U-1,SF-1} \end{bmatrix}. \qquad (3)$$

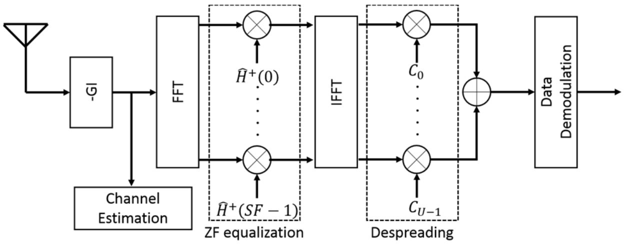
A code multiplexed signal vector $\boldsymbol{s}$ $(1 \times SF)$ is then generated by adding the s matrix $(U \times SF)$ components row by row. Furthermore, the last N_g components in the signal $\boldsymbol{s}$ are inserted as the guard interval (GI) placed at the beginning of the signal.

This extended signal $\boldsymbol{s}$ is transmitted over a frequency-selective fading channel. The fading channel is assumed to be composed of chip-spaced L discrete propagation paths. We assume a block fading, where the path gains remain constant during one DS-CDMA frame. The propagation channel matrix H is defined by

$$H = \begin{bmatrix} & h_{L-1} & \cdots & h_0 & & 0 \\ & & \ddots & & \ddots & \\ 0 & & \cdots & h_{L-1} & \cdots & h_0 \end{bmatrix}, \qquad (4)$$



(a) Transmitter



(b) Receiver

Figure 1. Conventional system.

where $h_i (i = 0, \dots, L-1)$ is channel impulse response. The influence of frequency-selective fading can be expressed, if $N_g T > \tau$ is satisfied, where T is the length of time allotted of one chip and τ the maximum delay time. In this case, the propagation channel matrix can be expressed as a cyclic matrix H_{cyc} $(SF \times SF)$.

$$H_{cyc} = \begin{bmatrix} h_0 & & & h_{L-1} & \cdots & h_1 \\ \vdots & \ddots & & & \ddots & \vdots \\ \vdots & & h_0 & 0 & & h_{L-1} \\ h_{L-1} & & & \ddots & & \\ & \ddots & & & \ddots & \\ 0 & & h_{L-1} & \cdots & \cdots & h_0 \end{bmatrix}. \quad (5)$$

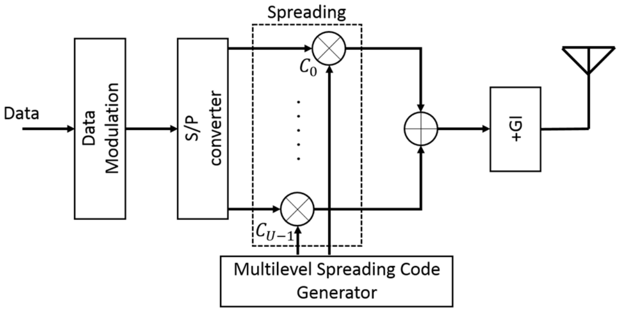
The received signal $\boldsymbol{r}$ $(SF \times 1)$ after removal of GI can be expressed by

$$\boldsymbol{r} = H_{cyc}\boldsymbol{s}^T + \boldsymbol{n}, \quad (6)$$

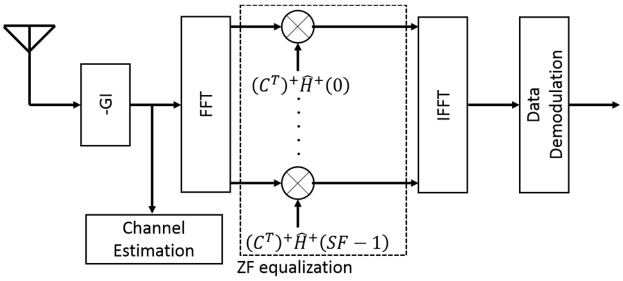
where $\boldsymbol{n}$ is the additive white Gaussian noise (AWGN) and $[\ ]^T$ denotes transposition. The ZF equalization is performed on the received signal by using the estimated propagation matrix. The signal can be expressed by

$$\begin{aligned} \boldsymbol{r_{ZF}} &= \hat{H}_{cyc}^{+}\boldsymbol{r} \\ &= \hat{H}_{cyc}^{+}\left(H_{cyc}\boldsymbol{s}^T + \boldsymbol{n}\right) \\ &= \boldsymbol{s}^T + \hat{H}_{cyc}^{+}\boldsymbol{n}, \end{aligned} \quad (7)$$

where $\hat{H}_{cir}^{+}$ is the estimated propagation channel matrix. When the propagation channel matrix is ill-conditioned, the resulting large $\det\left(\hat{H}_{cyc}^{+}\right)$ causes the noise enhancement.



(a) Transmitter



(b) Receiver

Figure 2. Proposed system.

3. Proposed System

3.1 MIMO-like Representation of the Transmitted and Received Signals

The transmitter (Fig. 2(a)) is the same as that of the conventional system (Fig. 1(a)) except for a new multilevel spreading code generator, which will be described in the next subsection. The received signal is given by

$$\begin{aligned} \boldsymbol{r} &= H_{cyc}\boldsymbol{s}^T + \boldsymbol{n} \\ &= H_{cyc}C^T d^T + \boldsymbol{n}. \end{aligned} \quad (8)$$

The proposed system regards the $H_{cyc}C^T$ as multiple-input multiple-output (MIMO) propagation channel and simultaneously perform ZF-FDE and despreading. Then, the received signal expressed by Eq. (8) is multiplied by $(C^T)^+\hat{H}_{cyc}^{+}$. The resulting signal can be expressed by

$$\begin{aligned} \boldsymbol{r'_{ZF}} &= (C^T)^+\hat{H}_{cyc}^{+}\boldsymbol{r} \\ &= (C^T)^+\hat{H}_{cyc}^{+}\left(H_{ciyc}\boldsymbol{s}^T + \boldsymbol{n}\right) \\ &= (C^T)^+\hat{H}_{cyc}^{+}H_{cyc}C^T d^T \\ &\qquad + (C^+)^T\hat{H}_{cyc}^{+}\boldsymbol{n} \\ &= d^T + (C^T)^+\hat{H}_{cyc}^{+}\boldsymbol{n}, \end{aligned} \quad (9)$$

where $(C^T)^+\hat{H}_{cyc}^{+}$ is the FDE weight. Equation (9) shows that the noise enhancement can be suppressed, if $\det(C^T)$ is maximized and thus $\det((C^T)^+)$is minimized.

3.2 Generation of the Multilevel Spreading Code

Maximization of the determinant of the spreading code matrix is carried out as follows. First, the spreading code matrix is singular-value decomposed as

$$C = V\Sigma U^H, \tag{10}$$

where Σ is a diagonal matrix whose diagonal elements are singular values, V and U are unitary matrix, namely

$$|\det(V)| = |\det(U)| = 1, \tag{11}$$

and $[\ ]^H$ denotes Hermitian transposition. The determinant of the C is

$$\det(C) = \det(V\Sigma U^H) = \det(\Sigma). \tag{12}$$

Then all the singular values in Σ are replaced by a positive constant, and a reverse operation to the singular value decomposition generates a new matrix C' given by

$$C' = V\Sigma' U^H, \tag{13}$$

where V and U are the same matrix in Eq. (10). Matrix C represents a pseudo random spreading code, so does C' automatically. This new code is not binary but multilevel, even when C is binary. Figure 3 shows an example of a Gold code and a proposed multilevel code derived therefrom.

4. Simulation

The influence of channel estimation error on the bit error rate (BER) of the proposed system and the conventional system by computer simulation, whereas ideal channel estimation was assumed in [5]. We adopt Gaussian error model [6] in channel estimation, expressed by

$$\hat{h}_i = h_i + e_i \qquad (i = 0, \ldots, L-1), \tag{14}$$

where $\hat{h}_i$ is the estimated channel impulse response and e_i the independent zero-mean complex Gaussian variables with the variance of $2\sigma^2_{error}$. Simulation conditions are listed in Table 1. An L-path frequency-selective fading channel having uniform power delay profile is assumed. The time delay τ_L of the Lth path is assumed to be $\tau_L = LT$. The conventional system uses Gold codes.

The BER performances of the conventional (DS-CDMA using the ZF-FDE) and the proposed systems were simulated, when as function of the signal-to-noise power ratio (SNR), when $\sigma^2_{error} = 0$ (ideal channel estimation), 0.0001, and 0.0005. With an increase in σ^2_{error}, the BER performances of both systems deteriorates, particularly with high SNRs. However, the proposed system outperforms the conventional system in all cases at high

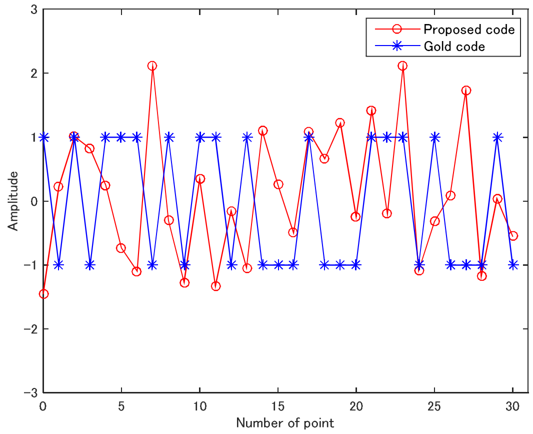


Figure 3. A Gold code and a proposed multilevel code derived therefrom.

Table 1. Simulation parameters

Modulation	BPSK
Spreading codes	Gold code and Proposed code
Spreading Factor	31
Guard interval	10
Channel	Frequency-selective fading
Number of paths	10
Interval of path	1

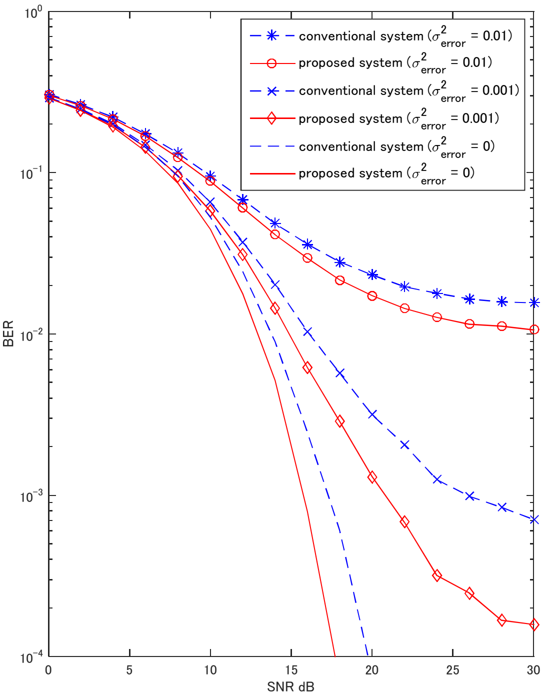


Figure 4. Simulated BER performances.

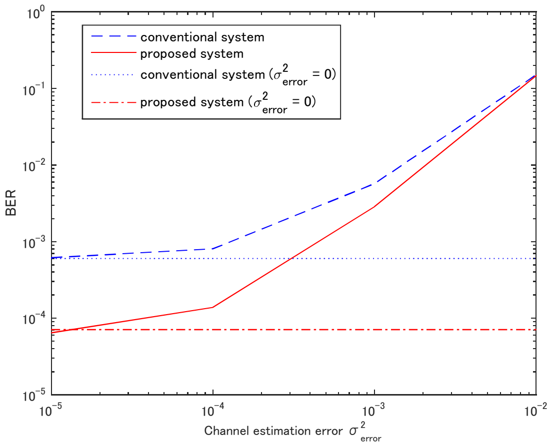


Figure 5. Influence of channel estimation error (SNR = 20 dB).

SNRs, as shown in Fig. 4. This is attributed to the suppression effect of noise enhancement with use of the proposed codes.

Figure 5 shows the BER performance of the conventional and the proposed system as a function of the channel estimation error σ^2_{error}. For comparison, the BER performance of $\sigma^2_{error} = 0$ (ideal channel estimation) is also plotted. As can be seen from this figure, the proposed system is effective up to about $\sigma^2_{error} = 0.001$, when SNR = 20 dB.

5. Conclusion

This paper has studied the influence of channel estimation error on noise enhancement suppression with multilevel spreading codes. The channel estimation error was modeled as the Gaussian approximation. It was found that the proposed system and multilevel codes are more effective than the conventional system even if some degree of channel estimation error occurs.

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