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Dependence of the Effective Length of a Receiving Antenna on the Space Charge Distribution in a Model Source of Quasi-Electrostatic Chorus Emissions

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Abstract

Recently it was shown on the basis of general theory of antennas in plasmas that the effective length of a receiving dipole antenna can be much greater than its geometric length in the case of reception of quasi-electrostatic chorus emissions propagating close to the resonance cone. For such calculations one needs to specify a model wave source, since the actual source—receiver geometry is not well known in the case of chorus emissions generated in a distributed region by a complicated nonlinear process. In this paper, we analyze how the effective length of a spacecraft-borne receiving antenna depends on the parameters of a model source of quasi-electrostatic chorus emissions. In particular, we study the dependence of the effective length on characteristic scales of the space charge distribution in the source and the distance between the source and the receiving antenna.

1 Introduction

One of the most important problems that concern the wave electric field measurements using antennas in plasmas is calculation of the effective length l_{eff} of a receiving antenna. This length is defined [1] according to equality

$$|U| = |E \cos \Upsilon| l_{\mathrm{eff}}, \tag{1}$$

where E is a complex amplitude of the electric field in the incident wave, U is a complex amplitude of voltage induced on the receiving antenna, and Υ is an angle between the electric field vector and the antenna axis.

It is known [2, 3] that, due to reradiation, this length may differ significantly from the antenna geometric length l_{rec} when the quasi-electrostatic waves (i.e., the waves propagating in a direction close to the resonance cone) are received by a short (as compared to the wave length λ_{em} of a parallel propagating whistler mode wave) electric dipole. Therefore, obtaining the electric field values, that are important to know, from voltage data, usually available from observations using spacecraft-borne antennas in space plasmas, is nontrivial in this case.

In [4, 5], an analytical expression for l_{eff} has been found for a case of quasi-electrostatic whistler mode waves and applied to spacecraft observations of oblique chorus waves. In order to specify the electric field of the incident wave and, consequently, to calculate l_{eff}, a model (i.e., fictitious) source of quasi-electrostatic chorus emissions propagating close to the resonance cone has been proposed and assumed to be the other electric dipole.

In this paper, we analyze this model source in more detail. It is investigated how its characteristic spatial scales and the distance between this source and the receiving antenna affect the receiving antenna effective length.

2 Expression for the Effective Length

By using the approach based on the reciprocity theorem and developed in [3], the following expression for the effective length of the short ($l_{\mathrm{rec}} \ll \lambda_{\mathrm{em}}$) dipole antenna receiving quasi-electrostatic whistler mode waves at a frequency ω was obtained in [4, 5]:

$$l_{\mathrm{eff}}(\omega) = \frac{64\lambda_{\mathrm{em}}^2}{\sqrt{\varepsilon(\varepsilon+|\eta|)}\, l_{\mathrm{tr}}^2 \sin\theta_{\mathrm{res}} |\cos\Upsilon|} \times \frac{\left| \int\limits_0^{+\infty} \int\limits_0^{2\pi} k^{-1/2} \rho_{\mathrm{tr}}(k) \rho_{0\vec{k}}(k,\psi) e^{iq(\omega-\omega_0)\tau_0 k} \mathrm{d}k \mathrm{d}\psi \right|}{\left| \int\limits_0^{+\infty} k^{1/2} \rho_{\mathrm{tr}}(k) e^{iq(\omega-\omega_0)\tau_0 k} \mathrm{d}k \right|}. \tag{2}$$

Here, ε and η are the transverse and longitudinal components of relative permittivity tensor, respectively, calculated for the carrier frequency ω_0 (we assume that a wave packet of the observed waves is quasi-monochromatic), θ_{res} is the resonance angle, $\cot^2\theta_{\mathrm{res}} = |\varepsilon/\eta|$, k is a wave number, ψ is the azimuth angle in $\vec{k}$ space (in the plane transversal to k_z, where z axis is parallel to the geomagnetic field), $q = (\partial\mu/\partial\omega)|_{\omega=\omega_0} \times (1+[\mu(\omega_0)]^2)^{-1}$, $\mu(\omega) = \sqrt{|\varepsilon(\omega)/\eta(\omega)|}$, τ_0 is the distance between the source of quasi-electrostatic waves and the receiving antenna (see Figure 1; since the quasi-electrostatic waves are analyzed here, the source center is assumed to be situated on the group velocity resonance cone direction going through the receiving antenna gap at the carrier frequency), $\rho_{\mathrm{tr}}(k) = \rho_{\mathrm{tr}}(\vec{k})|_{\theta_{\mathrm{res}}}$ is spectrum of the charge distribution $\rho_{\mathrm{tr}}(\vec{r})$ along the transmitting dipole (of length l_{tr}) calculated at the resonance angle, and $\rho_{0\vec{k}}(k,\psi) = \rho_{0\vec{k}}(\vec{k})|_{\theta_{\mathrm{res}}}$ is spectrum of the auxiliary charge distribution (with an amplitude of the total half-dipole charge that equals 1) along the receiving antenna calculated at the resonance angle. This charge distribution is used in the reciprocity theorem [3] required for obtaining (2) and therefore does not correspond to the charge induced on the receiver surface by the incident wave. If the receiving dipole consists of 2 thin straight rods with a gap between them, then

$$\rho_{0\vec{k}}(k,\psi) = -\frac{8i\exp[-ikR_0(\psi)]}{\gamma(\psi)kl_{\mathrm{rec}}}\sin^2\left[\frac{\gamma(\psi)kl_{\mathrm{rec}}}{4}\right], \quad (3)$$

and if the receiver consists of 2 small (as compared to the distance between them) spherical conductors, then

$$\rho_{0\vec{k}}(k,\psi) = -2i\exp[-ikR_0(\psi)]\sin\left[\frac{\gamma(\psi)kl_{\mathrm{rec}}}{2}\right]. \quad (4)$$

In (3) and (4),

$$\gamma(\psi) = \sin\alpha\sin\theta_{\mathrm{res}}\cos(\psi-\beta)+\cos\alpha\cos\theta_{\mathrm{res}}, \quad (5)$$

$$R_0(\psi) = \tau_0\sin\theta_{\mathrm{res}}\cos\theta_{\mathrm{res}}\left[\cos(\psi-\varphi_{\mathrm{obs}})+1\right], \quad (6)$$

φ_{obs} is the azimuth angle of the wave, incident on the antenna, in $\vec{k}$ space, α and β are the receiver orientation angles: α is the angle between the geomagnetic field and the dipole axis, and β is the azimuth angle of the dipole (see Figure 1). According to (5), $\cos\Upsilon = \gamma(\varphi_{\mathrm{obs}})$. Relation (3) corresponds to a piecewise constant charge distribution along the thin dipole, and (4) corresponds to 2 point charges approximating 2 small spheres.

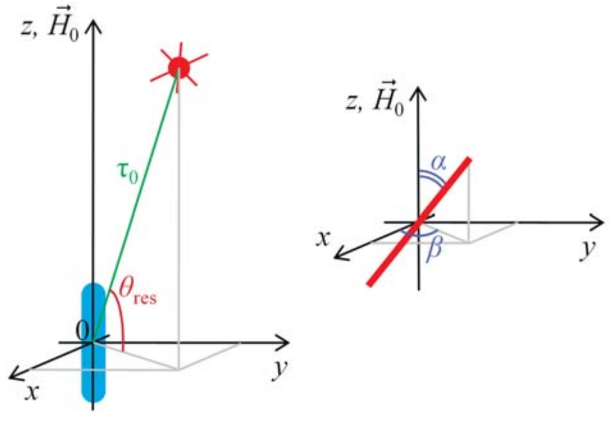

Figure 1. (left) Mutual positions of the source (blue) and spacecraft (red) along the group velocity resonance cone direction (green); and (right) antenna orientation.

3 Characteristics of the Source

The spectrum of charge distribution along the transmitting dipole $\rho_{\mathrm{tr}}(k)$, that enters (2), can be easily specified if the observed waves are transmitted by a metal antenna with known geometry and orientation such as in OEDIPUS-C experiment in the Earth's ionosphere [3]. A different situation takes place for emissions of natural origin such as chorus emissions [6], that are, according to the generally accepted theory [7, 8, 9, 10], generated due to the electron cyclotron instability in the radiation belts. In this case, $\rho_{\mathrm{tr}}(k)$ can be specified as a charge distribution that corresponds to a model (fictitious) transmitting antenna generating wave packets with the parameters corresponding to the observed ones. In order to apply the obtained expression for l_{eff} in this case, one should make an appropriate choice of ω_0 for each spectral component at each moment of time. Importantly, an amplitude of the total half-dipole charge Q_{tr} corresponding to this transmitter does not enter (2) because $\rho_{\mathrm{tr}}(k)$ enters both the numerator and the denominator in there. The latter fact, obviously, is related to the linear nature of the problem.

In [4, 5], where spacecraft observations of quasi-electrostatic chorus emissions have been considered, the following source model has been used:

$$\rho_{\mathrm{tr}}(\vec{r}) = -\frac{8Q_{\mathrm{tr}}}{l_{\mathrm{tr}}^2}z\exp\left(-\frac{4z^2}{l_{\mathrm{tr}}^2}\right)\delta(x)\delta(y), \quad (7)$$

where x,y,z are the Cartesian coordinates with z axis parallel to geomagnetic field, and $\delta(x)$ is the delta function. The corresponding spectrum calculated at the resonance angle is equal to

$$\rho_{\mathrm{tr}}(k) = \frac{i}{2}Q_{\mathrm{tr}}\sqrt{\pi}kl_{\mathrm{tr}}\cos\theta_{\mathrm{res}}\exp\left(-\frac{k^2l_{\mathrm{tr}}^2\cos^2\theta_{\mathrm{res}}}{16}\right). \quad (8)$$

Length l_{tr} of the fictitious transmitter has been calculated from the wave number k_{obs} that corresponds to the observed spectral maximum (i.e., k_{obs} satisfies equation $\mathrm{d}\rho_{\mathrm{tr}}/\mathrm{d}k = 0$): $k_{\mathrm{obs}}l_{\mathrm{tr}}\cos\theta_{\mathrm{res}} = 2\sqrt{2}$. Therefore, model (8) provides the wave field with measured parameters though this source does not coincide with an actual chorus source.

Such a choice of the fictitious transmitter means that the source has no sharp boundaries along z axis, and the extremum point k_{obs} of $\rho_{\mathrm{tr}}(k)$ is equal to its characteristic scale so function $\rho_{\mathrm{tr}}(k)$, up to a constant factor Q_{tr}, has only one parameter, i.e., $l_{\mathrm{tr}}\cos\theta_{\mathrm{res}}$.

The integrals over k in (2) are evaluated analytically [4]. The resulting expressions in case both (3) and (4) are very complicated and not presented here.

In this paper, we consider a more general source model:

$$\rho_{\mathrm{tr}}(k) = kQ\exp\left[-\frac{(k-k_0)^2}{\Delta^2}\right], \quad (9)$$

where $Q = \mathrm{const}$ is proportional to Q_{tr}, Δ is the characteristic scale, and parameter k_0 is found from equation $(\mathrm{d}\rho_{\mathrm{tr}}/\mathrm{d}k)|_{k=k_{\mathrm{obs}}} = 0$:

$$k_0 = k_{\mathrm{obs}}\left(1-\frac{\Delta^2}{2k_{\mathrm{obs}}^2}\right). \quad (10)$$

In this model, up to a constant factor Q, there are two parameters of the source charge distribution, i.e., Δ and k_0.

Spectrum (9) is written down for $k \geq 0$ only. If $k < 0$, spectrum can be defined as $\rho_{\rm tr}(k) = -\rho_{\rm tr}(-k)$ in order to make functions $\rho_{\rm tr}(k)$ and, consequently, $\rho_{\rm tr}(z)$ odd (it corresponds to a dipole distribution along z axis). However, according to (2), the effective length is determined by the region where $k \geq 0$ only so a form of $\rho_{\rm tr}(k)$ for $k < 0$ is not important for calculations at all.

When $k_0 = 0$ or $\Delta \approx k_{\rm obs} \approx k_0$, the situation is similar to (8). The other interesting case is that $\Delta \ll k_{\rm obs} \approx k_0$. It corresponds to a very narrow (in k space) wave packet with $k_{\rm obs} \approx k_0$ propagating in all azimuthal directions (i.e., $0 \leq \psi < 2\pi$) along the resonance cone (the limiting case $\Delta \to 0$ corresponds to $\rho_{\rm tr}(k) \propto \Delta\delta(k - k_{\rm obs})$). In this case when $\omega = \omega_0$, the integrals over k in (2) are evaluated approximately using the saddle-point method:

$$l_{\rm eff}(\omega_0) = \frac{64\lambda_{\rm em}^2 \left| \int\limits_0^{2\pi} \rho_{0\vec{k}}(k_0, \psi) {\rm d}\psi \right|}{\sqrt{\varepsilon(\varepsilon + |\eta|)} l_{\rm tr}^2 \sin\theta_{\rm res} |\cos\Upsilon| k_0}. \qquad (11)$$

In both models, (8) and (9), $k_{\rm obs}$ and, consequently, $l_{\rm tr}$ are defined by the measured wave field parameters and calculated from the dispersion relation and the observed wave normal angle $\theta_{\rm obs}$ that is found from the wave magnetic field measurements using the singular value decomposition method [11].

The other characteristic of the source charge distribution, both for (8) and (9), is distance τ_0 between the source and the receiving antenna (along the resonance cone for group velocity). In [4, 5], some preliminary estimates concerning this distance have been done. The minimal allowed value of τ_0 is obviously $l_{\rm tr}$ otherwise the proposed source model becomes doubtful: indeed, if $\tau_0 \lesssim l_{\rm tr}$, then the receiving antenna is very close to the source region or even within it, and a more fine-structured source model should probably be used. In this paper, dependence $l_{\rm eff}$ on τ_0 (for $\tau_0 > l_{\rm tr}$) is studied in more detail.

4 Calculation Results and Discussion

Here we apply the obtained formulas to chorus wave quasi-electrostatic fields. In order to specify realistic wave parameters, we employ THEMIS data reported in [12]. Specifically, we use the parameters of the chorus burst observed by THEMIS A on 26-11-2008 at 03:18:23 UT at the geomagnetic equator and L-shell $= 5$ ($\omega_0 = 15\,708$ s^{-1}, $\theta_{\rm res} = 64.7^\circ$, angle $\theta_{\rm obs} = 58.0^\circ$ has been found from the Search Coil Magnetometer (SCM) data, i.e., wave magnetic field data). This event, among others, has been considered in [12] in relation to electron energization.

Each THEMIS spacecraft [13] is equipped with the Electric Field Instrument (EFI) that consists of the three independent orthogonal dipole antennas of a half-length of 24.8 m, 20.2 m, and 3.47 m. We will refer to them as dipoles A, B, and C, respectively. Dipoles A and B consist of 2 small spheres, and dipole C consists of 2 straight thin rods. Their orientation angles α and β have been found from the Flux Gate Magnetometer (FGM) data that provide the results of geomagnetic field measurements. Dipoles A and C are considered in this paper in more detail.

Dependence of the effective length $l_{\rm eff}$ on distance τ_0 between the source and the receiver is shown in Figure 2 for $\omega = \omega_0$. As it can be seen, $l_{\rm eff}$ decreases as τ_0 increases. Function $l_{\rm eff}(\tau_0)$ appears to be close to a power function $l_{\rm eff} \propto \tau_0^{-1/2}$. This is probably can be explained by the following fact. On the one hand, the electric dipole radiation field $E_{\rm pl}$ decreases in a magnetoplasma as $E_{\rm pl} \propto \tau_0^{-1/2}$ [14]. On the other hand, the corresponding dependence in a vacuum is $E_{\rm v} \propto \tau_0^{-1}$, where τ_0 should be treated as a distance from the dipole in any direction [1]. Then $E_{\rm pl}/E_{\rm v} \propto \tau_0^{1/2}$. Furthermore, the receiving antenna effective length is equal to $\chi l_{\rm rec}$ in a vacuum, where factor $\chi = $ const depends on the receiving antenna geometry and wave length only [1]. Therefore, ratio $l_{\rm eff}/l_{\rm rec}$ can be treated, up to a constant factor, as a ratio of the receiver effective lengths in a magnetoplasma and vacuum. Thus, according to (1), $l_{\rm eff}/l_{\rm rec} \propto E_{\rm v}/E_{\rm pl} \propto \tau_0^{-1/2}$, and this is in agreement with Figure 2.

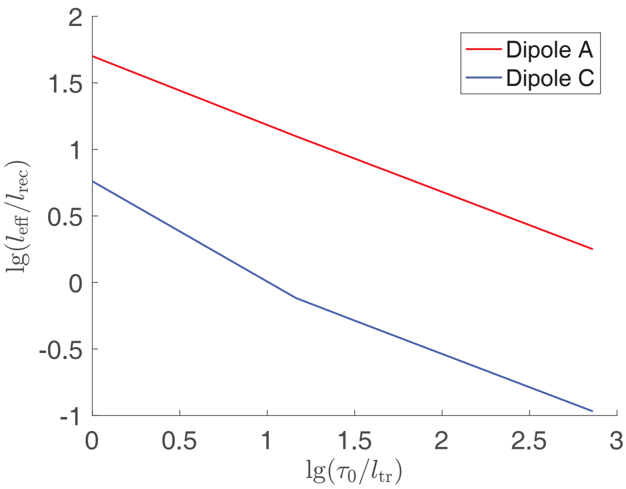

Figure 2. Dependence $l_{\rm eff}(\tau_0)$ for dipoles A and C.

Dependence of the effective length $l_{\rm eff}$ on the spectrum characteristic scale Δ is shown in Figure 3 for $\tau_0 \approx 15 l_{\rm tr}$. If $\Delta \lesssim k_{\rm obs}$, then $l_{\rm eff}$ is almost constant, i.e., it does not depend on Δ. This case corresponds to a wave packet with a broad range of k propagating in the direction close to the resonance cone, and this is typical for quasi-electrostatic waves excited by antennas in plasmas [14]. If $\Delta \ll k_{\rm obs}$, then $l_{\rm eff}$ increases as compared to $\Delta \lesssim k_{\rm obs}$. Perhaps this can be explained by the fact that a quasi-electrostatic wave with a single given wave number k is reradiated by the receiving dipole, probably, more effectively than a wave packet with a broad range of k. In the latter case, each harmonic with given k interferes with all other harmonics, and superposition of the currents, that are induced on the receiver by each harmonic and produce the reradiation field, is probably smaller as compared to a single harmonic. We also note that a case of $\Delta \ll k_{\mathrm{obs}}$ may correspond to a situation when refraction is significant, i.e., one harmonic (with single k) propagates in the resonance direction while all other harmonics propagate in other directions (closer to the geomagnetic field line) due to refraction.

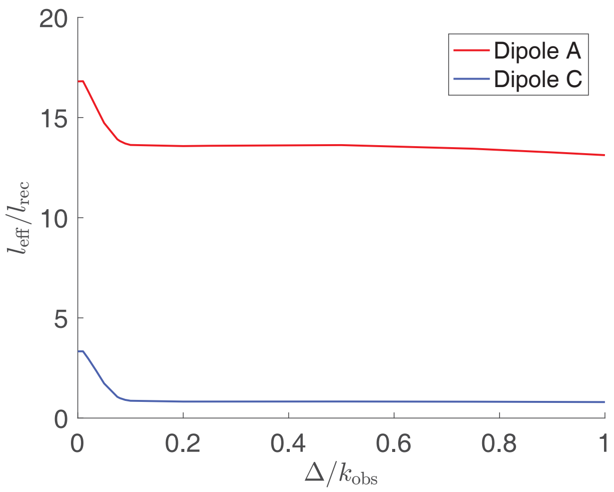

Figure 3. Dependence $l_{\mathrm{eff}}(\Delta)$ for dipoles A and C.

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