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Numerical Analysis of Antenna Excitation of Quasi-Electrostatic Waves: Application to Probing of the Near-Earth Plasma

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Abstract

In this paper, it is investigated how the numerical method of moments can be used for analysis of antenna excitation of quasi-electrostatic waves, propagating in cold magnetoplasmas close to the resonance cone. The main attention is focused on the properties of the integro-differential equation, that describes the charge distribution along the antenna wire, and the kernel of this equation. The calculations are performed for the parameters typical for the near-Earth plasma. Hence, the results can be used for design of the near-Earth plasma probing missions.

1 Introduction

It is known that quasi-electrostatic waves can propagate close to the resonance cone in a magnetoplasma, particularly in the whistler mode frequency range [1]. Their wave numbers (a) are much larger than the inverse length $\lambda_{\rm em}$ of the parallel (with respect to the ambient magnetic field) propagating electromagnetic wave and (b) generally, in a collisionless plasma, can be arbitrarily large. Consequently, these waves are effectively radiated by the short electric dipoles of length $2L \ll \lambda_{\rm em}$ [2]. Importantly, such dipoles are generally easier to place on spacecraft than a half-wave dipole because of their size.

Hence, the short electric dipoles are suitable for the near-Earth plasma probing missions. For example, such dipoles were used both for transmission and reception of waves (in particular, at the frequency of 100 kHz) on the tethered sounding rocket double payload OEDIPUS-C that reached an apogee altitude of 824 km over northern Alaska [3]. Now the VLF (1–10 kHz) transmitter POPRAD for polar orbiting LEO satellites is proposed in order to perform systematic probing of the plasmasphere by transmitting impulses that are powerful enough to reach the other hemisphere propagating along the magnetic field lines [4].

It is known that the electrodynamic characteristics (e.g., the input impedance and directivity) of antennas in magnetoplasmas in the resonance conditions (i.e., when the resonance cone exists) differ from the vacuum case both qualitatively and quantitatively. Indeed, the expression for the input impedance of the short antenna in the resonance magnetoplasma contains a significant real term which is equivalent to the radiation impedance $R > 0$ [5, 6] while $R \approx 0$ for a short dipole in a vacuum [7]. This is because a plasma wave is effectively radiated. Consequently, the dipole field pattern in the resonance magnetoplasma is very different from the vacuum case: the electric field has the maximum value on the resonance cone [8].

Therefore, for practical applications and spacecraft design, it is important to be able to calculate these electrodynamic characteristics for antennas of arbitrary geometry. In order to do this, one should find the current (charge) distribution on the antenna surface first of all. (This distribution is also important for a problem of electromagnetic compatibility of the spacecraft sensors, that may be sensible to strong currents and fields.) This problem is reduced to solution of the corresponding integro-differential equation that follows from the electromagnetic boundary conditions. However, this can be done analytically only for antennas of very simple geometry, such as straight cylindrical dipoles [6, 9]. Therefore, numerical calculations of such systems gain in importance. Typically, this integro-differential equation is solved using the numerical method of moments that works very well for antennas in a vacuum case [10, 11]. Unfortunately, this method is not yet adapted for antennas in anisotropic media such as resonance magnetoplasmas when the kernel of this equation (the Green's function) is singular not only at the source point but on the resonance cone surface too. In this paper, it is investigated how this method can be used for analysis of antenna excitation of quasi-electrostatic waves, propagating in cold magnetoplasmas close to the resonance cone.

2 Problem Formulation and Basic Equations

A symmetric center-fed dipole of length $2L \ll \lambda_{\rm em}$ and radius $a \ll 2L$, immersed in a cold magnetoplasma and parallel to the ambient magnetic field $\vec{H}_0$, is considered (see Fig. 1). Its radiation frequency ω is such that the resonance cone exists. Hence, its radiation electric field $\Re[\vec{E}(\vec{r})\exp(-i\omega t)]$ is quasi-electrostatic at each moment of time, $\vec{E}(\vec{r}) = -\nabla\Phi(\vec{r})$, and satisfies equation $\mathrm{div}[\hat{\boldsymbol{\varepsilon}}\nabla\Phi(\vec{r})] = 0$ in ambient space (i.e., outside the dipole), or

$$\varepsilon\frac{\partial^2\Phi}{\partial x^2} + \varepsilon\frac{\partial^2\Phi}{\partial y^2} + \eta\frac{\partial^2\Phi}{\partial z^2} = 0, \qquad (1)$$

where ε and η are the transverse and longitudinal components of the dielectric tensor $\hat{\boldsymbol{\varepsilon}}$, respectively. Importantly, $\varepsilon\eta < 0$ throughout the paper because the resonance conditions are considered.

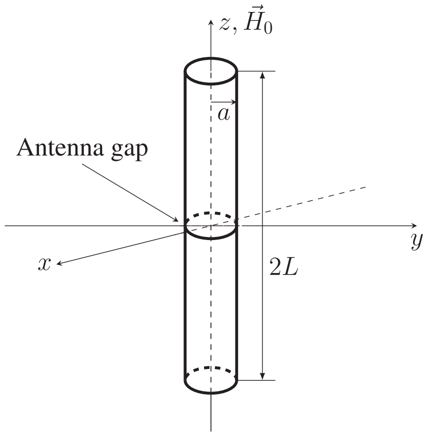

Figure 1. Geometry of the problem.

The Green's function of (1) that corresponds to the outgoing waves is [6]

$$G(\vec{r},\vec{r}\,') = \begin{cases} \dfrac{|\varepsilon\eta|^{-1/2}\,\mathrm{sgn}\,\varepsilon}{\sqrt{-(x-x')^2-(y-y')^2+\mu^2(z-z')^2}}, \\ \quad (x-x')^2+(y-y')^2 < \mu^2(z-z')^2; \\ -\dfrac{i\,|\varepsilon\eta|^{-1/2}}{\sqrt{(x-x')^2+(y-y')^2-\mu^2(z-z')^2}}, \\ \quad (x-x')^2+(y-y')^2 \geq \mu^2(z-z')^2, \end{cases} \tag{2}$$

where $\mu^2 = |\varepsilon/\eta|$. Note that this function is singular on the resonance cone surface $(x-x')^2+(y-y')^2 = \mu^2(z-z')^2$ and becomes purely imaginary when crossing this surface.

From (1), it is possible to represent $\Phi(\vec{r})$ as a functional of the charge density distribution $\sigma(\vec{r})$ on the antenna surface S:

$$\Phi(\vec{r}) = \oiint\limits_S \sigma(\vec{r}\,')G(\vec{r},\vec{r}\,')\,\mathrm{d}S'. \tag{3}$$

Representing the total electric field as a superposition of (a) the radiation field $\vec{E}$ and (b) the electric field $\vec{E}^{\mathrm{emf}}(z) = U_0\delta(z)\vec{z}_0$ of the given electromotive force U_0 (where $\delta(z)$ is the delta function, and $\vec{z}_0 \parallel \vec{H}_0$ is a unit vector), one gets

$$[E_z(z) + E_z^{\mathrm{emf}}(z)]|_{\vec{r}\in S} = 0. \tag{4}$$

Here the thin-wire approximation [12] and the boundary condition for the tangential component of electric field on the antenna surface are used. Consequently, using the boundary condition $\kappa(\pm L \mp 0) = 0$ for the linear charge density $\kappa(z)$, one finds for a thin wire that

$$E_z = -\frac{\partial\Phi}{\partial z} = -\frac{\partial}{\partial z}\int_{-L}^{L}\kappa(z')G(z-z')\,\mathrm{d}z' = -\int_{-L}^{L}G(z-z')\frac{\partial}{\partial z'}\kappa(z')\,\mathrm{d}z'.$$

Here

$$G(z-z') = G(\vec{r},\vec{r}\,')|_{x=y=0,\,\vec{r}\,'\in S} = \begin{cases} \dfrac{|\varepsilon\eta|^{-1/2}\,\mathrm{sgn}\,\varepsilon}{\sqrt{-a^2+\mu^2(z-z')^2}}, & a^2 < \mu^2(z-z')^2; \\ -\dfrac{i\,|\varepsilon\eta|^{-1/2}}{\sqrt{a^2-\mu^2(z-z')^2}}, & a^2 \geq \mu^2(z-z')^2. \end{cases} \tag{5}$$

Note that the charge density does not depend on the azimuth angle in *xy*-plane because current has only *z*-component in the thin-wire approximation.

Hence, the final form of the integro-differential equation that describes the unknown charge density distribution along the antenna wire is

$$\int_{-L}^{L}G(z-z')\frac{\partial}{\partial z'}\kappa(z')\,\mathrm{d}z' = U_0\delta(z). \tag{6}$$

Unlike a vacuum case, kernel $G(z-z')$ of this equation is singular here not at the source point only but on the resonance cone surface too. Validity of (5) and (6) is discussed in [13] in more detail.

3 Method of Moments

In order to find an approximate solution of (6), one should represent function $\sigma(z)$ as a finite linear combination of the basis functions $f_n(z)$, where $n = 1,\ldots,N-1$, with unknown coefficients κ_n:

$$\kappa(z) = \sum_{n=1}^{N-1}\kappa_n f_n(z). \tag{7}$$

One of the possible ways to define the basis functions is to separate the antenna wire into N segments collocating at points z_n, where $n = 2,\ldots,N$, and points $z_1 = -L$ and $z_{N+1} = L$ are the wire ends. In general, segmentation may be nonuniform and asymmetric. However, it is worthwhile to make symmetric (with respect to $z = 0$) segmentation for a symmetric dipole. In this paper, the basis functions are defined as

$$f_n(z) = 2H(z-z_{n+1}) - H(z-z_n) - H(z-z_{n+2}), \tag{8}$$

where $H(z)$ is the Heaviside step function (see Fig. 2).

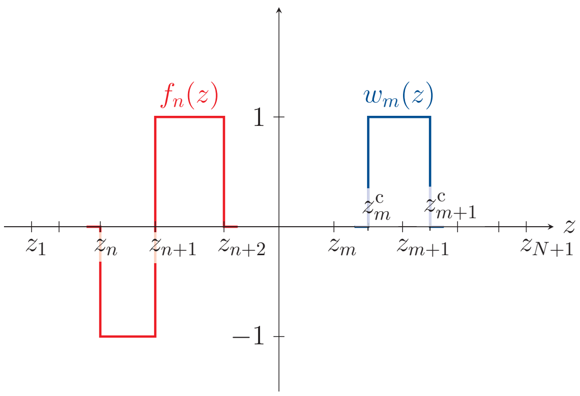

Figure 2. Basis (red) and weight (blue) functions.

Substitution of (7) to (6) gives

$$\sum_{n=1}^{N-1} \kappa_n \left[2G(z-z_{n+1}) - G(z-z_n) - G(z-z_{n+2})\right] = U_0\delta(z). \quad (9)$$

Importantly, linearity of equation (6) is taken into account here.

To proceed, one should introduce $N-1$ points z^c_m located in the middle of segments:

$$z^c_m = (z_m + z_{m+1})/2, \quad (10)$$

where $m = 1,\dots,N-1$. Then the weight functions are defined as

$$w_m(z) = H(z - z^c_m) - H(z - z^c_{m+1}) \quad (11)$$

(see Fig. 2).

Finally, introducing the inner product of functions as

$$\langle q_1(z)q_2(z)\rangle = \int_{-L}^{L} q_1(z)q_2(z)\,\mathrm{d}z, \quad (12)$$

and making this inner product of each side in (9) and the weight functions $w_m(z)$, one gets the following system of $N-1$ linear algebraic equations:

$$\mathbf{A}\vec{\kappa} = \vec{B}, \quad (13)$$

where $\mathbf{A}$ is an $(N-1)\times(N-1)$ matrix:

$$A_{m,n} = \int_{z^c_m}^{z^c_{m+1}} \left[2G(z-z_{n+1}) - G(z-z_n) - G(z-z_{n+2})\right]\mathrm{d}z, \quad (14)$$

$\vec{\kappa}$ is a vector of the unknown coefficients $\kappa_1,\dots,\kappa_{N-1}$, and $\vec{B}$ is a vector of $N-1$ elements. If segmentation is symmetric and N is even, all elements of $\vec{B}$ equal zero except the middle element which equals U_0. Note that $A_{m,n}$ is evaluated analytically. However, the resulting expression is quite complicated and not presented here.

Hence, the integro-differential equation (6) is reduced to the system of linear algebraic equations (13) that can be solved numerically.

4 Calculation Results

Calculations have been performed for $a = 1$ mm, $L = 5$ m, $\varepsilon = 1.2$, $\eta = -39$, and $\omega/(2\pi) = 100$ kHz. Plasma parameters and the radiation frequency correspond to the OEDIPUS-C experiment [3]. The calculated charge distribution $\kappa(z)$ is shown in Fig. 3 where 40 segments are used. It can be seen that charge is almost constant along each dipole arm, and this is in agreement with the theory [6, 9]. A significant change of $\kappa(z)$ near $z = 0$ and $z = \pm L$ is because of the edge effects.

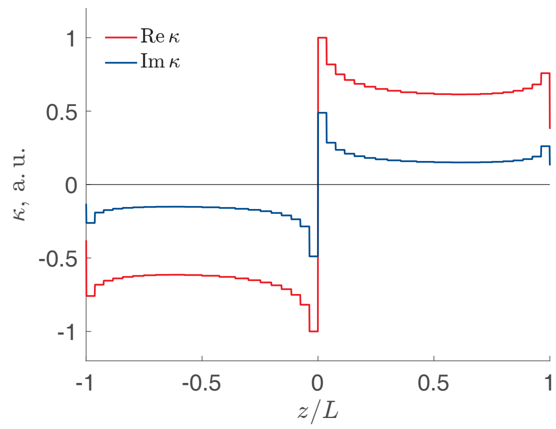

Figure 3. Calculated distribution $\kappa(z)$.

Since the calculated function $\kappa(z)$ is piecewise constant, it is easy to calculate the current distribution $I(z)$ from the the continuity equation:

$$I(z) = i\omega \int_{-L}^{z} \kappa(z')\,\mathrm{d}z'. \quad (15)$$

Then the input impedance, defined [7] as $Z = U_0/I(0)$, is calculated, and this value is used for accuracy study. The corresponding theoretical values Z_{th}, that are used for comparison, are found in [5, 6, 9]. Figure 4 shows how the relative error

$$\xi = (\Re Z - \Re Z_{\text{th}})/\Re Z_{\text{th}} \quad (16)$$

of computing the radiation impedance $\Re Z$ (with respect to $\Re Z_{\text{th}}$) depends on the number of segments. It may be surprising that $\xi = 0$ for $N = 2$, i.e., when the segmentation is very rough (1 segment for each dipole arm). However, this is simply because the theoretical values Z_{th} used in this study are calculated for the constant charge distribution along each arm. Obviously the method of moments allows one to calculate $\kappa(z)$ (see Fig. 3) and, consequently, Z with a higher level of precision. Hence, Figure 4 should be treated from this point of view. Anyway, the accuracy is quite good ($\xi \sim 1\,\%$ when $N \sim 10$). It is also important that the condition number K of matrix $\mathbf{A}$ is not very large ($K \sim 10$) when $N \sim 10$. That means that this matrix is not really ill-conditioned.

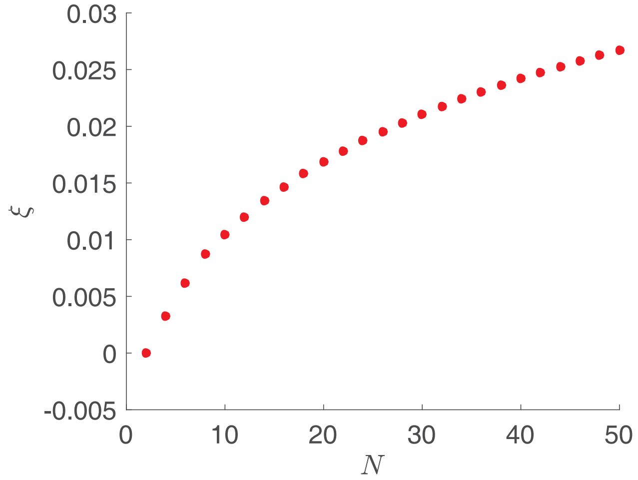

Figure 4. Dependence $\xi(N)$.

5 Conclusion

In this paper, it has been shown that the method of moments can be used for analysis of antenna excitation of quasi-electrostatic waves, propagating in cold magnetoplasmas close to the resonance cone. The corresponding relative error can be quite small (about 1 %). Hence, the method of moments can be used for calculations when antennas have more complex geometry and the spacecraft surface should be taken into account.

The same technique is obviously can be used for analysis of receiving antennas in resonance magnetoplasmas. In this case, one should write in (4) the incident electric field value (at the receiver location) instead of $\vec{E}^{\mathrm{emf}}$. This analysis is very important, particularly, for calculation of the receiving antenna effective length L_{eff}. The analytical expressions have been obtained for the antennas of very simple geometry only, and it appears that L_{eff} can be significantly greater than the geometric length when the quasi-electrostatic waves are received (see [14] and the references therein).

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