2nd URSI AT-RASC, Gran Canaria, 28 May - 1 June 2018

Optimal Terminating Impadances for Maximizing the Gains of a Four-coil WPT Link

Franco Mastri(1), Mauro Mongiardo(2), Giuseppina Monti(3), Luciano Tarricone(3),
Laura Corchia(3) and Alessandra Costanzo*(1)
(1) Department of Electrical, Electronic and Information Engineering “Guglielmo Marconi”, University of Bologna, Bologna, Italy; e-mail: franco.mastri@unibo.it, alessandra.costanzo@unibo.it
(2) Department of Engineering, University of Perugia, Perugia, Italy; e-mail: mauro.mongiardo@unipg.it
(3) Department of Engineering for Innovation, University of Salento, Lecce, Italy; e-mail: giuseppina.monti@unisalento.it, luciano.tarricone@unisalento.it, laura.corchia@unisalento.it

Abstract

In this paper, a Wireless Power Transfer (WPT) link consisting of four magnetically coupled resonators (a transmitting and a receiving resonator coupled through two relay elements) is analyzed. Standard gain definitions are used for describing the performance of the link and the analytical expressions of the terminating impedances providing the optimal operating conditions are derived.

1 Introduction

Resonant inductive Wireless Power Transfer (WPT) is an attractive technology enabling the wireless recharge of electronic systems [1–6].
The basic configuration uses just two magnetically coupled resonant loops [7, 8]: a transmitting (TX) and a receiving (RX) resonator. However, improved performance can be obtained by using additional resonators (relay resonators) not directly connected to the source and the load [9–16]. In this regard, two possible schemes of interest are the one using three resonators (i.e., a TX, an RX and one relay resonators) [13, 14] and the one using four resonators (i.e., a TX, an RX and two relay resonators) [15].

In particular, in [17] the scheme with a single relay element has been solved referring to the problem of maximizing the gains of the link. It is demonstrated that, for a given link, the three power gains (i.e., the power gain, the available gain and the transducer gain) can be simultaneously maximized by setting the terminating impedances equal to the *image resistances* of the link (i.e., the conjugate image impedances of the two–port network which for a three-resonator WPT link are purely resistive). In the present paper, the problem of finding the optimal terminating impedances of a four–resonator WPT link is analyzed. It is shown that, as in the case of a three–resonator scheme, the power gains are maximized when the link is terminated on its *image resistances*.

2 Analyzed problem

The problem analyzed in this paper is schematized in Fig. 1. A WPT link consisting of four resonators is considered; it is assumed that each resonator consists of a distributed inductor L_i loaded by an appropriate compensating capacitor C_i realizing the resonance condition at the angular resonant frequency ω_0. The resistors R_i model the resonator losses. The parameters summarized in Table 1 are used for the analysis and the power gains (Power Gain, G_P, Available Gain, G_A, and Transducer Gain, G_T) as defined in the context of active two–port network are adopted for describing the performance of the link. The definitions and the expressions in terms of impedance matrix representation of the network are summarized in Table 2.

2.1 Gains maximization for a four–resonator WPT link: analytical data

By specializing the expressions reported in the second column of Table 1 for a four–coil WPT link, the following results can be obtained for the gains:

$$G_P = \frac{\chi_{14}^2 \chi_{43}^2 \chi_{32T}^2}{\left[\left(\chi_{14}^2+1\right)\left(\chi_{32T}^2+1\right)+\chi_{43}^2\right]\left(\chi_{32T}^2+\rho\right)} \frac{Q_{2T}}{Q_L}, \tag{1}$$

$$G_A = \frac{Q_{1T}}{Q_G} \frac{\chi_{1T4}^2 \chi_{43}^2 \chi_{32}^2}{\left[\left(\chi_{1T4}^2+1\right)\left(\chi_{32}^2+1\right)+\chi_{43}^2\right]\left(\chi_{1T4}^2+\rho\right)}, \tag{2}$$

$$G_T = \frac{Q_{1T}}{Q_G} \frac{4\chi_{1T4}^2 \chi_{43}^2 \chi_{32T}^2}{\left[\left(\chi_{1T4}^2+1\right)\left(\chi_{32T}^2+1\right)+\chi_{43}^2\right]^2} \frac{Q_{2T}}{Q_L}. \tag{3}$$

According to the analysis reported in [18], for a generic two–port network the terminating impedances which allow to simultaneously maximize the three power gains are the *conjugate image impedances* of the network, Z_{ci} $(i = 1,2)$, which, in general, are complex quantities. For the analyzed

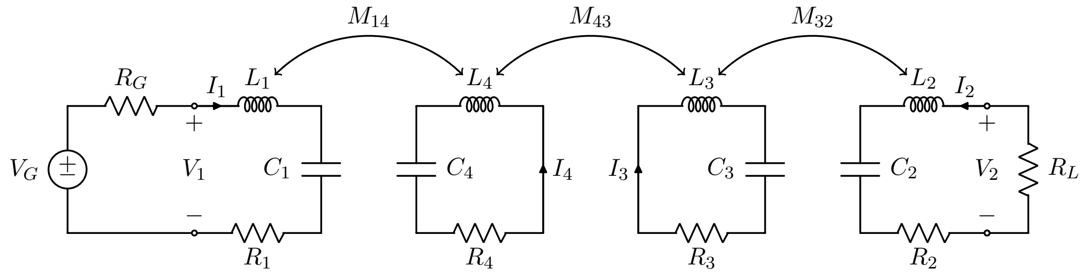

Figure 1. Equivalent representation of a WPT link using two relay elements.

Table 1. Definitions.

$X_0 = \omega_0 L_2$	$n_{ij} = \sqrt{\frac{L_i}{L_j}}$	$k_{ij} = \frac{M_{ij}}{\sqrt{L_i L_j}}$	$Q_i = \frac{\omega_0 L_i}{R_i}$	$Q_G = \frac{\omega_0 L_1}{R_G}$	$Q_L = \frac{\omega_0 L_2}{R_L}$
$Q_{1T} = \frac{Q_1 Q_G}{Q_1 + Q_G}$	$Q_{2T} = \frac{Q_2 Q_L}{Q_2 + Q_L}$	$\chi_{ij} = \sqrt{Q_i Q_j} k_{ij}$	$\rho = Q_4 Q_3 k_{43}^2 + 1$	$\chi_{1T4} = \sqrt{Q_{1T} Q_4} k_{14}$	$\chi_{32T} = \sqrt{Q_3 Q_{2T}} k_{32}$

Table 2. Gains definitions and expressions.

	Definition	Expression
G_P	$\frac{P_L}{P_{in}}$	$\frac{R_L}{R_{in}} \left\| \frac{z_{21}}{z_{22}+Z_L} \right\|^2$
G_A	$\frac{P_A}{P_{AG}}$	$\frac{R_G}{R_{out}} \left\| \frac{z_{21}}{z_{11}+Z_G} \right\|^2$
G_T	$\frac{P_L}{P_{AG}}$	$\frac{4\|z_{21}\|^2 R_G R_L}{\|(z_{11}+Z_G)(z_{22}+Z_L)-z_{12}z_{21}\|^2}$

problem, the following expressions can be derived for the conjugate image impedances:

$$Z_{c1} = \frac{X_0 n_{12}^2}{Q_1} \sqrt{\frac{\left[\left(\chi_{14}^2+1\right)\left(\chi_{32}^2+1\right)+\chi_{43}^2\right]\left(\chi_{14}^2+\rho\right)}{\rho\left(\chi_{32}^2+\rho\right)}}, \quad (4)$$

$$Z_{c1} = \frac{X_0}{Q_2} \sqrt{\frac{\left[\left(\chi_{14}^2+1\right)\left(\chi_{32}^2+1\right)+\chi_{43}^2\right]\left(\chi_{32}^2+\rho\right)}{\rho\left(\chi_{14}^2+\rho\right)}}. \quad (5)$$

It can be noticed that, as for the case of a three–resonator link, the conjugate image impedances are purely resistive quantities (i.e., $Z_{c1} = R_{i1}$ and $Z_{c2} = R_{i2}$), so that they can be referred as *image resistances*. By setting $Z_G = R_{i1}$ and $Z_L = R_{i2}$, the three gains assume their maximum realizable value, the so called *ultimate gain*: $G_P = G_A = G_T = G_u$. In particular, for the present case G_u assumes the following expression:

$$G_u = \frac{\left(\sqrt{\left(\chi_{14}^2+\rho\right)\left(\chi_{32}^2+\rho\right)} - \sqrt{\left[\left(\chi_{14}^2+1\right)\left(\chi_{32}^2+1\right)+\chi_{43}^2\right]\rho}\right)^2}{\chi_{14}^2 \chi_{43}^2 \chi_{32}^2}. \quad (6)$$

2.2 Validation

In order to verify the theory, circuital simulations have been performed with the commercial tool NI AWR design environment. The parameters of the analyzed resonators are summarized in Table 3 and have been taken from [15] (see

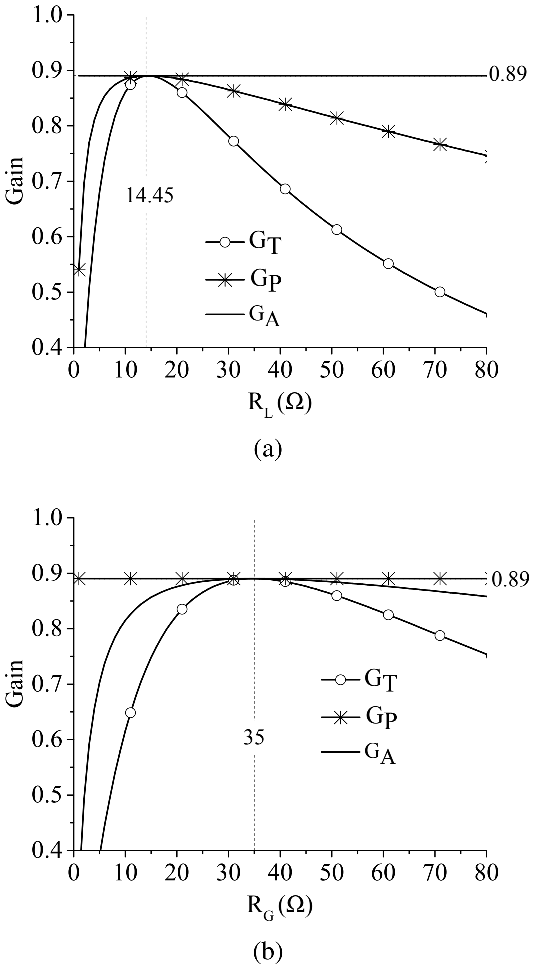

Figure 2. Results obtained from circuital simulations for G_T, G_P and G_A by varying: a) the value of R_L for $R_G = R_{i1}$, b) the value of R_G for $R_L = R_{i2}$. The values assumed for the couplings are: $k_{14} = 0.2$, $k_{43} = 0.1$ and $k_{32} = 0.22$.

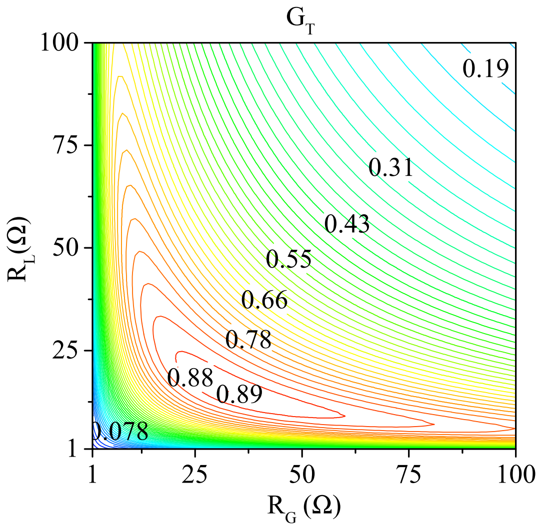

Figure 3. Results obtained from circuital simulations for G_T by varying R_G and R_L. The values assumed for the couplings are: $k_{14} = 0.2$, $k_{43} = 0.1$ and $k_{32} = 0.22$.

Table 3. Data of the analyzed example.

$L_1 = L_4$ (µH)	$L_2 = L_3$ (µH)	$C_1 = C_4$ (pF)	$C_2 = C_3$ (pF)	k_{43}	k_{14}	k_{32}
0.9	0.4	153.07	344.40	0.1	0.2	0.22

Table II). The frequency of resonance is 13.56 MHz. For the analyzed case the values of the image resistances are: $R_{i1} = 14.45\ 14.45\,\Omega$, $R_{i2} = 35\ 14.45\,\Omega$. Simulations have been performed by varying the values of R_G and R_L; the results achieved this way are given in Figs. 2–3. From these figures it is evident that the three gains are simultaneously maximized for $R_G = R_{i1}$ and $R_L = R_{i2}$.

Additionally, as expected in view of the general expressions reported in the second column of Table 2, it can be seen that the available gain only depends on the generator impedance R_G, while the power gain G_P only depends on the load impedance R_L. As per the transducer gain, the dependence on both R_G and R_L is highlighted in Fig. 3.

3 Conclusion

A resonant inductive WPT link using two relay elements has been analyzed. The link has been modeled as a two–port linear network described by well–established power gain definitions. Closed form analytical expressions have been reported for both the three gains of interest (the power gain, the available gain and the transducer gain) and the optimal terminating impedance.

4 Acknowledgements

This work was supported by "Fondo di Sviluppo e Coesione 2007-2013 - APQ Ricerca Regione Puglia Programma regionale a sostegno della specializzazione intelligente e della sostenibilità sociale ed ambientale - Future In Research".

References

[1] A. Karalis, J. D. Joannopoulos, and M. Soljačić, "Efficient wireless non-radiative mid-range energy transfer," *Annals of Physics*, vol. 323, no. 1, pp. 34–48, Jan. 2008.

[2] A. Costanzo, M. Dionigi, M. Masotti, M. Mongiardo, G. Monti, L. Tarricone, and R. Sorrentino, "Electromagnetic energy harvesting and wireless power transmission: A unified approach," *Proceedings of the IEEE*, vol. 102, no. 11, pp. 1692–1711, Nov 2014.

[3] G. Monti, M. Dionigi, M. Mongiardo, and R. Perfetti, "Optimal design of wireless energy transfer to multiple receivers: Power maximization," *IEEE Transactions on Microwave Theory and Technique*, vol. 65, no. 1, pp. 260–269, Jan 2017.

[4] T. S. Bird, N. Rypkema, and K. W. Smart, "Antenna impedance matching for maximum power transfer in wireless sensor networks," *IEEE Sensors*, pp. 916–919, 2009.

[5] G. Monti, P. Arcuti, and L. Tarricone, "Resonant Inductive Link for Remote Powering of Pacemakers," *IEEE Transactions on Microwave Theory and Techniques*, vol. 63, no. 11, pp. 3814–3822, Oct. 2015.

[6] M. Mongiardo, Q. Wang, W. Che, M. Dionigi, R. Perfetti, Y. Chang, and G. Monti, "Wireless Power Transfer Between One Transmitter and Two Receivers: Optimal Analytical Solution," in *Asia-Pacific Microwave Conference*, 2015.

[7] M. Dionigi, M. Mongiardo, and R. Perfetti, "Rigorous network and full-wave electromagnetic modeling of wireless power transfer links," *IEEE Transactions on Microwave Theory and Techniques*, vol. 63, no. 1, pp. 65–75, Jan 2015.

[8] G. Monti, A. Costanzo, F. Mastri, M. Mongiardo, and L. Tarricone, "Rigorous design of matched wireless power transfer links based on inductive coupling," *Radio Science*, vol. 51, no. 6, pp. 858–867, June 2016.

[9] F. Zhang, S. A. Hackworth, W. Fu, C. Li, Z. Mao, and M. Sun, "Relay effect of wireless power transfer using strongly coupled magnetic resonances," *IEEE Transactions on Magnetics*, vol. 47, no. 5, pp. 1478–1481, May 2011.

[10] G. Monti, L. Corchia, L. Tarricone, and M. Mongiardo, "A network approach for wireless resonant energy links using relay resonators," *IEEE*

Transactions on Microwave Theory and Techniques, vol. 64, no. 10, pp. 3271 – 3279, Oct 2016.

[11] T. Sun, X. Xie, G. Li, Y. Gu, Y. Deng, and Z. Wang, "System with an efficiency-enhanced power receiver for motion-free capsule endoscopy inspection," *IEEE Transactions on Biomedical Engineering*, vol. 59, no. 11, pp. 3247–3253, November 2012.

[12] A. Kurs, A. Karalis, R. Moffatt, J. D. Joannopoulos, P. Fisher, and M. Soljacic, "Wireless power transfer via strongly coupled magnetic resonances," *Science*, vol. 317, no. 5834, p. 83, 2007.

[13] J. Kim, H.-C. Son, K.-H. Kim, and Y.-J. Park, "Efficiency Analysis of Magnetic Resonance Wireless Power Transfer With Intermediate Resonant Coil," *IEEE Antennas and Wireless Propagation Letters*, vol. 10, pp. 389–392, Oct. 2011.

[14] W. Zhong, C. Zhang, X. Liu, and S. Hui, "A methodology for making a three-coil wireless power transfer system more energy efficient than a two-coil counterpart for extended transfer distance," *IEEE Transactions on Power Electronics*, vol. 30, no. 2, pp. 3288 – 3297, February 2015.

[15] M. Kiani, U.-M. Jow, and M. Ghovanloo, "Design and Optimization of a 3-Coil Inductive Link for Efficient Wireless Power Transmission," *IEEE Transactions on Biomedical Circuits and Systems*, vol. 5, no. 6, pp. 579–591, 2011.

[16] G. Monti, L. Tarricone, M. Dionigi, and M. Mongiardo, "Magnetically coupled resonant wireless power transmission: An artificial transmission line approach," in *Proceedings of the European Microwave Conference (EuMC)*, Oct. 2012, pp. 233–236.

[17] F. Mastri, M. Mongiardo, G. Monti, M. Dionigi, and L. Tarricone, "Gain expressions for resonant inductive wireless power transfer links with one relay element," *Wireless Power Transfer*, pp. 1–15, Oct. 2017. [Online]. Available: http://dx.doi.org/10.1017/wpt.2017.13

[18] S. Roberts, "Conjugate-Image Impedances," in *Proceedings of the IRE*, 1946, pp. 198–204.