



## Scaling Rain Attenuation as a Function of the Link Elevation

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### Abstract

The effect of the attenuation due to rain in satellite links increases rapidly for frequencies above 10 GHz. The development of tools to mitigate this impairment relies on the good knowledge on the behavior of the rain attenuation, and in particular on its relationship with frequency, polarization and link elevation. The severe increase in the rain attenuation at low elevations has a strong impact on the design of telecommunications (TLC) and data download systems based on low earth orbit (LEO) satellites. Due to the lack of long-term measurements of rain attenuation on a large range of link elevations, the use of scaling techniques for the complementary cumulative distribution functions (CCDF) of  $A(\theta)$  becomes essential. This paper presents a method to scale the CCDF of rain attenuation starting from the CCDF along the zenithal path.

### 1. Introduction

In the last decade, satellite communications are facing an increasing demand for high data rates. Along with the saturation of the conventional frequency bands (e.g. Ku), this increasing demand is pushing the systems to operate at higher frequencies, such as Ka and Q/V bands. The propagation of electromagnetic (EM) waves at these frequencies is severely affected by the tropospheric impairments, with rain attenuation ( $A_R$ ) being the most detrimental one. The losses induced by rain are caused by EM energy absorption and scattering by raindrops, with the last one increasing significantly with frequency because the diameter of raindrops is more and more comparable to the wavelength. For frequencies above 10 GHz the raindrop cannot be considered as a lossless dielectric due to the fact that the imaginary part of its complex permittivity increases [1].

Aiming to minimize these effects, the design of systems operating at these frequencies should be based on Propagation Impairments Mitigation Techniques (PIMTs). However, due to the scarcity of long-term measurements of rain attenuation on global basis, the system design mostly rely on models of the tropospheric impairments that are able to estimate the amount of attenuation which the Earth-space link is going to be subject to.

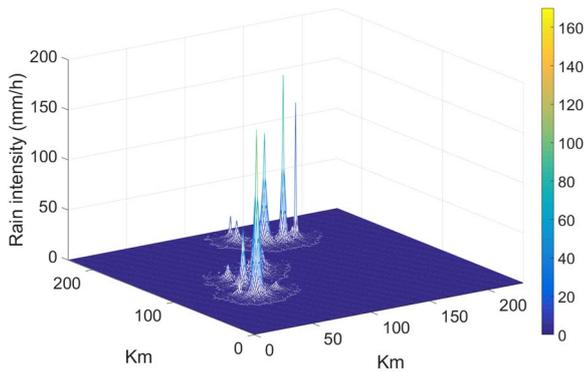
Rain attenuation is strongly dependent on the link elevation as with the decrease of the elevation angle the path across rain definitely increases as well, which, in turn, enhances the probability that the Earth-space path will cross multiple rain structures experiencing deeper fades [2]. This a key element to be considered, especially in applications involving LEO satellites and deep-space probes, in which the link elevation angle changes from very low to high during each satellite pass. Unfortunately, few measurements on rain attenuation at low elevation angles currently exist [3] which calls for the needs of models to assess the impact of precipitation in the frame of applications involving non-geostationary space vehicles.

This paper presents a methodology aimed at scaling the yearly CCDF of rain attenuation as a function of the elevation angle. The model is derived from processing a pool of rain attenuation CCDFs obtained from simulations that exploit the rain maps synthesized by the MultiEXCELL model [4]. The paper is organized as follows: section 2 describes the database used as well as the procedure adopted to perform the simulations starting from the synthetic rain maps; section 3 presents the proposed model; in section 4, some preliminary results are shown while section 5 draws some conclusions.

### 2. Simulations

#### 2.1 Database

In this work we used a reference database consisting of 578 square rain maps with lateral dimension of 230 km and a spatial resolution of 1 km×1 km. These maps were generated by the propagation-oriented MultiEXCELL rain field model described in [4]. The model enables the generation of realistic rain fields composed by aggregates of synthetic rain cells, each one with an exponential profile. MultiEXCELL was largely validated in several propagation-oriented applications (see e.g. [5], [6] and [7]). More specifically, MultiEXCELL considers that rain cells tend to aggregate into larger rain structures. Each aggregate consists of at least two rain cells with peak rain rate exceeding 5 mm/h. One or more aggregates are usually present in a rain map. Figure 1 shows a sample map. The maps used in this work refer to the site of Spino D'Adda, Italy (45.4° N, 9.5° E).



**Figure 1.** Sample rain map generated by the MultiEXCELL model.

This complex representation of the rain field is particularly important for the performance evaluation of Earth-space links pointing at non-geostationary satellites, in which the path length across rain becomes longer and longer as the elevation angle decreases.

## 2.2 Procedure

In order to calculate the attenuation induced by rain from MultiEXCELL maps, we apply the well-known expression in (1) [8].

$$A(\text{dB}) = \int_0^{L_s} \kappa R^\alpha dL \quad (1).$$

In (1),  $L_s$  is the slant-path length across rain (km) that depends on the elevation angle and the rain height – typically approximated as the difference between the mean annual 0 °C isotherm height above mean sea level and the height of the site. The latter is here retrieved from ITU-R Recommendation P.839 [9]. Moreover,  $R$  represents the rain intensity extracted from the synthetic rain maps;  $k$  and  $\alpha$  are parameters dependent on frequency, polarization and elevation angle, and are extracted from ITU-R Recommendation P.838-3 [10]. The power law  $kR^\alpha$  yields the specific attenuation ( $\gamma$ ) in dB/km. The simulations were performed for frequency  $f = 39.6$  GHz with circular polarization, according to one of the three beacon signals emitted by the ITALSAT experimental satellite (Spino d’Adda, 1994 to 2000) [11]: in fact, we use the CCDF at 39.6 GHz (with elevation angle  $\theta = 37.7^\circ$ ), to validate the procedure.

The simulations were performed by placing the ground station at each pixel of each map, and then by evaluating the attenuation for each elevation angle from  $1^\circ$  to  $90^\circ$ . In this way we obtained a large number of samples, which increase the statistical meaningfulness of the results.

At low elevations, depending on the position of the ground station, the horizontal projection  $L_s$  exceeds the boundary of the map: it is then necessary to extend the rain field,

which is achieved by placing alongside the first map a second one with approximately the same rain coverage (which is acceptable when considering no significant changes in the local orography). The solution adopted is to randomly pick, within the database, a map with a similar rain coverage (with 20% of tolerance).

After collecting all the attenuation samples, we obtained the CCDF of rain attenuation for each elevation angle: Figure 2 shows the CCDFs for values of  $\theta$ .

## 3. Modelling of the Scaling Factor

The scaling factor was obtained with a procedure similar to the one described in [12] for cloud attenuation. First, it was necessary to model the variation of  $P_0$ , the probability to have rain attenuation along the link, which increases with the decrease in the link elevation.

The scaling factor ( $SF_0(\theta)$ ) is defined as:

$$SF_0(\theta) = \frac{P_0(90^\circ)}{P_0(\theta)} \quad (2)$$

where  $P_0(90^\circ)$  and  $P_0(\theta)$  are the long-term probability to have rain attenuation along the zenith path, and along a slant path ( $\theta$  ranging from  $1^\circ$  to  $89^\circ$ ), respectively. A satisfactory model for the scaling factor ( $SF_0(\theta)$ ) is given by the following simple formula:

$$SF_0(\theta) = 1.741\theta^{-0.6015} + 0.8931 \quad (3).$$

with  $\theta$  is expressed in degrees. Figure 3 shows the values of  $SF_0$  obtained from the simulations and from the power law model, which indicates a very good fitting accuracy.

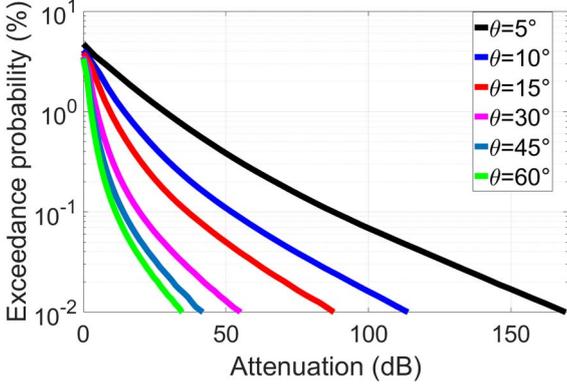
We used the same scaling factor concept above to model the variation of the attenuation CCDF at different probability levels as a function of the elevation variation. In this case, as shown in (4), the scaling factor  $SF(P, \theta)$  is defined as the ratio between the attenuation exceeded at elevation  $\theta$  and the one predicted with the simple cosecant law from the zenith path:

$$SF(P, \theta) = \frac{A_{dB}(P, \theta)}{\csc(\theta)A_{dB}(P, 90^\circ)} \quad (4).$$

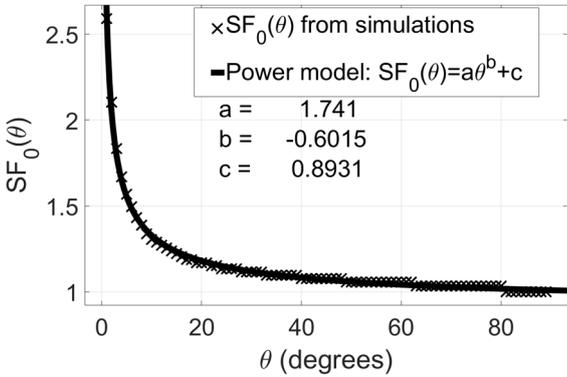
In (4),  $A_{dB}(P, \theta)$  is the attenuation exceeded with probability  $P$  at the elevation  $\theta$ .

The scaling factor was calculated at the following probability levels: [1, 0.5, 0.4, 0.3, 0.2, 0.1, 0.05, 0.04, 0.03, 0.02, 0.01] %.  $SF(P, \theta)$  is then modelled using the polynomial relationships in (5):

$$SF(p, \theta) = \alpha_1(\theta)P^3 + \alpha_2(\theta)P^2 + \alpha_3(\theta)P + \alpha_4(\theta) \quad (5).$$



**Figure 2.** CCDFs of rain attenuation for different elevation angles.



**Figure 3.** Scaling factor for the probability to have rain attenuation along the link.

The coefficients  $\alpha_i$  dependent on the elevation angle and are calculated using (6) and Table 1.

$$\alpha_i(\theta) = a_i e^{b_i \theta} + c_i e^{d_i \theta} \quad \text{for } i < 4$$

$$\alpha_4(\theta) = \frac{0.9551\theta^2 - 0.9108\theta + 3.213}{\theta^2 - 1.998 + 5.921} \quad (6).$$

Table 1. Coefficients for the expression in (6)

$i$	$a$	$b$	$c$	$d$
1	0.09608	-0.2675	-0.07699	-0.0319
2	-0.4054	-0.03019	0.5544	-0.1964
3	-0.4024	-0.02615	0.9201	-0.1177

## 4. Results

In order to evaluate scaling model accuracy, we used the figure of merit proposed in ITU-R Recommendation P.311-15 [13] given in (7) and its RMS value in (8).

In (7),  $A_{sc}$  is value scaled using  $SF$  and  $A_{si}$  is the value obtained from the reference simulated curves;  $p$  is the probability level;  $\mu_\epsilon$  and  $\sigma_\epsilon$  are respectively the mean and the standard deviation of  $\epsilon$ . The result obtained from the

model are then compared with the one obtained from the customary cosecant scaling.

$$\epsilon = 100 \ln \left( \frac{A_{sc}(p)}{A_{si}(p)} \right) \left( \frac{A_{si}(p)}{10} \right)^{0.2} \quad \text{for } A_{si} < 10 \text{ dB} \quad (7).$$

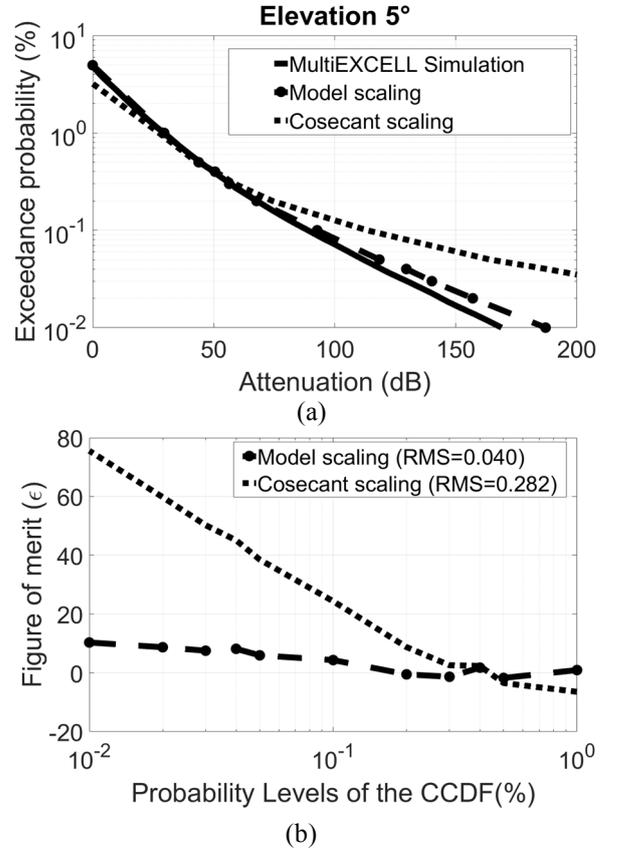
$$\epsilon = 100 \ln \left( \frac{A_{sc}(p)}{A_{si}(p)} \right) \quad \text{for } A_{si} > 10 \text{ dB}$$

$$RMS(\epsilon) = (\mu_\epsilon^2 + \sigma_\epsilon^2)^{0.5} \quad (8).$$

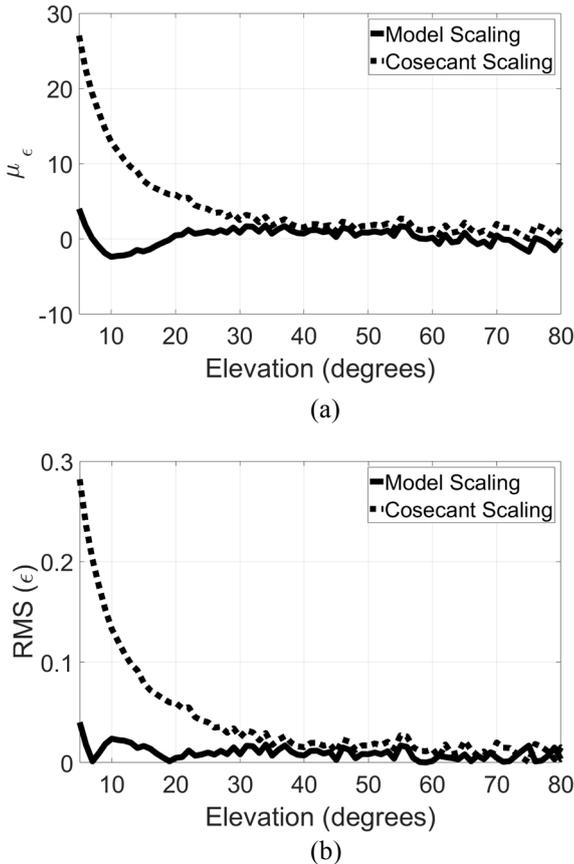
As an example, Figure 4 shows the CCDF obtained at 5° from the simulation and the one scaled (a), as well as the figure of merit as a function of the probabilities (b).

Figure 5 summarizes the results by showing the figure of merit as a function of the elevation angle (ranging from 5° to 80°): findings indicate that the proposed model outperforms the simple cosecant scaling in the range spanning from 5° to approximately 40°.

Using MultiEXCELL, we have also performed simulations for a different frequency (18.7 GHz) and a different site (Miami, tropical climate): the results are similar to those presented here, but not shown for the sake of brevity.



**Figure 4.** Scaling factor applied to 5°: (a) CCDFs; (b) figure of merit.



**Figure 5.** Mean and RMS of the figure of merit for different elevations: (a)  $\mu_\epsilon$ ; (b) RMS.

## 5. Conclusions

In this paper, we have presented a model aiming at scaling the CCDF of the rain attenuation with link elevation  $\theta$  covering the full range of values ( $5^\circ \leq \theta \leq 90^\circ$ ). The proposed scaling methodology, which takes advantage of the same modeling approach already employed to scale cloud attenuation statistics, shows a definite improvement in the prediction accuracy when compared to the simple customary cosecant scaling law. Further tests will be carried out in order to provide a more comprehensive picture of the accuracy of the proposed model, for example as a function of different electrical (change in the frequency) and meteorological (rain height and temperate/tropical/equatorial precipitation regime) parameters of the simulations.

## 6. Acknowledgements

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## 7. References

1. J. E. Allnutt, *Satellite-to-Ground Radiowave Propagation*, London: The Institution of Engineering and Technology, 2011.
2. D. G. Michelson and W. Liu, "Simulation of Rain Fading and Scintillation on Ka-Band Earth-LEO Satellite Links," in *Canadian Conference on Electrical and Computer Engineering*, St. John's, NL, 2009.
3. T. Tjelta, M. Rytir, L. E. Bråten, P. M. Grotthing, M. Cheffena and J. E. Håkegård, "Results of a Ka band campaign for the characterisation of propagation conditions for SatCom systems at high latitudes," in *11th European Conference on Antennas and Propagation (EUCAP)*, Paris, 2017.
4. L. Luini and C. Capsoni, "MultiEXCELL: A new rain field model for propagation applications," *IEEE Transactions on Antennas and Propagation*, November 2011.
5. L. Luini and C. Capsoni, "A physically based methodology for the evaluation of the rain attenuation on terrestrial radio links," in *EuCAP*, Barcelona, 2010.
6. C. Capsoni, L. Luini and M. D'Amico, "The MultiEXCELL model for the prediction of the radio interference due to hydrometeor scattering," in *EuCAP*, Barcelona, 2010.
7. L. Luini and C. Capsoni, "A rain cell model for the simulation and performance evaluation of site diversity schemes," vol. 12, pp. 1327-1330, 2013.
8. L. Ippolito, *Radiowave propagation in satellite communications*, New York: Van Nostrand Reinhold Company Inc., 1986.
9. ITU-R, "Recommendation P.839-4 - 'Rain height model for prediction methods'," 2013.
10. ITU-R, "Recommendation P.838-3: 'Specific attenuation model for rain for use in prediction methods'," 2005.
11. S. Bertorelli, "Analysis of the tropospheric radio channel behavior for the project of satellite systems based on the principle of shared resource," Politecnico di Milano, Milan, 2006.
12. L. Luini and C. Capsoni, "Scaling Cloud Attenuation Statistics With Link Elevation in Earth-Space Applications," *IEEE Transactions on Antennas and Propagation*, vol. 64, no. 3, pp. 1089-1095, March 2016.
13. ITU-R, "Recommendation P.311-15 - 'Acquisition, presentation and analysis of data in studies of radiowave propagation'," ITU, Geneva, 2015.