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Positioning of dynamic objects by means of systems of multi-frequency sounding of HF radio paths with the use of FMCW signals

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Abstract

The paper presents methods and algorithms of dynamic objects positioning by means of oblique-incidence sounding of HF radio paths with the use of FMCW signals. Experimental studies showed that the average errors in determining the distance to an object and its location using developed algorithms do not exceed 1%. Requirements for sounding signal parameters were obtained

1. Introduction

Currently the main radio navigation tools are the GNSS and GPS satellite systems. However, their performance can be disrupted during the period of military conflicts. In this case, radio-technical tools of the HF range keep playing an important role in positioning of complex dynamic objects, such as aircrafts. Their disadvantage is much less accuracy of positioning due to the random and quasiregular variations (diurnal, seasonal, etc.) of a propagation medium (ionosphere) which reflects signal under long-range (over-the-horizon) wave propagation. Thus, positioning of complex dynamic objects is usually performed under conditions of uncertainty caused by the properties of a propagation medium. Ionosphere is a medium where the wave velocity differs from the light velocity and depends on the wave frequency. Thus, it is crucial to take into account the state of a propagation medium in order to increase the accuracy of positioning. This approach should improve the method of determining the distance to an object. Multi-frequency sounding of the "earth-ionosphere-dynamic object" radio path using FMCW signal could be applied to solve the problem. Besides, FMCW signal allows to decrease emitting power and minimize ionosonde dimensions, which is crucial in this case.

The improvement of theoretical approaches aimed at solving the problem requires verification of their use in full-scale experiments. We shall note that the possibility of experimental studies of this urgent problem has been limited for a long time due to the insufficient efficiency of sounding tools and accuracy of synchronization of transmitting and receiving terminals, which is necessary requirement for measuring signal propagation time from a transmitter to a receiver. Currently, there are many possibilities for studying the proposed method, such as ionosondes which use complex signals with linear frequency modulation (LFM) and have high time delay resolution, as well as small atomic time devices [1-3]. The frequency dependences of a signal delay are obtained by element-by-element optimal processing of the continuous signal. High accuracy of determining signal propagation time of each element is achieved by signal compression in the frequency domain.

The aim of the research is to develop and study effective methods of positioning a dynamic object using data from multi-frequency sounding of "earth-ionosphere-dynamic object" radio path by means of FMCW ionosonde.

2. General framework of the model

Mathematical model of the regular ionosphere in the framework of geometrical optics approximation is used to solve the problem. We used a spherically layered symmetric model of the ionosphere in which case the electron density distribution model was defined as a piecewise continuous function of height corresponding to ray propagation in a free space and in the ionosphere. We have assumed that in a free space and ionospheric layers, which are below the F layer, the refractive index is equal to 1, and the F-layer is represented by quasi-parabolic approximation for electron density [4-6]. This approach allows to obtain analytical equations for a problem solution.

We have assumed that the absolute delay τ_k at different operating frequencies f_k, the maximum usable frequency (MUF) f_{MUF} and the lowest usable frequency (LUF) are known values and are determined from experimental data obtained by FMCW sounding of a radio path. Let us suppose that a signal propagates from a transmitter to a receiver with a single hop. The method of equal hops should be used for multiple-hop propagation.

The equation system (1) was obtained in the framework of the quasi-parabolic ionospheric F-layer model to determine the distance D, taking into account the curvature of the Earth with parameters: h_o - height of a lower boundary of a reflective layer; y_m - layer half thickness; $h_m = h_o + y_m$ - layer peak height from the Earth; f_o – layer penetration frequency.

$$\begin{cases} D_E(r_o, y_m, a_k, f_k/f_o) = D \\ P_g(r_o, y_m, a_k, f_k/f_o) = c\tau_k \\ \dfrac{\partial D_E}{\partial a}(r_o, y_m, a_{MUF}, f_{MUF}/f_o) = 0 \end{cases} \quad (1).$$

where $1 \le k \le n$, k – order number of operating frequency, $n \in N$ (natural number) – number of operating frequencies, c – light velocity, τ_{MUF} – MUF group delay time, R – Earth's radius, a_k, a_{MUF} – radiation angles corresponding to the frequencies f_k and f_{MUF}, $r_o = h_o + R$, D_E – distance (range) and P_g – group path are described with the use of Croft's T.A and Hugasyan's G. equations [6]. Apart from D the unknown parameters in the system are r_o, y_m, f_o, a, a_{MUF}. Equation (2) expresses one of the boundary conditions.

$$\frac{\partial D_E}{\partial a}(r_o, y_m, a_{MUF}, f_{MUF}/f_o) = 0 \quad (2).$$

Search for a system solution (1) is an incorrect task and, in general, such tasks don't give an unambiguous solution. The solution is complicated due to the fact that the relationship between the parameters being identified and the data are non-linear. To solve these problems and get an unambiguous solution for the system (1), it was proposed to use an algorithm [7] which identifies the initial value of parameters, h_o, y_m, f_o and D, located near the true values which are obtained from solving the problem within a plane-stratified ionosphere and parabolic ionospheric F-layer with the same parameters.

Since the experimental values τ_k contain a random component (contaminated with random errors), the value τ_k must be smoothed in order to obtain stable solutions of (1), because the mathematical model based on the approximation of geometrical optics leaves it out of account. Thus, methods for filtering experimental frequency dependencies of the delay in the elements of a continuous signal were developed. Smoothing function $t(f)$ was selected from a set of functions of frequency f which are continuously differentiable on the interval (LUF, MUF) and have a stationary point with the coordinates (f_{MUF}, b):

$$t = t(f) = \varphi^{-1}(f) \quad (3).$$

where $f = \phi(t) = a(t-b)^m + f_{MUF}$, m – positive integer.

The main requirement for plotting a smoothing function is to select an integer degree m in order to minimize the mean square error (RMS) σ_D in determining a distance using the proposed method. Studies suggest that cubic degree m=3 is an optimal filtering parameter for radio paths with the length of 1000-3200 km. The equation for the distance between a transmitter and a receiver takes into account the sphericity of the Earth with the polar axis emerging from the center of the Earth in the direction of the north pole.

Rho-rho navigation was used to calculate the coordinates of a dynamic object. It was assumed that the location of two FMCW signal transmitters was known (ψ_i; φ_i) (ψ_i - latitude, φ_i - longitude of the i-th point, i = 2,3), and the location of the receiving station was unknown i = 1, (ψ_1; φ_1), (see Figure 1). In accordance with rho-rho navigation the desired location of an object is defined as a point of intersection of two circles with radiuses equal to the distances D_{1i} from an object to the transmitting stations, which were determined with the use of the discussed above method, i.e. from the equation system (1).

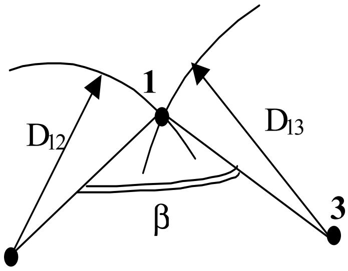

Figure 1. Scheme of an object positioning

According to this logic, a system of equations for determining coordinates of an object ψ_1 and φ_1 can be represented as follows:

$$D_{12} = R \cdot \arccos(\cos\psi_1 \cos\psi_2 \cdot \cdot \cos(\varphi_1 - \varphi_2) + \sin\psi_1 \sin\psi_2) \quad (4a).$$

$$D_{13} = R \cdot arccos(cos\,\psi_1\, cos\,\psi_3 \cdot \cdot cos(\varphi_1 - \varphi_3) + sin\,\psi_1\, sin\,\psi_3) \quad (4b).$$

If the terrain clearance of an object is given, it is easy to find the third coordinate of a three-dimensional object - the radius-vector of the point (object).

3. The findings of full-scale experiments

Experimental studies were carried out using a simulation method, when a stationary receiving station developed for sounding of a radio path "transmitting station-ionosphere-receiving station" was used instead of a dynamic object [2, 8]. It is permissible when the movement of an object is insignificant during the sounding time. The receiving station was located in Yoshkar-Ola. Foreign transmitters intended for ionospheric sounding (ionosondes) were used as the transmitting stations. They emit continuous FMCW signals in the range of 3-30 MHz and are located in the Cyprus, Spitsbergen island and Inskip (see Table 1). We shall note that the radio paths 2-1 and 3-1 are mid-latitude and the radio path 4-1 is subpolar.

The time delay resolution is 10 μs, and timing accuracy among transmitting and receiving parts of ionosonde does not exceed ± 5 μs that is ± 1.5 km in distances. Verification procedure of rho-rho navigation was conducted for three pairs radio paths presented in table 1. The first pair is 2-1 and 3-1, the second is 2-1 and 4-1 and the third is 3-1 and 4-1.

Table 1. Data on experimental radio paths

Radio path number	Transmitting point - receiving point	Path length, km
2-1	Cyprus – Yoshkar-Ola	2552
3-1	Svalbard (Spitsbergen) – Yoshkar-Ola	2715
4-1	Inskip (England) – Yoshkar-Ola	3156

The angles β between the radio paths are 74.1º, 128.8º, 56.4º respectively. Experimental data (800 sounding sessions) were obtained for different seasons of the year and time of the day. Studies showed that this amount of data is adequate to obtain representative estimates. The obtained results were compared with the true coordinates (56.6°N, 47.9°E) of the receiving station, which allowed us to estimate the error of the positioning method. We estimated: absolute distance errors averaged per day $\langle\Delta D\rangle$, latitude $\langle\Delta\psi\rangle$ and longitude $\langle\Delta\varphi\rangle$; their maximum absolute errors; distance relative errors $\langle\delta D\rangle$the average value of a location error, calculated as the distance between a true location and a found in experiments $\langle r\rangle$; maximum and relative location errors $\langle\delta r\rangle$; the distance between the true and the averaged (measured per day) location of an unknown point $\langle\Delta R\rangle$, as well as probabilistic statistical estimates of the errors in determining the distance and object location. Generalized results of studying seasonal and daily accuracy characteristics of the proposed method are presented in table 2 for different path lengths.

Table 2. Generalized results of studying seasonal and daily accuracy characteristics of the method of determining the distance to the object

Path	$\langle D\rangle$, km	$\langle\Delta D\rangle$, km	$\langle\delta D\rangle$, %	max δD, %	$\langle\sigma_D\rangle$, km
2-1	2552	11	0.5	1.5	14
3-1	2715	12	0.5	1.3	14
4-1	3156	12	0.4	1.2	15

It was identified that the average daily errors $\langle\Delta D\rangle$, $\langle\delta D\rangle$ for all considered paths did not exceed, respectively, 15 km and 0.6%. The maximum absolute and relative errors did not exceed, respectively, 40 km and 1.5%. In this case, the method errors significantly depend on the path length. The method's systematic errors for all considered paths did not exceed ~ ± 3 km. The method's systematic errors, averaged over the whole time of the experiment, were minus 0.3 km. The study suggests that RMS of random errors in determining the distance varies between 11 - 17 km. Thus, the potential accuracy of determining the distance in the experiment is about 11 km. The weighted RMS value of the distance from all paths was about 15 km.

Table 2 shows that the modulus of the difference between the true and the average distance $\langle D\rangle$, measured for the time of the numerical experiment, for all considered paths did not exceed 0.5 km, that is, the arithmetic average of the distance calculation results for paths with different length, converges to its true value. Figure 2, a, b, c shows, respectively, bar charts of error distribution $\Delta\varphi$, $\Delta\psi$ in determining longitude, latitude and an object location errors, for all the considered pairs of paths during the experiment.

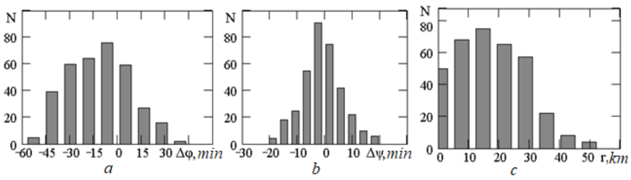

Figure 2. Bar charts of the errors distribution in determining $\boldsymbol{\varphi}$, $\boldsymbol{\psi}$ and *r*

Thus, the systematic errors of the method in determining longitude are ~ minus 5.0 minutes; latitude ~ 1.0 minutes; the potential accuracy of determining longitude in the experiment is about 12 minutes, latitude ~ 4 minutes. Weighted RMS values of latitude and longitude for all pairs of paths are respectively $\sigma_\psi \approx 6.0$ minutes, $\sigma_\varphi \approx 18.0$ minutes. Average location errors did not exceed 1% of the length of the lowest path in the pair. The difference between errors in determining the ψ and φ is caused, firstly, by the fact that the relationship between the linear and degree measures is different; secondly, by the presence of the horizontal gradient of the electron density, which appears when the terminator moves, which affects the results of determining longitude at the transition time of the day. This hypothesis has been confirmed by the experimental data on error ellipses shown, respectively, in figure 3 a, b, c for the pairs of paths: 1st pair is 2-1 and 3-1, 2nd pair is 2-1 and 4-1, 3rd pair is 3-1 and 4-1.

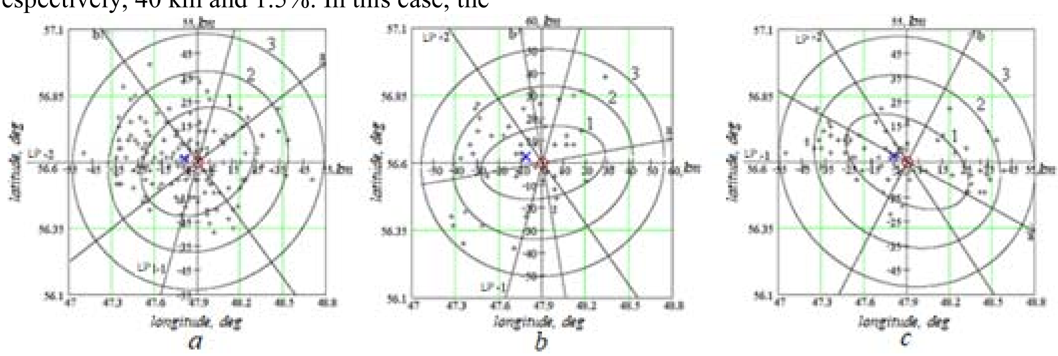

Figure 3. Error ellipses for the considered pairs of paths

The numbers 1, 2 and 3 correspond to the probabilities of 0.68, 0.95 and 0.997 that a location point of an object is inside the ellipse. These data suggest that the accuracy of determining the distances are almost equal. The RMS for all paths is ~ 15 km. It means that a major axis of an ellipse coincides with the bisector of the acute angle β. Thus, the errors of the method depend on the path lengths and the angle between the paths. It is confirmed by the generalized data presented in table 3.

Table 3. Generalized data on accuracy characteristics of the positioning method

Season	Path pair	1	2	3
During the experiment	$\langle\Delta\psi\rangle$, minutes	6.0	4.0	7.0
	max $\Delta\psi$, minutes	17.0	11.0	24.0
	RMS ψ, minutes	7.2	4.5	8.2
	$\langle\Delta\varphi\rangle$, minutes	12.0	21.3	12.0
	max $\Delta\varphi$, minutes	34.0	53.0	42.0
	RMS φ, minutes	14.5	26.0	15.2
	$\langle r\rangle$, km	18	24	20
	max r, km	40	50	50
	$\langle\delta r\rangle$, (%)	0.7	0.9	0.7
	RMS r, km	21	25	23
	$\langle\Delta R\rangle$, km	4	3	4

Movement of the receiving terminal leads to the deformation of the error ellipse in the moving direction. It is crucial to select the parameters of a sounding signal in such a way that the movement of a receiving terminal does not exceed a location error. Let us suppose that the frequency sweep rate of a FMCW signal is 100 ... 1000 kHz/sec. Thus, sounding of a HF range with a 27 MHz frequency band will take 270 ... 27 sec. If an object moves with a speed equal to 850 ... 2500 km/h, then the object will move to a distance equal to 64 ... 190 km during the sounding time with a frequency sweep rate 100 kHz/sec. The distance will be much smaller if the frequency sweep rate is 1000 kHz/sec. The sounding time can be reduced by decreasing the range of sounding frequencies. It is possible, because the sounding range should be equal to the difference between the MUF and LUF. This range does not exceed 20 MHz for paths of various length. Decrease in the sounding frequency range by 26% leads to the same decrease in the object movement i.e. it is, respectively, about 5 km and 14 km. Studies showed that such values are adequate.

4. Conclusions

We proposed and theoretically justified the algorithms for determine the distance to an object and its location which are based on using the data from multi-frequency oblique-incidence sounding of ionospheric radio channels by FMCW signals. We developed a method for filtering experimental frequency dependencies of element delays of a continuous signal, which allows to find stable solutions of the problem of determining the distance to an object and its location. We also determined the form of a smoothing function, and obtained its optimal parameters for the paths with the length of 1000 - 3200 km. The errors of the proposed positioning method were experimentally obtained with the use of FMCW ionosonde. It is shown that the errors in determining the distance for paths with the length of 2500 - 3200 km do not exceed 1%. The location error does not exceed 1% of the length of the lowest path in the pair. Frequency sweep rate of a sounding FMCW signal should be at least 1 MHz/sec, and the sounding frequencies range should not exceed 20 MHz.

5. Acknowledgements

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